GROUP THEORY 2018–19, SYLLABUS
BERNOULLI INSTITUTE, RUG

STEFFEN MÜLLER

Course description

Abstract groups are one of the basic concepts in linear algebra but they are also used, e.g., in cryptography in the context of decomposition of large numbers to prime factors. However, the notion of a group is essential in almost all branches of abstract mathematics and it has many applications in fundamental physics, where groups arise in the context of symmetry or invariance of models for elementary particles. The crystallographic groups are of interest in chemistry, physical chemistry, and solid state physics.

Besides giving you a solid background in the theory of groups and how to apply it to solve specific problems, the aim of this course is to develop your ability to design your own rigorous proofs of abstract statements. There will be very little computational work. Some results in the course are of a more abstract nature than those encountered in most other courses in the Groningen undergraduate curriculum. However, we will spend quite a bit of time on concrete examples.

All courses focusing on algebra (such as algebraic structures, advanced algebraic structures and security & coding), as well as some of the other courses in the undergraduate and graduate curriculum in mathematics at RUG crucially rely on the contents of this course.

Literature and pre-knowledge

We will follow a slightly modified version of the lecture notes "Group Theory" by Jaap Top. These notes (in the version we will use) will appear for downloading on 09-11-18 on Nestor and at http://www.rug.nl/staff/steffen.muller/lecture-notes-group18.pdf. They will also be available via http://uorder.nl.

Chapter I and some parts of Chapter II summarize material already covered in the part of the first year course 'Kaleidoscope Mathematics' (formerly known as 'Introduction to Mathematics') dealing with modular arithmetic. This will be assumed as pre-knowledge, and we will draw on this material as a source of concrete examples in the course. If you don't feel comfortable with modular arithmetic, you are strongly encouraged to spend some time reviewing it, including doing some exercises. I will also assume that you remember linear algebra.

The notes emphasize both proofs and examples, but for some students it might be useful to take additional literature into account. The following books are recommended for further reading and for alternative discussion and proofs.

- M.A. Armstrong: Groups and symmetry, Springer UTM, 2nd printing, 1997.
- D.S. Dummit, R.M. Foote: Abstract Algebra, Wiley, 3rd edition, 2003.
- N.C. Carter: Visual Group Theory, MAA Press, 2009.

Date: November 9, 2018.

In some of these books the notation may differ somewhat from the notation used in the course and the notes.

Homework

Homework is not mandatory, but highly recommended. Indeed, the only good way to really understand the concepts taught in the lectures is to go to the tutorials and do the homework problems. Most of the exercises will be from the notes, but there might also be some additional problems.

It is also possible to improve your grade in the final exam through your homework results. There will be four homework assignment, each comprising four problems. You get a grade between 1 and 10 for each assignment. If you want to get credit for your work, you have to follow the following rules:

- Hand in your work before the start of the tutorial at the indicated hand-in date.
- Staple your work: do not hand in loose sheets.
- You are encouraged to work in groups of two. However, if you are part of a pair of two students, the solutions to two out of the four problems should be in your own handwriting (if you hand in solutions to all four problems, that is. If you only hand in solutions to two or three problems, each student should write up the solution to at least one of the problems).
- Write your name(s) and student number(s) on each page.
- All answers need to be accompanied with an explanation or a calculation.

If there is a good reason why you are unable to stick to these rules for one or more of the assignments, contact your TA or me.

We will not publish solutions of the homework problems; such solutions will be discussed during the tutorials, and if you're interested in them, you should attend the tutorials.

Final exam and grade

The final exam takes place on 22-01-19 at 9am; there is no midterm exam. There will be 90 points on the final exam. The exam grade E is determined by the number of points P scored during the exam:

$$E = \frac{P + 10}{10}$$

Your final grade is determined by your exam grade E and your average homework grade H. Specifically, the grade is

$$\max(E, 0.75 \cdot E + 0.25 \cdot H).$$

If this number less than 5.5, you fail the course. Otherwise you pass, and the grade will be rounded to the nearest multiple of 0.5 (unless this multiple is 5.5, in which case your final grade is 6).

The resit will probably take place on 11-04-19, but this date is still preliminary.

Email address: steffen.muller@rug.nl