Coexistence of Competing Strategies
in Evolutionary Games

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university of
groningen

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Chapter 1

General Introduction and Thesis Outline

Cooperation, where one incurs a cost to confer a benefit on another, is regarded as a fundamental building block of all the life in nature and particularly human societies. However, it is hard to understand this seemingly altruistic behavior from the perspective of natural selection rule where maximizing one's benefits stems from a survival instinct of individual organisms in nature. Evolutionary game theory analyzes models of rational and selfish individuals acting in their own best interests, models of competition and cooperation between game players. The complex network theory, one of the big concerns for engineering and computer science researchers, also pays its attention towards networks that depict the gaming connections of populations. Combining the evolutionary game theory with the complex network theory yields an interdiscipline direction which has received a rapidly increasing amount of attention in recent years.

Moreover, individual heterogeneity and biological or social diversity are also well-known phenomena in nature and in social society of humans. It is a main focus whether and how biodiversity affects the emergence and transmission of strategy, disease, information, opinion and so on. The potential difficulties brought by individual heterogeneity in mathematical modeling, raise important challenges for existing theoretical models which have only considered simple individuals in games. However, many more studies concerning with the individual heterogeneity or diversity and their possible coexistence, in the framework evolutionary game theory, are expected in the near future. Only in this way could we gain more hints on cracking a series of perplexing puzzles about cooperative phenomena in the real social society.

In this dissertation, I apply the approaches from mathematics, statistical physics, computer science and engineering to explore the competing dynamics in the related populations involved in social dilemma situations. Herein I present a collection of evolutionary game theoretic models that help to explore questions related to the origins and maintenance of cooperative actions in nature. The basic line of this thesis is addressing the role of individual heterogeneity in promoting cooperation. In this thesis, I aim to contribute to our understanding about the interplay between the individual heterogeneity and human cooperation, by the aid of establishing effective theoretical models in the framework of evolutionary game theory. First, I endow the players with switching probabilities between strategies, and study whether and (if the answer is yes) how different switching probabilities affect the strategy evolution dynamics in the gaming population (Chapters 2 and 3). Second, I investigate the individual difference on the time scales in strategy updating process, hoping to shed light on how cooperation can be influenced by the individual diversity or heterogeneity (Chapter 4). Third and finally, I investigate the effects of insurance on the evolution of cooperation in two scenarios (Chapters 5 and 6).

This chapter serves as a brief introduction to the evolutionary problem of cooperative behaviors among selfish populations, complex networks employed as the structure of the gaming populations, and our motivation. Finally, I will give an overview of the aims and contents of this thesis.

1.1 Background and framework

Here, we first introduce some background information and motivation for the research work in this thesis.

- **The puzzle of altruism (cooperation)**

In biology, altruism can be defined as an individual performing an action which is harmful or at a cost to itself (e.g., pleasure and quality of life, time, probability of survival or reproduction), but benefits, either directly or indirectly, another third-party individual, without the expectation of reciprocity or compensation for that action (Moran 1962, Hamilton 1963, Axelrod 1984, Mukherji et al. 1996, Axelrod and

Dion 1988). In this sense, cooperation is deemed as the process where groups of organisms work or act altogether for the common or mutual benefits of the groups. It is commonly defined as any adaptation that has evolved, at least in part, to increase the reproductive success of the actor's social partners (Gardner et al. 2009). For instance, territorial choruses by male lions discourage intruders and then probably benefit all the contributors in the group (Clutton-Brock 2009a). This process contrasts with the intragroup competition where agents work against each other for some selfish reasons. The diversity of taxa that exhibits cooperation is quite large, ranging from zebra herds to pied babblers to African elephants. Many animal and plant species cooperate with both members of their own species and with members of other species.

In spite of the diversity of different contexts in which agents cooperate, researchers from multi-disciplinary areas incline to focus their studies on situations in which the benefits of an individual are opposed to the interests of the collective group. Under this kind of social dilemma situations, cooperative action means a cost for the actor and benefits others. From an evolutionary perspective, cooperative behavior is puzzling due to the fact that selfish individuals help others at their own cost or expense, and hence there is the potential or temptation for exploitation of cooperative individuals by free riders, or defectors who profit at others's expense. Theoretical analysis predicts that rational individuals, who aim to maximize their payoffs or benefits in games, should behave selfishly in such circumstances. However, cooperative situations are so widespread in real-life situations such as the animal kingdom and human societies.

How cooperation among non-relatives can persist in the face of cheating and the cruel rule of 'survival of the fittest' driven by natural selection, remains a puzzling, fascinating and broad-ranging unsolved question in evolutionary biology. Moreover, this interdisciplinary topic has also obtained plenty of attention, interest and research across disciplines, i.e. social sciences, behavioral sciences, psychology, physics, computer science, engineering and so on. Explaining the cooperation evolution is not only an issue of central importance to evolutionary biology but also one of hot interdisciplinary topics so far, since it is commonplace throughout all levels of the natural world.

Human societies are founded on cooperation, and psychologists and economists have explored what and how personal factors motivate agents to cooperate. It is plausible that the individual heterogeneity cannot be neglected when studying the cooperation of real agents in real social societies. And, the details of individual personal factors are so complicated that needs much more attention in the future study. These empirical studies complement a large body of theoretical work from evolutionary game theory by proposing some effective mechanisms. However, there still need much more effort to establish (possibly complicated) mathematical models involving the individual personal factors, also to verify the effectiveness of experimental results in the related works. This is also the focus and motivation of this thesis, to dig into the complex issues of cooperation that are overwhelming us from the perspective of individual heterogeneity, by the aid of mathematical analysis and agent based evolutionary simulations in computer science and engineering.

• Evolutionary game models

As mentioned, altruism refers to a costly behavior that benefits others. However, mutual cooperation is often found in nature even when selfish behavior is apparently rational for individuals. Thus, social dilemmas are situations in which the optimal decision of an individual contrasts with the optimal decision for the group. Why and under what circumstances, presumptively selfish agents cooperate is a question of longstanding interest to multidisciplinary research (Nash 1950b, Nash 1951, Axelrod 1980, Axelrod and Hamilton 1981, Axelrod 1984, Dawkins 1976, Axelrod and Dion 1988, Alexander and Irvine 1987, Colman 2006, Diggle et al. 2007, Doebeli et al. 2004, Bendor and Swistak 1995). Game theory is one of the key paradigms behind many scientific disciplines from biology to behavioral sciences to economics.

Past studies used simple game theory models, such as the classic prisoner's dilemma models, to determine decisions made by individuals in cooperative interactions. However, complicated interactions between individuals need more complicated concepts such as the Nash equilibrium (Nash 1950a, Nash 1951, Nash 1950b). The Nash equilibrium is frequently used in a type of non-cooperative game theory that assumes an individual's decision is affected by its knowledge of the strategies of other individuals. This theory is novel in considering the higher cognitive capabilities of gaming individuals. The evolutionarily stable strategy (Maynard Smith and Price 1973) is a refined version of the Nash equilibrium in that it assumes strategies are heritable and are subject to natural selection. Established by Maynard Smith and Price (Maynard Smith 1978, Maynard Smith 1979), evolutionary game theory provides a competent theoretical framework to address the subtleties of cooperation among selfish and unrelated individuals. Moreover, evolutionary game theory is an interdisciplinary mathematical tool which seems to be able to embody several relevant features of the problem and, as such, is used in much cooperation-oriented research. By the aid of evolutionary game theory, vast theoretical or experimental mechanisms for emergence and maintenance of cooperation in social dilemma games have been reported thus far (Clutton-Brock 2009b, dos Santos et al. 2011, Eldakar and Wilson 2008).

The referred social dilemmas are described as the situation where individual benefits are opposed to that of the group. In investigating the social dilemma problem, the standard framework utilized is evolutionary game theory together with its extensions involving evolutionary context. Since in this thesis I will not restrict the work to one specific form of social dilemma model, here I list some of them as follows for reference. In existing studies, the prisoner's dilemma game is unrivaled in popularity when it comes to studying the evolution of cooperation through pairwise interactions (Axelrod 1980, André and Day 2007, Nowak and Sigmund 1989, Andreoni and Varian 1999, Ashlock et al. 1996, Zhang, Chen, Zhang, Wang and Chu 2010a). The game promises a defector the highest payoff if encountering a cooperator. Meanwhile, the exploited cooperator is worse than a defector playing with another defector. In line with the principles of Darwinian's natural selection, defection will be the dominating strategy of the population.

Relaxing the inevitability of a social downfall resulted by the well-mixed prisoner's dilemma game is the snowdrift game or hawk-dove games (Ahmed and Elgazzar 2000). Other interesting games also constitute powerful metaphors to describe conflicting situations often encountered in natural and social sciences. For example, trust game (Anderhub et al. 2002), volunteer's dilemma (Archetti 2009b), donor-recipient games (Berger 2009), stag hunt dilemmas (Pacheco et al. 2009a, Pestelacci and Tomassini 2008, Skyrms 2004), predator–prey game (Abbott 2010) and so on. Whenever collective action of groups of individuals is at stake, N-

person games are appropriate. Many previous investigations of cooperation have employed the N-person prisoner's dilemma games in the form of public goods game (Andreoni 1995, Archetti 2009a, Barclay 2004, Zhang, Zhang and Chu 2011) to study the possibility of emergence of cooperation among groups of interacting agents. Chapter 2, entitled Strategy updating for evolution in interaction networks, employs the prisoner's dilemma game and snowdrift game for model setting and an effective comparison. Chapters 3 and 5 adopt the prisoner's dilemma game, snowdrift game and stag hunt game for a systematic study to ask whether the specific dilemma model affects the evolution dynamics of the populations. Chapter 4 uses the public goods game, a classical n-person paradigm for recurring game interactions, to investigate the four competing strategies in such settings.

- **Competing and coexistence of strategies**

Cooperation and defection are the two strategies that are at the heart or core of each social dilemma. Cooperators make contribution to the collective benefit at a personal cost or damage, while defectors make no contribution and take advantage of others' contributions. Since individual heterogeneity is a common phenomenon in nature and society, and real agents always face multiple strategy choices in the competition with others involved in social dilemma situations. This is particulary true in the context of human cooperation where human decision making is probably shaped by a wealth of individual factors.

Based on these considerations, aside from these two classical strategies which indicate obligatory participation, many different strategies (e.g., loner and punishment) have also been proposed to investigate their potential roles in resolving the cooperation dilemma problems. Voluntary participation (Hauert et al. 2002b, Hauert et al. 2002a) allows players to adopt a risk-aversion strategy, named as loner. A loner refuses to join in an unpromising public enterprise and instead relies on a small but fixed payoff. Cooperation can also be stabilized by punishment. In theory and in experiments, punishment has turned out to be a simple but effective mechanism to prevent cheating. There is now a rich literature on whether and how various forms of punishment are effective in bringing about cooperation (van den

Berg et al. 2012, Perc and Szolnoki 2010), peer punishment (Boyd et al. 2003, Hauert et al. 2007), pool punishment (Szolnoki et al. 2011), and anti-social punishment (Rand and Nowak 2011). Besides, our previous work proposes another role in game playing, named as insurance against punishment, enriching the potential strategy profiles for players (Zhang et al. 2013).

As for the cooperation problem, a major challenge for the involved researchers is to explain how cooperation is maintained or even dominates in a selfish population, by the aid of effective mechanisms which help the competition, invasion or domination of cooperators. However, reality suggests that a typical characteristic for the real societies and nature is the individual heterogeneity, social diversity and coexistence of competing partners. Such a society, from the perspective of evolution, is a society where the competition and cooperation coexist, and thus is a complicated system. Even if our genes may instruct us to be selfish, we are not necessarily compelled to obey them all our lives (Dawkins 1976). And importantly, the coexistence, not only competition, of multiple competing decisions indeed can be observed in real society and nature. Actually, agents often adopt multiple choices in decision making due to the internal personality factors or intervention of external factors, especially when facing the complicated cooperative dilemma situation. It may be reasonable and meaningful to share or split some attention to the coexistence of competing roles, when we rack our brains trying to figure out how cooperation can defeat all the other strategies, and dominate the population finally. This is also the starting point of my thesis.

Modeling the additional strategy options found in several real-life systems, has also evolved into a mushrooming avenue of research. Probing into more strategies not only stems from the need to provide new ways of fostering cooperation in situations constituting social dilemma, but also help us understanding the coexistence of multiple competing strategies or social diversity in nature. Inspired by this fact, we propose a new strategy named as speculator and comprehensively examine four kinds of strategies (cooperation, defection, loner, and speculators) in Chapter 4. These model settings are inspired by the existing insurance behaviors in economic systems. To fill in the gap between theoretical model and economic behaviors in real social society, we establish the mathematical model and focus on the evolution of evolutionary dynamics in this setting. It is remarkable that proposing more competing strategies in the gaming populations is still widely open to research and thus holds promises of exciting new discoveries. Moreover, it is worthy noting that we also relax the limitation of uniform players in the field of switching probability (Chapters 2 and 3) and time scales (Chapter 5), to study the competition or coexistence of different players in the games.

• History of cooperation research

One of the first references to animal cooperation was made by Charles Darwin, who noted it as a potential problem for his natural selection theory (Darwin 1859, Darwin 1871). He proposed many mechanisms that could help explain why cooperation could be favored over selfish behaviors. Nowadays, the mechanisms introduced by Darwin are still at the core stage of research for solving the puzzle of cooperation evolution. Prominent biologists, such as E.O. Wilson, and W.D. Hamilton, have also found the evolution of cooperation fascinating because natural selection favors those who obtains the greatest reproductive success while cooperative behavior often decreases or inhibits the reproductive success of the individual performing the cooperative action (Clutton-Brock 2009a). Additionally, some species have been found to perform cooperative behaviors that may even be detrimental or harmful to their own evolutionary fitness or survival benefits. For example, when a ground squirrel sounds an alarm call to warn other group members of a nearby coyote, it attracts the coyote's attention to itself and meanwhile increases its own risk of being eaten and death (Sherman 1977). Therefore, cooperation poses a fundamental puzzle to the traditional theory of natural selection, which rests on the assumption that individuals selfishly and vehemently compete to survive and maximize their reproductive successes in nature.

'How did cooperative behavior evolve' was regarded as one of the top 25 big questions facing science over the next quarter-century, proposed for celebrating the journal of *SCIENCE*'s 125 anniversary in 1995. And, Robert May began his last presidential address to the Royal Society on 30 November 2005 by saying:" The most important unanswered question in evolutionary biology, and more generally in the social sciences, is how cooperative behavior evolved and can be maintained in human or other animal groups and societies" (Colman 2006). In this sense, cooperation problem has already been an issue of central importance to many disciplines, including the active members of engineering, physics and computer science. More importantly and meaningfully, achieving a satisfactory understanding of the evolution of cooperation in social dilemmas is fundamental for elucidating many important problems in social sciences, such as the sustainability of social diversity, information or strategy spreading, public resources consumption, public project provision and so on.

Since widespread cooperation is crucial for the prosperity of society and is frequently encountered in nature, many studies and new approaches aimed towards resolving the social dilemma have been spawned. It is worthwhile to highlight some of them here to acknowledge their contributions, and also since some of them will be referred in the discussions of this thesis. For example, the kin selection theory focuses on cooperation among individuals that are genetically related. Darwin recognised that reciprocity may lead to or foster cooperation: when individuals interact with each other repeatedly, a cooperative action may be returned later when the games proceed. Theories of direct reciprocity thus focus on the selfish incentives for cooperation in bilateral long-term interactions (Hamilton 1964, Clutton-Brock 2002, Nowak 2006, Ohtsuki and Nowak 2007, Pacheco et al. 2008). The theories of indirect reciprocity and signalling show how cooperation in larger groups can emerge when the cooperators can build a reputation (Nowak and Sigmund 2005, Brandt and Sigmund 2005). Other effective mechanisms or rules see (Gross and Blasius 2008a, Perc and Szolnoki 2010) for a comprehensive review, and references therein.

Particularly vibrant in recent years has been the subject of evolutionary games on complex networks. The ubiquity and importance of complex networks raised quite naturally the question of how natural selection works on top of different kinds of network topologies of agents. It is well known that the evolution of cooperative behavior is dependant upon certain environmental conditions. And, in realistic multi-player systems players do not interact with all other players. One such condition that has been extensively studied is the use of a spatially structured population (Alonso-Sanz 2009, Amaral et al. 2000, Arapaki 2009, Barabási and Albert 1999, Zhang et al. 2014). The key concept of spatially structured populations is: agents are assigned to the vertices of a network, which can be a regular lattice or has a more complex structure. The edges denote links between players in terms of game dynamical interactions. Then, agents are constrained to interact only with their adjacent neighbors to play evolutionary games in which more successful strategies spread on the system, if a social dilemma is embedded in a richer game theoretical structure.

The preceding transitions from well-mixed populations to spatial grids and further to complex networks, and particularly their success in promoting the evolution of cooperation, invite further extensions of the theoretical framework. Further, a variety of studies suggest that coevolution, including migration, is also a relevant factor to take into account in as much as they may enhance strong altruism (Szabó and Fáth 2007, Ohtsuki et al. 2006, Zhang, Zhang and Chu 2011). And the evolution and coevolutiion of dynamics in multi-layer complex networks has added a new wrinkle to this transatlantic research on cooperation.

Here is a very brief introduction about the complex network and the networked gaming populations:

- Node: the node is the principle unit of the network. A networks consists of a number of nodes connected by edges. In a typical setup of spatial evolutionary games, agents are assigned to the nodes of the network.
- Neighbors: two nodes are said to be neighbors if they are connected by a link or edge.
- Link: a link is a connection between two nodes in the networks. In the common setup of spatial evolutionary games, the edges denote links between the corresponding players in terms of game dynamical interactions.
- Degree: the degree of a node is the number of closest neighbors to which a node is interacted with. The average degree of the network is the mean of the individual degrees of all the nodes in the network.
- Dynamics: depending on the context, the word dynamics is used in the literature to refer to a temporal change of either the state or the topology of a network. In the common setup of spatial evolutionary games, it denotes the evolutionary game dynamics occurring on the interactions, being subject to

the specific strategy updating rules or the introduced coevolution dynamics between networks and strategies.

The integration of the microscopic patterns of interactions among the agents composing a large population into the evolutionary setting provides a way out for cooperation to survive in paradigmatic scenarios. This is also an extremely hot topic in recent years, and attracts plenty of attention of researchers especially from engineering and computer science. The body of literature devoted to this topic is extensive, from game dynamics on static networks to evolving complex networks, from regular lattice network to complex real-world networks. Along this booming line, many more studies concerning with the individual heterogeneity or diversity on complex network are expected in the near future. The most often employed networks are: random regular network (Wormald 1981), lattice network (Nowak et al. 1994), small-world networks (Watts and Strogatz 1998, Watts 1999, Newman and Watts 1999), scale-free graphs (Barabási and Albert 1999), evolving networks (Skyrms and Pemantle 2000) and so on. Based on this consideration, in Chapters 2 and 3 of this thesis, the random regular graph and BA scale-free networks are both employed for investigating the competing strategies among the structured populations.

- **Dynamics of evolution**

A model in evolutionary game theory is made complete by postulating the game dynamics, i.e., the rules that describe the update of strategies in the population. Depending on the actual problem, different kinds of dynamics can be appropriate. The game dynamics can be continuous or discrete, deterministic or stochastic, and within these major categories a large number of different rules can be formulated depending on the situation under investigation. On the macroscopic level, by far the most studied continuous evolutionary dynamics is the replicator dynamics. It was originally introduced by Taylor and Jonker (Taylor and Jonker 1978), and it has exceptional status in the models of biological evolution. On the phenomenological level the replicator dynamics can be postulated directly by the reasonable assumption that the per capita growth rate $\dot{\rho}_i/\rho_i$ of a given strategy type is proportional to the fitness difference (Szabó and Fáth 2007).

$$\frac{\dot{\rho}_i}{\rho_i} = \text{fitness of type i - average fitness}$$

The fitness is the individual's evolutionary success, i.e., in the game theory context the payoff of the game.

A large number of different population-level dynamics are discussed in the game theory literature. These can be either derived rigorously from microscopic (i.e., agent-based) strategy update rules in the large population limit, or they are simply posited as the starting point of the analysis on the aggregate (population, macro) level. Many of these share important properties with the replicator dynamics, others behave quite differently. An excellent review on the various game dynamics in (Hofbauer and Sigmund 2003).

Evolutionary game dynamics generally involve how players update their strategies as time evolves. The updating rules are therefore crucial and, until now, most of them are based on replication and imitation (Nowak and Sigmund 2004, Schlag 1999). The essence of replication rules is that a strategy with better performances has a higher replication rate. Imitation rules assume that a player can imitate her opponents' strategy with a probability when interacting with individuals having obtained higher payoffs.

One much studied approach to spatial games is based on a more detailed modelling of the networks of interacting players. Considering the simplest case, players situate at the nodes of a given lattice (Hofbauer and Sigmund 2003). At each of the (discrete) time steps t, each agent k participates in pairwise interactions with each of the partners l from some neighborhood $N(k)$. Each game yields a payoff $P(k, l)$, and player k's total payoff is determined by $P(k) := \Sigma_{l \neq k \in N(k)} P(k, l)$. Next, players make strategy updating for larger payoffs through some imitation rule. For example, player k compares payoff with all her neighbors $l \in N(k)$ and finally adopts the strategy of the best performer. Again, many variants are possible: in particular, the set of k's potential role models could be distinct from $N(k)$, the imitation rule could be stochastic rather than deterministic, the updating of the strategy could occur at different times for different players, the neighborhood lattice could evolve in time, etc. It is worth noting that Chapter 5 in this thesis proposes multiple time scales in strategy updating in theoretical game model, aiming to enhance our understanding of cooperation.

Hence, to apply such update rules, players have to know in general the exact magnitudes of the payoffs of all her opponents. In Chapters 2 and 3 of this thesis, we aim at decreasing the need for specific information by introducing switching probability endowed with players, and study the corresponding evolution dynamics underlying game theory.

1.2 Related approaches

1.2.1 Infinite populations

For infinite populations, the main analysis tools are the Lotka-Volterra equations. The competitive Lotka-Volterra equations, proposed by Lotka (Lotka 1922, Lotka 1925) and Volterra (Volterra 1926), are a simple model of the population dynamics of species competing for some common resource.

Here x represents the number of preys, and y denotes the number of predators. The Lotka-Volterra equations often takes the following form:

$$\begin{aligned}\frac{dx}{dt} &= x(\alpha - \beta y)\\ \frac{dy}{dt} &= -y(\gamma - \delta x),\end{aligned}$$

where α denotes the birth rate of preys, β represents the effect predators have on the population of preys, γ represents the death rate of predators, and δ means the the effect preys have on the population of predators.

1.2.2 Finite populations

It is plausible that the size of populations in real societies and nature are often finite. Stochastic noise will occur when employing the finite population with the deterministic dynamics equation, for example the above mentioned Lotka-Volterra equations. For finite populations, stochastic models often seem more realistic than deterministic ones. For computer simulations of multi-agent systems, this is the only natural approach to model the dynamics. In this case, the stochastic processes theory is effective in the dynamic analysis of evolutionary games in finite populations. The most often used examples for stochastic evolutionary game dynamics are Pairwise comparison process, Moran process and Wright-Fisher process (Traulsen and Hauert 2009). Here we give a brief introduction about these mentioned process.

- Moran process

  Assuming that A and B are the two available strategies in the gaming population. we focus on a population of size N, consisting of i A players and $N-i$ B players. The probability of an A encounters with another A is $i/(N-1)$, and the probability of A encounters with a B is $(N-i)/(N-1)$. For an agent B, its probability of encountering with an A is $i/(N-1)$, and the probability of encountering with another B is $(N-i-1)/(N-1)$. Thus, the expected payoffs of player A and B are respectively given by

  $$\begin{aligned} F_i &= \frac{a(i-1)+b(N-i)}{N-1} \\ G_i &= \frac{ci+d(N-i-1)}{N-1}. \end{aligned}$$

  In the above equations, F_i and G_i are the expected payoffs of player A and B when there are i A players in the investigated population. The fitness of agent A and B under natural selection are respectively described by

  $$\begin{aligned} f_i &= 1-\beta+\beta F_i \\ g_i &= 1-\beta+\beta G_i, \end{aligned}$$

  where the constant β is called the intensity of selection since $\beta \to \infty$ leads to strong selection where the probability for selecting fitter individual is 1 and when $\beta \ll 1$, the update reduces to the Moran process under weak selection (Nowak et al. 2004).

- Pairwise comparison process

In this kind of dynamics, two agents, a focal individual and a role model, are sampled randomly from the large population. The focal one imitates the strategy of the role model with probability p, which depends on the corresponding payoff comparison. If both individuals gain the same payoff, the focal individual randomizes between the two strategies. One common choice of a nonlinear function of the payoff difference for p is the Fermi function from statistical mechanisms, given by

$$p(s_x \leftarrow s_y) = \frac{1}{1 + \exp[(P_x - P_y)/\omega]}, \tag{1.1}$$

where the magnitude of ω characterizes the uncertainty related to the strategy update. P_x and P_y are the payoffs of agent x and y respectively. For finite positive values of ω, strategies performing worse may also be adopted based on unpredictable variations in payoffs or errors in the decision making. For weak selection $\omega \ll 1$, the probability p reduces to a linear function of the payoff difference. For strong selection $\omega \to \infty$, this process converges to the imitation dynamics. In this case, p becomes a step function being positive for $P_x < P_y$ and negative for $P_x > P_y$.

- Wright-Fisher process

  The Wright-Fisher process is also rooted in population genetics. Different from the selection dynamics in the Moran process, where only one individual reproduces at a time, the Wright-Fisher process represents discrete generations. In each generation, each member of the population with size N produce a large number of offsprings, proportional to their fitness. From the large offspring members, a new generation of size N will be sampled at random. In this situation, the population composition can change or update much faster. The population could go back to a single ancestor in a single generation. This suggests the fact that the Wright-Fisher process is a more general Markov process.

The main approaches we employed in the thesis are Discrete-time Markov chain, Stochastic process theory, Pair approximation analysis, Mean-field analysis, and Monte Carlo simulations. By the aid of them, this thesis mainly focuses on the coexistence of competing strategies, and the main factors in evolutionary game theory such as strategies and time scales, in evolutionary games.

1.3 Scope of this thesis

The objective of this thesis is manifold, it contains:

- the introduction of evolutionary game theory and the cooperative behaviors in structured populations,
- a new method based on that switching probability of competing strategies between players in structured populations,
- introduction of the expanding strategy profile in the evolutionary game theory and
- introduction of diversity of time scales in strategy updating process.

Individual heterogeneity is one of a most often observed phenomenon in realistic systems. Explaining the competition and coexistence of individual diversity is an open question. There are several ways how this feature can be built into a model. Henceforth we present an extensive, systematic study concentrating on the potential heterogeneity of individual behaviors. The proposed individual differences refer to strategy decisions, time scales, transfer probabilities of strategies. Depending on the microscopic details these features can either decrease or increase the frequency of cooperators in the gaming population, indeed influencing the evolutionary dynamic outcomes. Nonetheless, the investigation of evolutionary games on these topics is still widely open to research, and will lead to the exploration of new phenomena and thus raise a number of interesting questions.

It is worth recalling that some basic information (e.g., payoffs at least), are required in the strategy updating rules or dynamics. For example, players will imitate the strategy of those neighboring players (including themselves) who has scored the highest payoff. However, from the viewpoint of real societies, the traditional assumption is often unrealistic. Even in simple interactions between two individuals A and B, it is not easy to obtain full information of partners' decision making, as individuals usually acquire rather limited or even wrong information about the gaming partners or other reference objects. Simply stated, the information acquisition ability and results vary among different real social agents. In **Chapter 2** we tackle the problem of payoffs dependency issue in strategy updating, we introduce a new strategy changing updating rule, an intriguing feature of which is the absence of related payoff information. More specially, we propose the switching probability between competing strategies and employ them for strategy updating, and this novel approach can be successfully used in various specific gaming models. The results presented in this chapter have been published in (Zhang, Zhang, Cao and Weissing 2015)

Chapter 3 further extends the proposed switching probabilities to a more general case. Herein strategy switch happens among all the individuals, not only the restrictive case for the competing strategists (cooperator and defector). Therefore, we introduce and analyze an alternative way of establishing the strategy renewal for interacting players. The work may be helpful in reflecting the real phenomenon in social systems.

The time scales of gaming and strategy updating are also a crucial concept and feature responsible for the cooperative phenomena. **Chapter 4** of this thesis focuses on the multiple time scales in strategy updating. The corresponding work gives mathematical evidence that heterogeneity in time scales enriches the evolutionary dynamics and under simplifying conditions, the possible outcomes can be effectively predicted under suitable situations.

The above three chapters focus on updating rules. We show that details in updating rules, for example the number of neighbours for updating and the multiple time scales in updating, have significant effects on the evolution of strategies.

In the traditional settings of classical (rational) game theory, players have two options to choose from which are called cooperation and defection. For instance, people face frequently the situation of prisoner’s dilemmas in real life when they have to choose between to be selfish or altruistic, to keep the ethical norms or not, to work hard or lazy, etc. However, multiple strategy choices resulted in the complex decision making process are also notable reality in human society that can not be overlooked. Examples in previous studies include punisher, loner, and so on. Introducing more strategies combining the individual characteristics, will meaningfully help our understanding about how altruistic behavior occurs in many naturally occurring dilemma situations. In **Chapter 5** of this thesis, we introduce the insurance for cooperators into threshold publics goods games. We analyzed the conditions with different initial states and parameters. We find some scenarios where contribution to the public pool is promoted. The results presented in this chapter have been published in (Zhang, Zhang and Cao 2015)

In **Chapter 6** of this thesis, we will discuss the possibilities and conditions under which cooperative behavior can subsist in multi-agent models, with multiple strategies (cooperation, defection, loner and speculation) capable of representing a remarkably rich variety of decision choices in games. Our aim is to study the competition and coexistence of competing strategies in this productive framework.

1.4 Outline and contributions

This section briefly states the outline of the thesis and the topics of the chapters. The chapters are organized as follows:

Chapter 1 briefly introduces the background of cooperative dilemma problems and the gained research results, including the hot topic of cooperation study in complex-structured populations these years. It is the preliminary for the thesis work.

Chapter 2 provides a new approach to investigate strategy updating process in the framework of evolutionary games. In this work for two-strategy evolutionary games in structured populations, we remove the requirement for explicit information about exact payoffs, by encoding the payoffs into the willingness of any player to switch from her current strategy to the competing one. Moreover, the robustness of the proposed methods is verified in different types of game models such as the prisoner's dilemma game, snowdrift game and stag hunt game.

- Theoretical computations and numerical simulations indicate that the evolutionary dynamics are intrinsically regulated by contact relationships specified by the network topologies of the populations. More precisely, when each player plays simultaneously against more than one neighbor, strategies can easily coexist even when one strategy dominates the other in each base game. The results further reveal that the frequencies of the coexisting strategies can be calculated analytically.

- This work provides a new analysis tool in analyzing the competing dynamics of different strategies. And the results help us to find a viable escape hatch out of evolutionary stalemate.

Chapter 3 extends the individual player's switching probabilities between players, relaxing the restriction that strategy switch occurs between competing strategies. The critical ingredient that enables us discover new mechanisms for coexistence of strategies based on players' contact patterns, is exactly each player's probability of switching strategies that we have just described.

In previous studies, one of most-often used assumption is that natural selection acts on individuals at the same time scale, i.e. players renovate their strategies with the same frequency. Everyday phenomenon reminds us of the variation in learning rates within populations. Thus, evolutionary game theory may not necessarily be restricted to uniform time scales associated with the game interaction and strategy adaption evolution. In **Chapter 4** we focus our attention on a more realistic model where the population update strategies at non-uniform time scales. The basic message from results is that heterogeneity in time scales of individuals' updating will drastically enrich collective evolutionary dynamics.

- We remove the assumption of uniform time scales by dividing the population into fast and slow groups according to the players' updating frequencies. We aim to investigate how different strategy compositions of one group influence the evolutionary outcome of the other's fixation probabilities. Analytical analysis and numerical calculations are established to study the evolution dynamics of strategies in some typical classes of two-player games (Prisoner's dilemma game, snowdrift game and stag-hunt game here).
- Results show that heterogeneity in strategy-update time scales dramatically affects the dynamics of strategies. We provide a proximation formula of the fixation probability of mutant types in finite populations and study the evolution outcomes under weak selection. This work shows that heterogeneity in time scales enriches the evolutionary dynamics and under simplifying conditions, the more complicated possible outcomes can be effectively predicted in the premise that the population composition and payoff parameters are known.

Our previous work found scenarios where speculation either leads to the reduction of the basin of attraction of the cooperative equilibrium or even the loss of stability of this equilibrium, if the insurance costs are lower than the expected fines on defectors. In **Chapter 5** we extend the common binary-strategy combination of cooperation and defection by adding a third strategy, called insured cooperation, which corresponds to buying an insurance covering the potential loss resulted from the unsuccessful public goods game. We analyze the dynamics in such a three strategy system and find that insurance enhances the cooperation.

As an extension of our study proposing speculation strategy (Zhang et al. 2013), in **Chapter 6** we restrict our attention to the the analysis of replicator dynamics competed by four competing strategies: C (cooperators), D (defectors), S (speculators) and L (loners, i.e. nonparticipants). Our main interest is to probe into effective mechanisms for cooperation to get supported, when players face multiple decisions or choices. Moreover, we hope to gain more insight into the competition and coexistence of multiple strategies in nature, by the aid of this model settings.

- Results show that the evolutionary dynamic outcomes of the gaming population are closely related to the model parameters. Initialized from a three-strategy state, the system will evolve into the observed domination of some strategy or a rock-paper-scissors type of cycle, suggesting that the additional strategic options can radically alter the evolution of cooperation. And, larger multiplication factor and punishment on defectors can facilitate cooperation to be a dominant strategy in the absence of speculation. Results suggest that the option to abstain from the joint enterprise offers an escape from the social trap, leading to the decline of exploiters and allows the reemergence of cooperators.

- Moreover, public goods cooperation can also be fostered to be an equilibrium under moderate values of punishment and cost of insurance in the absence of loner. Further, cooperation fails to dominate the population in the competition with speculation and loner strategy, even though in the absence of defection. And, when the initial state consists of the four strategies, at least one strategy will go extinction within the evolution.

Finally, **Chapter 7** presents a concluding summary of the research and a collection of ideas for future work and investigation.

Chapter 2

Crucial Role of Strategy Updating for Evolution in Interaction Networks

Here, we start the main blocks of this thesis. In the beginning, I want to ask some questions: what is the essential attribute of competing among different strategies? Do you think the statement ”we are born selfish” (Dawkins 1976) is right or wrong? On the other hand, is the statement ”evolution is constructive because of cooperation, and that we might add natural cooperation as a third fundamental principle of evolution beside mutation and natural selection”(Nowak 2006) right or wrong? In this chapter, what I want to show is that it is really difficult to out-compete another strategy. In other words, it is easy to get coexistence for different strategies. Cooperation is important for different individuals, but defective behaviours are also of importance to be investigated because every strategy is difficult to be eliminated by other strategies (Zhang, Chen, Zhang, Wang and Chu 2010b, Zhang, Zhang and Chu 2010).

In this chapter, we start from a simple model with structured populations. We use complex networks for describing the structures of investigated populations. Network models are useful tools for studying the dynamics of social interactions in a structured population. After a round of interactions with the players in their local neighbourhood, players update their strategy based on the comparison of their own payoff with the payoff of one of their neighbours. Here we show that the assumptions made on strategy updating are of crucial importance for the strategy dynamics. In the first step, we demonstrate that seemingly small deviations from the standard assumptions on updating have major implications for the evolutionary outcome of two cooperation games: cooperation can more easily persist in a Prisoner’s Dilemma game, while it can go more easily extinct in a Snowdrift game. To explain these outcomes, we develop a general model for the updating of states in a network that allows us to derive conditions for the steady-state coexistence of states (or strategies). The analysis reveals that coexistence crucially depends on the number of agents consulted for updating. We conclude that updating rules are as important for evolution on a network as network structure and the nature of the interaction.

2.1 Introduction

Network theory has provided important insights into the dynamics of interactions in a structured population. In this framework, population structure is represented by a network, the nodes of which represent the individual agents while the links correspond to the possible interactions (Albert and Barabási 2002, Newman 2003, Boccaletti et al. 2006, Lieberman et al. 2005). The agents can be molecules, individual organisms, or groups of individuals, and the interactions can also be highly diverse, ranging from chemical reactions among molecules to the exchange of goods or knowledge among groups (Nowak and May 1992, Santos et al. 2008, Bastolla et al. 2009, Ohtsuki et al. 2006). To fix ideas, we will here focus on the evolution of social interactions among individuals. In this context, network models typically assume that each agent is endowed with a certain strategy (corresponding to the agent's "state") that determines the agent's behaviour in interactions with their neighbours in the network and the resulting payoffs. After the interaction phase, agents can update their strategies by comparing their own accumulated payoffs with the payoff of one of their neighbours (Szabó and Fáth 2007, Perc and Szolnoki 2010, Gross and Blasius 2008a).

Network models have revealed that network structure plays an important role for the evolutionary dynamics of behaviour in a social interaction. Take, for example, the Prisoner's Dilemma game (PDG) (Weibull 1995, McNamara and Weissing 2010), where mutual cooperation is favoured to mutual defection by both players. Yet, cooperation is outcompeted by defection in a well-mixed population, since defection is a dominant strategy. When interactions take place on a network, however, cooperation can get established, but this strongly depends on the network structure; cooperation gets easily off the ground in heterogeneous networks (e.g., scale-free networks), while it will not easily evolve in homogeneous networks (e.g. random-regular networks) (Santos and Pacheco 2005, Santos et al. 2006). In a Snowdrift game (SDG), another prototype example for the evolution of cooperation, the coexistence of cooperation and defection is expected in a well-mixed population, while network models predict the fixation of either cooperation or defection under a wide range of conditions (Axelrod 1984, Roca et al. 2009, Doebeli and Hauert 2005, Hauert and Doebeli 2004, Ramazi and Cao 2014).

Until now, the discussion on evolutionary games on networks has mainly focused on network structure and the nature of the game (Claussen and Traulsen 2008, Melbinger et al. 2010, Ohtsuki, Nowak and Pacheco 2007, Pacheco et al. 2006, Roca et al. 2006a, Riehl and Cao 2014, Zhang, Zhang, Chu and Perc 2011, Riehl and Cao 2015). Here we will scrutinize the role of strategy updating.

Instead of investigating the influences of cognitive processes or incomplete information on strategy dynamics in large populations, we introduce individual player's switching probabilities between competing strategies to investigate how strategies are being taken in structured populations. In the simplest scenario where two players interact with each other, say a cooperator and a defector, it is safe to assume that the cooperator has a larger probability to switch its strategy than that of the defector because of the advantage of defection over cooperation. In a real society, the probability that a defector turns into a cooperator is small but not zero. It is difficult to get the precise values of the two probabilities; however, it is relatively easier to know which is greater than the other. When a player interacts with more than one players, the problem of how she adjusts her strategy becomes complicated. In this study, we provide a framework to investigate the evolution of how players in large structured populations choose from two competing strategies after repeatedly playing games with their neighbors. The critical ingredient that enables us discover new mechanisms for coexistence of strategies based on players' contact patterns is exactly the introduction of each player's probability of switching strategies that we have just described.

2.2 Basic model

For simplicity, we consider games with two pure strategies, like the PDG or the SDG. At each point in time, an agent employs one of the two strategies. The payoff obtained by an agent using strategy i in an interaction with an agent using strategy j is given by m_{ij}, where $M = (m_{ij})$ is the 2×2 payoff matrix characterizing the game. For example, the payoff matrices of a PDG and an SDG are given by

$$M_{PDG} = \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}, M_{SDG} = \begin{pmatrix} b-\frac{c}{2} & b-c \\ b & 0 \end{pmatrix}, \tag{2.1}$$

where b and c ($b > c$) indicate the benefits and costs of cooperation, respectively.

Typically, strategy updating is modelled as follows (Szabó and Fáth 2007, Perc and Szolnoki 2010, Gross and Blasius 2008a): an agent having used strategy A and accumulated payoff π_A in the previous interactions randomly selects another agent from her neighbourhood; if that agent happens to have used the alternative strategy B and accumulated payoff π_B, then the focal agent will switch from A to B with a probability $u_{A\to B}$ that reflects the payoff difference $\pi_B - \pi_A$. This probability may, for example, be given by the Fermi function

$$u_{A\to B} = \left(1 + e^{-\beta(\pi_B - \pi_A)}\right)^{-1}. \tag{2.2}$$

In the above equation, β controls the intensity of selection.

2.3 Evolutionary dynamic results

The red curves in Figure 1 show the evolution of cooperation in the PDG and the SDG for $b = 1$ and a spectrum of c-values for this updating rule. The results confirm that cooperation in a PDG can evolve in a scale-free network (for $c < 0.1$) but not in a random-regular network, and that cooperation in the SDG will spread to fixation more easily in a scale-free network (for $c < 0.2$) than in a random-regular network (for $c < 0.6$). The other curves in Figure 2.1 illustrate what happens if the strategy updating is not based on the consultation of one other agent, but on the consultation of two or more other agents. In these cases, a focal agent compares her payoff with that of m other agents and switches from A to B whenever any of these comparisons would result in such a switch in the standard updating scenario ($m = 1$) considered above.

Figure 2.1 clearly shows that such a change in strategy updating has a major effect on the evolutionary outcome. Now cooperation in the PDG can also get off the ground in a random-regular network ($m = 2$: $c < 0.2$; $m = 4$: $c < 0.9$; $m = 10$: all c). Most strikingly, for larger values of m, fixation for either cooperation or defection gives rise to the stable coexistence of these strategies. Moreover, for large values of m, the evolutionary outcome is relatively independent of the type of interaction (i.e. PDG versus SDG) and the structure of the network (i.e. random-regular versus scale-free).

We also considered still another updating rule: agents interact sequentially with their neighbours (in random sequence) and update their strategy as above, but now updating takes place after each individual interaction. In other words, the switching probability is given by Eq. (2.2), but now the payoffs of the A- and the B-players are not accumulated over several interactions, but given by $\pi_A = m_{AB}$ and $\pi_B = m_{BA}$. In other words, the two switching probabilities ($u_{A \to B}$ and $u_{B \to A}$) only depend on the payoff matrix of a specific game. In addition, we can investigate the evolutionary dynamics of strategies in a more general scenario where the values of the two switching probabilities are given without a payoff matrix although we know that the switching probabilities are essentially determined by the payoff matrix. Many simulations for a large variety of payoff matrices M have revealed that - when $m = 1$, irrespective of the structure of the network - the evolutionary outcome is only dependent on the sign of $m_{12} - m_{21}$: if $m_{12} > m_{21}$, strategy 1 will spread to fixation; if $m_{12} < m_{21}$, strategy 2 will spread to fixation; and both strategies will coexist at equal frequencies if $m_{12} = m_{21}$. Hence, coexistence is very unlikely. But again, we arrive at the conclusion that the evolutionary outcome is more strongly affected by the updating rule than by the nature of the interaction (which is crucially dependent on the payoff parameters m_{12} and m_{21}) or the structure of the network. Next, we describe in detail this general model and the results from computer simulations and analytical approaches.

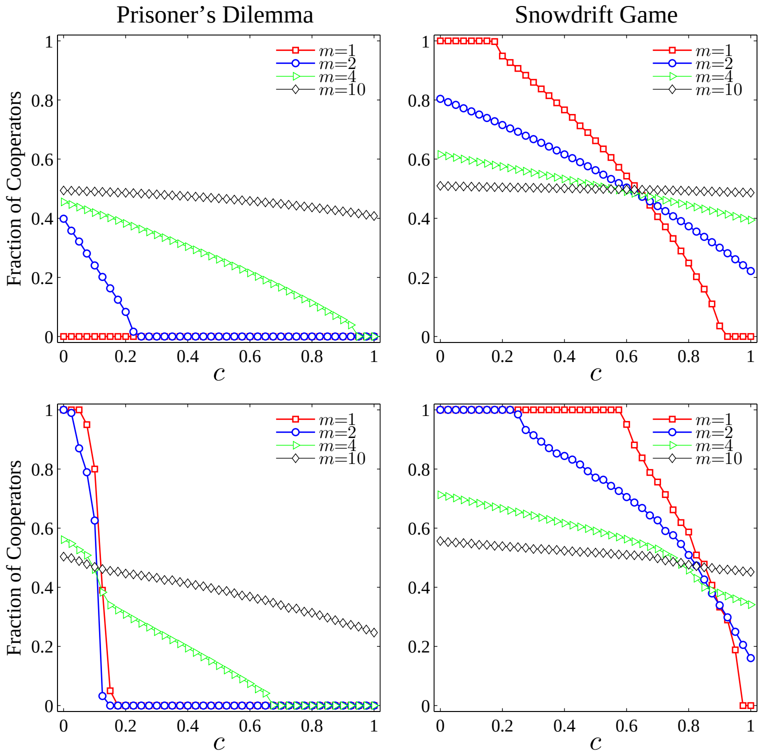


Figure 2.1: Equilibrium fraction of cooperators in the Prisoner's Dilemma game (PDG) and the Snowdrift game (SDG) as a function of the costs c of cooperation for four values of m, the number of agents consulted for strategy updating. Left panels: PDG; right panels: SDG; upper panels: random-regular network with degree 10; lower panel: Barabási-Albert scale-free network with average degree 10. The benefit of cooperation was kept constant at $b = 1$. In our simulations on scale-free networks, if a player's degree was smaller than m, she chose all of her neighours for consultation.

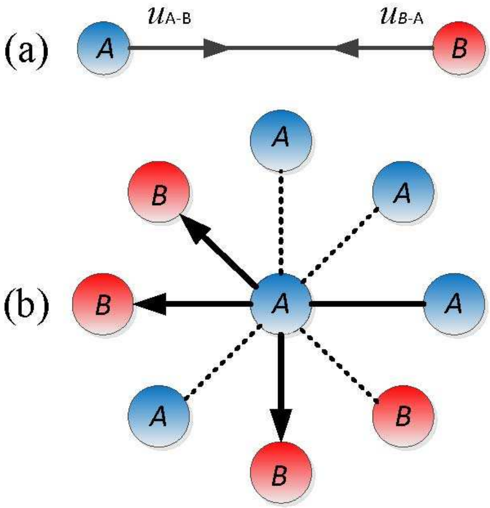


Figure 2.2: (a) A single updating when two different individuals encounter each other. Strategy A (blue) has a probability $u_{A \to B}$ to switch to B (red), while B switches to A with probability $u_{B \to A}$. (b) Diagrams illustrating an updating event in a network scenario where each player chooses m of her neighbours randomly for updating. Each arrow specifies a probabilistic switching because it is formed by different strategies. Dot lines indicate neighbours which are not selected at the current time step.

To explain the results mentioned above, we now take a more mathematical approach that is applicable beyond the context of evolutionary games. This approach is based on the two transition probabilities $u_{_{A \to B}}$ and $u_{_{B \to A}}$, which are viewed as given model parameters that do not necessarily reflect a fitness comparison. As above, $u_{_{A \to B}}$ denotes the probability that an agent using strategy A will switch to the alternative strategy B when this agent happens to consult a B-player. Fig. 2.2(a) shows the setup for the case $m = 1$ where a single updating takes place between two players with different strategies. Fig. 2.2(b) represents a network scenario in which the focal player chooses m neighbours for updating. In this specific case, $m = 4$ and three of the four chosen neighbours maintain different strategies. If $p_{_\Omega}$ denotes the probability that a neighbour of agent i uses strategy A, we can now calculate the probabilities $U^i_{_{A \to B}}$ and $U^i_{_{B \to A}}$ with which, after consulting m neighbours, agent i

would switch from A to B or from B to A, respectively:

$$\begin{cases} 1 - U^i_{A\to B} = \big(1 - u_{A\to B}(1 - p_{\Omega_i})\big)^m \\ \\ 1 - U^i_{B\to A} = \big(1 - u_{B\to A} p_{\Omega_i}\big)^m . \end{cases} \tag{2.3}$$

For example, $u_{B\to A} p_{\Omega_i}$ is the probability that agent i, when having played B, is consulting an A-playing neighbour that induces agent i to switch to A; $1 - u_{B\to A} p_{\Omega_i}$ is the probability that any given neighbour does not induce agent i to switch when having played B, and $\big(1 - u_{B\to A} p_{\Omega_i}\big)^m$ is the probability that none of m consulted neighbours will induce player i to switch to A. By definition, the latter probability corresponds to $1 - U^i_{B\to A}$.

We can now derive a recurrence equation for the probability $p_i(t)$ that a given agent i will employ strategy A at time t:

$$p_i(t+1) \quad = \quad p_i(t)[1 - U^i_{A\to B}(t)] + [1 - p_i(t)] U^i_{B\to A}(t). \tag{2.4}$$

2.3.1. THEOREM. *Under the dynamics of (2.4), it holds that*

$$\begin{cases} \text{A will persist if } u_{B\to A} > \frac{1-(1-u_{A\to B})^m}{m}; \\ \\ \text{B will persist if } u_{A\to B} > \frac{1-(1-u_{B\to A})^m}{m}. \end{cases} \tag{2.5}$$

Proof: The first term on the right-hand side of Eq. (2.4) corresponds to the joint probability of having played A in the previous time step and not having switched to B, while the second term corresponds to the probability of having played B at time t but having switched to A. An equilibrium $p_i(t+1) = p_i(t) = \hat{p}_i$ of (2.4) is characterized by

$$\hat{p}_i \cdot \hat{U}^i_{A\to B} = (1 - \hat{p}_i) \cdot \hat{U}^i_{B\to A}. \tag{2.6}$$

For a homogeneous network, such as a random-regular network, it is plausible to assume that the probability to use strategy A will converge to the same value $\hat{p}_i = \hat{p}_{\Omega_i} = \hat{p}_i$ for all i. Inserting Eq. (2.3) in Eq. (2.6) yields an implicit equation for $\hat{p}$:

$$\begin{aligned} &\hat{p}_i \cdot \left[1 - (1 - u_{A\to B}(1 - \hat{p}))^m\right] \\ &= (1 - \hat{p}) \cdot \left[1 - (1 - u_{B\to A}\hat{p})^m\right]. \end{aligned} \tag{2.7}$$

When $u_{B\to A} \to u^*_{B\to A}$, the probability that a player i adopts strategy A is $\hat{p}_i$, where $0 < \hat{p}_i \ll 1$. Assume that after a long time evolution, $\hat{p}_i \approx \hat{p} \approx \hat{p}_{\Omega_i}$, then after substituting this into Eq. (2.6), and neglecting the high-order terms in $\hat{p}_i$, we get

$$m\hat{p}u^*_{B\to A} = \hat{p} - \hat{p}(1 - u_{A\to B})^m. \tag{2.8}$$

For fixed values of $u_{A\to B}$ and m, we have

$$u^*_{B\to A} = \frac{1-(1-u_{A\to B})^m}{m}. \tag{2.9}$$

Similarly, strategy B will persist for a given value of $u_{B\to A}$ whenever $u_{A\to B}$ is larger than a threshold value $u^*_{A\to B}$, which can be obtained from Eq. (2.6) by taking the limit $\hat{p} \to 1$. The result is:

$$\begin{cases} \text{A will persist if } u_{B\to A} > \frac{1-(1-u_{A\to B})^m}{m}; \\ \text{B will persist if } u_{A\to B} > \frac{1-(1-u_{B\to A})^m}{m}. \end{cases}$$

□

For $m = 1$, this immediately implies that equilibrium coexistence of both strategies (i.e. $0 < \hat{p} < 1$) is possible if, and only if, $u_{A\to B} = u_{B\to A}$. This explains our earlier results that strategy updating after each individual interaction will only lead to the coexistence of the two strategies if $m_{12} = m_{21}$. It also implies that in a homogeneous network strategy coexistence requires that, at equilibrium, both strategies have the same payoffs: $\pi_A(\hat{p}) = \pi_B(\hat{p})$.

Figure 2.3 illustrates that for $m > 1$ the coexistence of A and B is easy to achieve. For two values of $u_{A\to B}$, this figure shows the equilibrium frequency $\hat{p}$ of strategy A for a spectrum of values $u_{B\to A}$ and the outcome of simulations that are in excellent agreement with the equilibrium value predicted by Eq. (2.6). For a given value of $u_{A\to B}$, strategy A will persist in the population whenever $u_{B\to A}$ is larger than a certain threshold value $u^*_{B\to A}$. This minimum value for $\hat{p} > 0$ can be calculated by taking the limit $\hat{p} \to 0$ in Eq. (2.6).

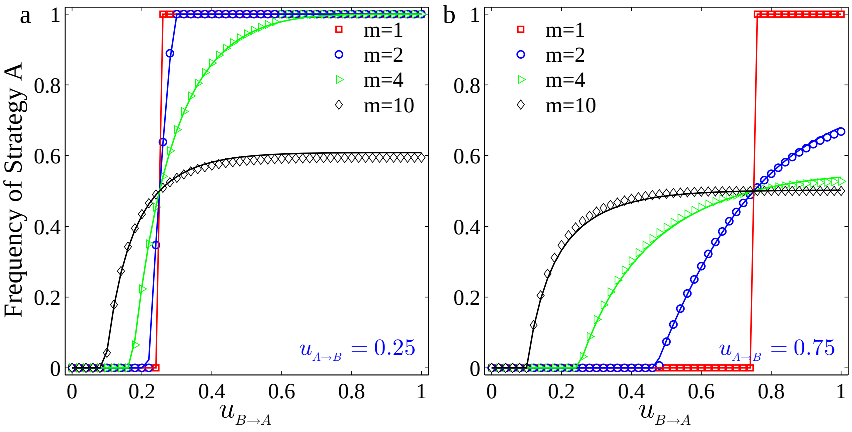


Figure 2.3: Equilibrium frequency of strategy A in a random-regular network of degree 10 for four values of m, the number of agents consulted for strategy updating. For (a) $u_{A\to B} = 0.25$ and (b) $u_{A\to B} = 0.75$, the analytical predictions based on Eq. (2.6) (solid lines) and the outcome of simulations (symbols) are shown for a spectrum of values of $u_{B\to A}$. Both panels clearly indicate that a larger value of m favours the equilibrium coexistence of both strategies. In our simulations time is discretized in time steps and in each step players choose to be an A or a B-player with the probability determined when finishing the previous step. We start from a configuration in which each player adopts strategy A with a probability chosen uniformly from the range [0, 1]. In each round, player i updates her strategy and is correspondingly associated with a probability that she is an A-player in the next round. Each simulation result corresponds to a result of averaging over 10^3 generations after a transient period of 10^4 rounds in 100 independent realizations with the population size 10^4.

Coexistence of A and B will occur if both conditions of Eq. (2.5) are satisfied. Figure 2.4 illustrates that simulations in random-regular networks are in excellent agreement with this prediction and that the coexistence region becomes very large already for moderate values of m (e.g. $m = 4$).

Our analytical results do not directly apply to heterogeneous networks, since the equilibrium value $\hat{p}_i$ of the probability to use strategy A will depend on the degree of player i. As a rule, $\hat{p}_i$ will more likely be between 0 and 1 when the degree of player i is higher. Qualitatively, however, our basic insight that a larger value of m favours polymorphism for a broad range of values of $u_{A\to B}$ and $u_{B\to A}$ also applies to heterogeneous networks. This is illustrated by Figure 2.5 that indicates for a Barabási-Albert scale-free network that the conditions for the coexistence of competing strategies are even less stringent than in a random-regular network. Here, the critical values of $u_{B\rightarrow A}$ for a given $u_{A\rightarrow B}$ in scale-free networks are smaller than that in random-regular networks.

To show the validity of the approach here discussed, we have also performed more computer simulations on different networks for the sake of comparison. To model the evolutionary dynamics on the described topologies we incorporate a model in which, at each time step, each node has a probability of strategy A. In our simulations time is discretized in time steps and each simulation starts with a random probability P_i ranging in [0, 1]. The simulation runs until a stationary state for the average frequency of strategy A (f_A) of the whole population. Notably, the network topology employed on top of which the evolutionary game is evolved is regular ring, thus enabling an efficient comparison of different average degrees $\bar{k}$.

Our results on ring networks with different $u_{B\rightarrow A}$, where individuals share homogeneous interaction degree. From Fig. 2.6-2.8, we can be see that the simulation results are in accordance with the theoretical analysis, where larger average degrees are beneficial for the maintenance and diversity of the two strategies. For example, when $k = 100$, f_A can be close to 0.5 at high $u_{A\rightarrow B}$ no matter the value of $u_{B\rightarrow A}$. This result also transfer a clue that interactions play a relatively high impact on the coexistence of different strategies.

Until now, we have shown that the essential factor is the interaction number for each player. However, we still believe that the two probabilities which indicate the shifting rates to each other are also important for describing a specific game. For example, if we consider a Prisoner's Dilemma game (PDG), the probability of defection to cooperation should be much smaller than the opposite. Whereas, in a Snowdrift game (SDG), the two probabilities should have smaller difference than that in a PDG. In other words, the two probabilities should not be independent of each other. Motivated by the above considerations, we now study a very simple situation where $u_{B\rightarrow A} = 1 - u_{A\rightarrow B}$.

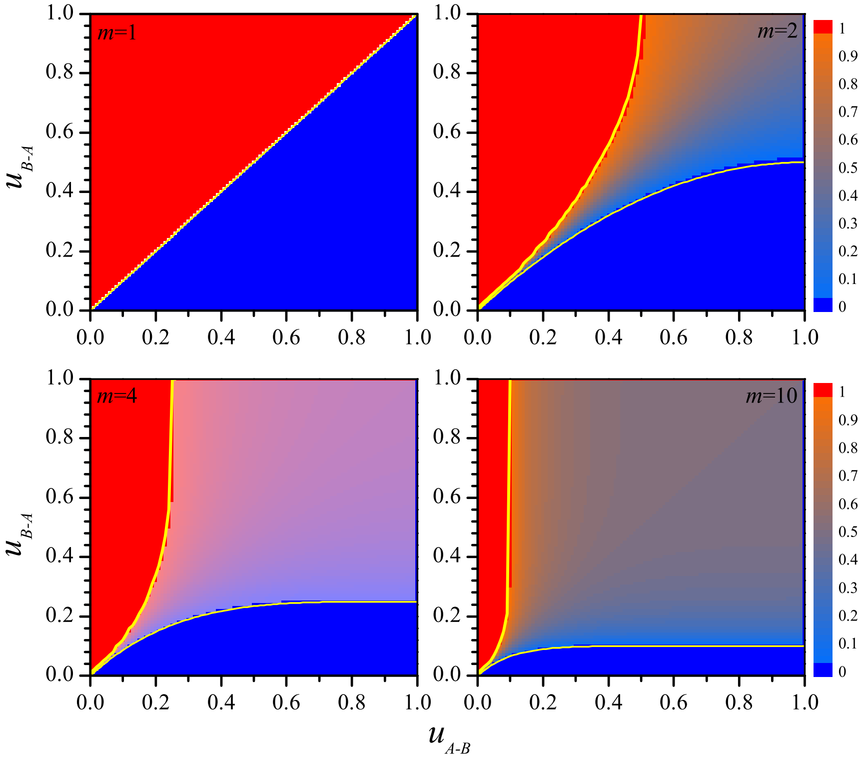


Figure 2.4: Equilibrium coexistence of strategies A and B as a function of the updating probabilities $u_{A\to B}$ and $u_{B\to A}$ for four values of m, the number of agents consulted for strategy updating: (a) $m = 1$; (b) $m = 2$; (c) $m = 4$; (d) $m = 10$. Red: fixation of strategy A; blue: fixation of strategy B; yellow lines: boundaries of coexistence region based on Eq. (2.7); all other colours: frequency of A ($0 < \hat{p} < 1$) resulting from Eq. (2.6).

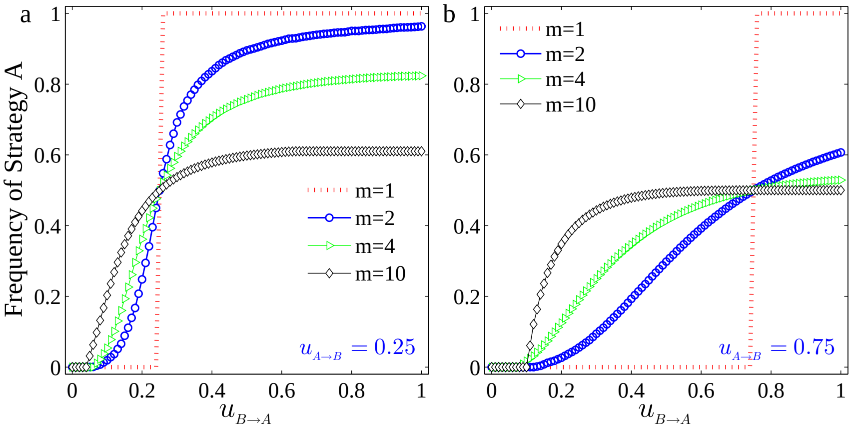


Figure 2.5: Equilibrium frequency of strategy A in a Barabási-Albert scale-free network with average degree 10 for four values of m, the number of agents consulted for strategy updating. For (a) $u_{A\to B} = 0.25$ and (b) $u_{A\to B} = 0.75$, the outcome of simulations is shown for a spectrum of values of $u_{B\to A}$. As in case of a random-regular network (Fig. 2.3), a larger value of m favours the equilibrium coexistence of both strategies.

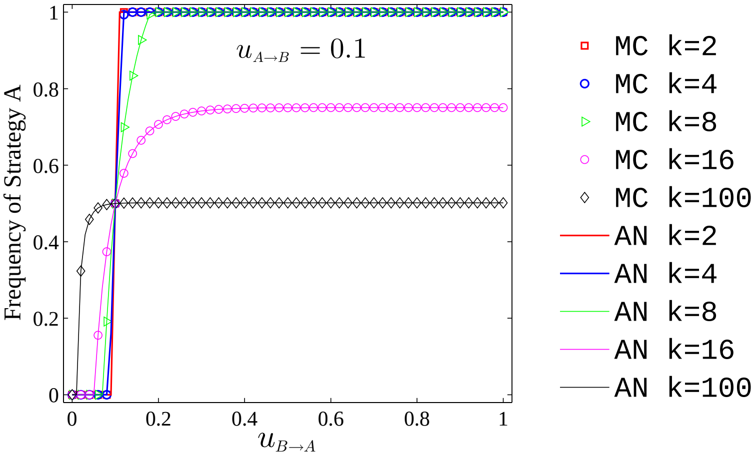


Figure 2.6: Phase diagram for the interaction model in Ring networks for different values of $u_{B\to A}$ and average degree k when values of $u_{A\to B} = 0.1$. The networks are made up of $N = 10^4$ nodes. MC results are averages over 100 realizations. Solid lines corresponds to the theoretical prediction and symbols to MC results.

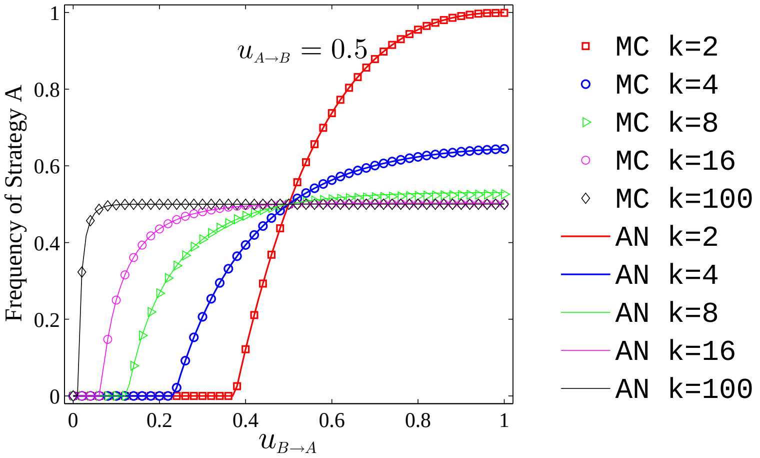


Figure 2.7: Phase diagram for the interaction model in Ring networks for different values of $u_{B\to A}$ and average degree k when values of $u_{A\to B} = 0.5$. The networks are made up of $N = 10^4$ nodes. MC results are averages over 100 realizations. Solid lines corresponds to the theoretical prediction and symbols to MC results.

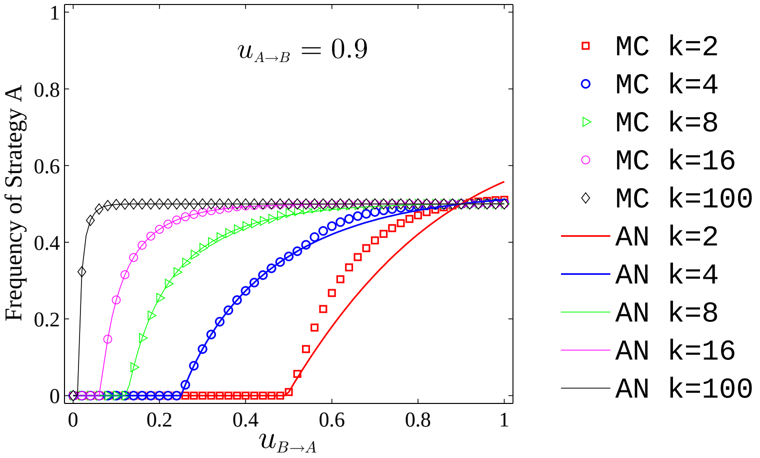


Figure 2.8: Phase diagram for the interaction model in Ring networks for different values of $u_{B\to A}$ and average degree k when values of $u_{A\to B} = 0.9$. The networks are made up of $N = 10^4$ nodes. MC results are averages over 100 realizations. Solid lines corresponds to the theoretical prediction and symbols to MC results.

Here, a big $u_{B\to A}$ (larger than 0.5) means that strategy A has an advantage over strategy B, and vice versa. Thus, we get

$$2p - p(1 - u_{B\to A}p)^m + p(u_{B\to A} + (1 - u_{B\to A})p)^m + (1 - u_{B\to A}p)^m - 1 = 0 \tag{2.10}$$

Figure 2.9 presents the results on regular random networks and BA scale-free networks respectively. In BA scale-free networks, we still suppose that each player uses all of her neighbours for updating. Still, simulation results are in great agreement with that obtained through theoretical analysis. The results also confirm that it is easier for achieving coexistence of different strategies when players choose more neighbours for updating. Different than the results shown in Fig. 2.2, the fractions of A-players are much larger than 0.5 even when the value of m is large (e.g. $m = 10$). The reason is that the two probabilities are correlated. This plays a positive feedback effect on the dynamics of strategies.

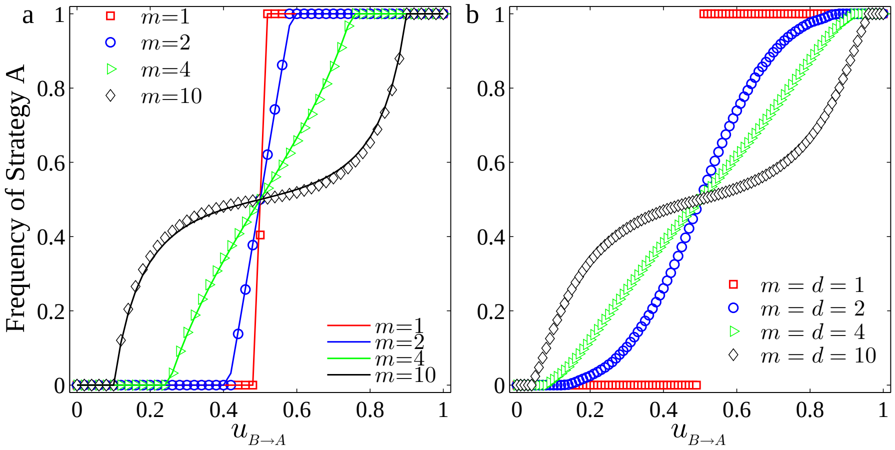


Figure 2.9: Frequency of A-players for the interaction model in random regular (RR) networks (left panel) and BA scale-free (SF) networks (right panel), with $u_{B\to A} + u_{A\to B} = 1$. In RR networks, each player chooses m neighbours for updating while in SF networks, one chooses all of her neighbours for updating. The settings are the same as in Fig 2.2. Solid lines correspond to the theoretical prediction and symbols are for simulation results.

2.4 Conclusion remarks

In conclusion, we have shown that evolution on an interaction network can be as strongly affected by the strategy updating procedure as by the network structure and the payoff matrix. In this paper for two-strategy evolutionary games in structured populations, we follow a different approach, bypassing the requirement for explicit knowledge of the exact payoffs, by encoding the payoffs into the willingness of any player to switch from her current strategy to the competing one.

Theoretical computations and numerical simulations show that the evolutionary dynamics are intrinsically regulated by contact relationships specified by the network topologies of the populations. We demonstrate that updating rules are of crucial importance for the steady state distribution of states. On the basis of general arguments, we show that the coexistence of different states strongly depends on the number m of agents that determine the updating of a given agent: if $m = 1$, as typically assumed, coexistence is difficult to achieve, while coexistence occurs under mild conditions when $m > 1$. By means of two cooperation games, we show that this general insight has important implications for the strategy dynamics of games on a network. In comparison to earlier models, cooperation can more easily persist in a Prisoner's Dilemma game, while it can go more easily extinct in a Snowdrift game. This implies that strategy updating deserves more attention in empirical and theoretical studies.

The main conclusion of this chapter is that updating rules are crucial for the evolution of competing strategies and every strategy is difficult to be eliminated by other strategies. In other words, it is difficult to say which strategy is better than the others because even a strategy earns lower payoff, it can spread in the population in some scenarios.

Chapter 3

An Extended Model for Strategy Updating in Interaction Networks

In chapter 2, we introduced a model with switching probabilities. In this chapter, we will extend this model to make it more general. First, let us look again at the model in Chapter 2. The main idea is that every player can switch when she meets another player with different strategy. It is quite realistic because it is the case that players are adopting for looking for better strategies. When an individual finds that another one is playing a game with a different strategy but with a higher payoff, she may shift her own strategy. However, if we go further, the assumption that individuals can switch if and only if they meet players with different strategies is not realistic because that if a player knows the information of the game, they may choose a better strategy without the information. Here, we want to extend the model in Chapter 2 by relaxing the limitation that only different strategies can lead switching. Players can switch even they meet the same strategies. Although it is an extension of Chapter 2, you will find that the two chapters are of different essences.

Still, we will explain this work in a typical way in the framework of evolutionary game theory. Models for evolutionary games have traditionally assumed that players imitate their successful neighbours by comparison of payoffs. In this chapter for two-strategy evolutionary games in structured populations, we follow a different approach, bypassing the requirement for explicit knowledge of the exact payoffs, by encoding the payoffs into the willingness of any player to switch from her current strategy to the competing one. Theoretical computations and numerical simulations show that the evolutionary dynamics are intrinsically regulated by contact relationships specified by the network topologies of the populations. In particular, when each player plays simultaneously against more than one neighbor, strategies can easily coexist even when one strategy dominates the other in each base game. The results further reveal that the frequencies of the coexisting strategies can be calculated analytically. This provides new insight into why and how different strategies coexist in large populations.

3.1 Introduction

Despite its ubiquity in nature and human societies, the survival of cooperative behavior among selfish individuals when defection is the most advantageous strategy is not fully understood (Griffin et al. 2004, Axelrod 1984, West et al. 2007, Nowak 2006). The study of complex networks has provided new grounds to the understanding of evolutionary dynamics (Albert and Barabási 2002, Newman 2003, Boccaletti et al. 2006, Zhang, Zhang, Chu and Chen 2010). The integration of the microscopic patterns of interactions among players becomes a central topic to study population dynamics in paradigmatic scenarios. Effects of network topologies, or equivalently population structures, on the evolutionary processes have been discussed intensively (Nowak and May 1992, Santos et al. 2008, Bastolla et al. 2009), and with the development of complex network theory, these effects are gradually unraveled (Szabó and Fáth 2007, Gross and Blasius 2008b, Perc and Szolnoki 2010, Lieberman et al. 2005, G. and Ye 2009, Melbinger et al. 2010). In particular, complex network theory has paved the way for exploring many real-world large-scale networks, and describing and understanding various processes that evolve in typical such networks (Santos and Pacheco 2005, Hauert 2006, Roca et al. 2009, Ohtsuki, Nowak and Pacheco 2007, Pacheco et al. 2006).

Evolutionary game dynamics generally involve how players update their strategies as time evolves. The updating rules are therefore crucial and, until now, most of them are based on replication and imitation (Nowak and Sigmund 2004, Schlag 1999). The essence of replication rules is that a strategy with better performances has a higher replication rate. Imitation rules assume that a player can imitate her opponents' strategy with some probability when interacting with individuals having achieved higher payoffs. Hence, to apply such update rules, players have to know in general the exact magnitudes of the payoffs of all her opponents. However, inferring payoffs may not be as easy as it is often assumed: individuals' bounded rationality implies limited cognition and decision-making capabilities (Helbing 1996, Blume 2003); in addition, computations might be cognitively expensive and thus unfavorable. Until now, it remains a hotly debated topic on how cognitive processes with limited information take place in game playing.

In this chapter, instead of investigating the influences of cognitive processes or incomplete information on strategy dynamics in large populations, we introduce individual player's switching probabilities between competing strategies to investigate how strategies are being taken in structured populations. In the simplest scenario where two players interact with each other, say a cooperator and a defector, it is safe to assume that the cooperator has a larger probability to switch its strategy than that of the defector because of the advantage of defection over cooperation. In a real society, the probability that a defector turns into a cooperator is small but not zero. It is difficult to get the precise values of the two probabilities; however, it is relatively easier to know which is greater than the other. When a player interacts with more than one players, the problem of how she adjusts her strategy becomes complicated. In this study, we provide a framework to investigate the evolution of how players in large structured populations choose from two competing strategies after repeatedly playing games with their neighbors. The critical ingredient that enables us discover new mechanisms for coexistence of strategies based on players' contact patterns is exactly the introduction of each player's probability of switching strategies that we have just described.

3.2 Set-up of basic model

Consider a network of N ($N \gg 1$) players, labeled by $1, \ldots, N$, each of whom has two candidate strategies A and B to play against one another. They actually play with mixed strategies. We thus use p_i to denote the probability that player i chooses strategy A, and obviously she plays strategy B with probability $1 - p_i$.

Different from classical game descriptions, we encode the payoffs associated with a game between two players into the willingness of a player to shift her current strategy to the other one after interacting with her opponent; more specifically, we denote by $u_{B \to A|A}$ the willingness that a B-player shifts her strategy from B to A right after she encounters an A-player, and correspondingly $u_{A \to B|B}$ the willingness that an A-player adopts strategy B after playing with a B-player. Similarly, $u_{A \to B|A}$

denotes the probability that an agent using strategy A will switch to the alternative strategy B when this agent happens to consult an A-player. $u_{B\to A|B}$ denotes the probability that an agent using strategy B will switch to the alternative strategy A when this agent happens to consult a B-player.

The network topology determines completely who meets whom, and we use the N-by-N adjacency matrix $(a_{ij})_{N\times N}$ to describe the players' interaction pattern, where $a_{ij} = 1$ if and only if players i and j may play against each other in the network and $a_{ij} = 0$ otherwise. Hence, the adjacency matrix of a network is symmetric, e.g. $a_{ij} = a_{ji}$.

Following standard setups for evolutionary games in structured populations, players interact as time evolves and P_i changes with time. It is the goal of this paper to study in this N-player network, whether the competing strategies A and B may coexist in the long run; in addition, if, to the contrary of intuition, the answer to this question is yes, how they coexist. Towards this end, we first look into the discrete-time model for the evolution of the probability that any player i plays with strategy A

$$P_i(t+1) = P_i(t)[1 - U^i_{A\to B}(t)] + [1 - P_i(t)]U^i_{B\to A}(t), \tag{3.1}$$

where $U^i_{A\to B}(t)$ is the tendency that player i's strategy switch to B if she is an A-player before the current game round and similarly $U^i_{B\to A}(t)$ is the tendency that player i's strategy switch to A if she is a B-player before the current game round. The above equation assumes that the update rule is a Markov process with no memory. Then for all $i = 1, \dots, N$, $U^i_{A\to B}(t)$ and $U^i_{B\to A}(t)$ are crucial factors. The above assumptions are general enough to incorporate a great variety of possible game scenarios. This equation is similar with Eq. (2.4) in chapter 2. The main idea in the two equations are same. That is each player may shift her strategy when she meets other players. However, we know that how to calculate the switching probabilities plays an essential role in the two equations. We will find that the two equations are essentially different because they show different evolutionary dynamics.

The generalized version, at the steady state reads as

$$P_i = P_i[1 - U^i_{A\to B}] + [1 - P_i]U^i_{B\to A}, \tag{3.2}$$

3.3 Evolutionary dynamic results

In what follows, we concentrate on a specific model on how the games in each round take place. We consider the situation where each player chooses m of her nearest neighbors randomly for updating after each game round. Fig. 3.1(a) shows the setup for a single updating event between two players with identical or different strategies. We extend our previous work by considering that players can change her strategy even when she encounters a neighbor with same strategy with her at the current time step. Fig. 3.1(b) represents a network scenario in which the focal player choose more neighbours for updating. In this specific case, she chooses four neighbours including three of them maintain different strategies. In general, we get the switching probabilities for a specific player i are

$$U^i_{A\to B} = 1 - \sum_{k=0}^{m} \binom{m}{k} P^k_{\Omega_i}(1 - P_{\Omega_i})^{m-k}(1 - U_{A\to B|A})^k(1 - U_{A\to B|B})^{m-k}, \quad (3.3)$$

$$U^i_{B\to A} = 1 - \sum_{k=0}^{m} \binom{m}{k} P^k_{\Omega_i}(1 - P_{\Omega_i})^{m-k}(1 - U_{B\to A|A})^k(1 - U_{B\to A|B})^{m-k}. \quad (3.4)$$

Here, $P_{\Omega_i}(t)$ is the density of A-players in i's neighbourhood at time t. $\binom{m}{k}$ represents a binomial coefficient when $k \leq m$ and is defined as zero when $k > m$. From the above equations, we get

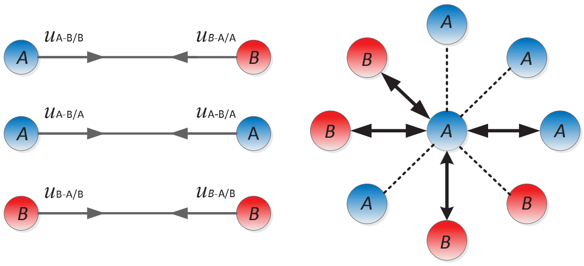
$$\begin{aligned} U^i_{A\to B} &= 1 - [P_{\Omega_i}(1 - U_{A\to B|A}) + (1 - P_{\Omega_i})(1 - U_{A\to B|B})]^m \\ &= 1 - [1 - U_{A\to B|B} + P_{\Omega_i}(U_{A\to B|B} - U_{A\to B|A})]^m, \end{aligned} \quad (3.5)$$

$$\begin{aligned} U^i_{B\to A} &= 1 - [P_{\Omega_i}(1 - U_{B\to A|A}) + (1 - P_{\Omega_i})(1 - U_{B\to A|B})]^m \\ &= 1 - [1 - U_{B\to A|B} + P_{\Omega_i}(U_{B\to A|B} - U_{B\to A|A})]^m. \end{aligned} \quad (3.6)$$

We assume that after a long enough evolution, the system converges to an equilibrium state where we have

$$\hat{p}^i = \hat{p}^i(1 - U^i_{A\to B}) + (1 - \hat{p}^i)U^i_{B\to A}. \quad (3.7)$$

It always has the solutions $\hat{p}_i = 0$ or $1, \forall i = 1, ..., N$, which indicates that only one strategy survives in the whole population. Here, we assume that neighbours with identical or different strategies can both induce a player to switch her strategy. This means that if $\hat{p}_i = 1$ for every i, we have $U^i_{A \to B} = 0$ because there is no B-players in the population. Similarly, when $\hat{p}_i = 0$ for every i, one gets $U^i_{B \to A} = 0$.



(a) (b)

Figure 3.1: (a) A single updating when two different individuals encounter each other, the two agents can have identical or different strategies. (b) Diagrams illustrating an probabilistic updating event in a network scenario where each player chooses some of her neighbours randomly for updating. In this specific case, the focal player has eight neighbours but she only chooses four of them as updating references. Valid lines indicate the selected neighbours, and three of them are of arrows. Each arrow specifies a probabilistic switching even if it is formed by same strategies. Dot lines indicate neighbours which are not selected at the current time step.

Let $\hat{p}$ denote the frequency of A-players in the population at an equilibrium state, namely $\hat{p} = \frac{1}{N}\sum_{i=1}^{N} \hat{p}_i$. We assume that at an equilibrium state, the population is homogenous in probabilities of playing strategy A. It means that $0 < \hat{p} \approx \hat{p}_i \approx \hat{p}_{\Omega_i} \ll 1$. We get

$$\begin{aligned} U_{A \to B} &= 1 - [1 - U_{_{A \to B}|B} + P_{\Omega_i}(U_{_{A \to B}|B} - U_{_{A \to B}|A})]^m \\ &= 1 - (1 - U_{_{A \to B}|B})^m - mp(1 - U_{_{A \to B}|B})^{m-1}(U_{_{A \to B}|B} - U_{_{A \to B}|A}) + O(p) \end{aligned} \tag{3.8}$$

$$
\begin{aligned}
U_{B\to A} &= 1-[1-U_{B\to A|B}+P_{\Omega_i}(U_{B\to A|B}-U_{B\to A|A})]^m \\
&= 1-(1-U_{B\to A|B})^m - mp(1-U_{B\to A|B})^{m-1}(U_{B\to A|B}-U_{B\to A|A})+O(p)
\end{aligned}
\tag{3.9}
$$

After inserting Eqs. (3.8) and (3.9) into Eq. (3.7), we obtain

$$
\hat{p}[1-(1-U_{A\to B}(1-\hat{p})^m)] = (1-\hat{p})[1-(1-U_{B\to A}\hat{p})^m]. \tag{3.10}
$$

Thus,

$$
U_{B\to A} - pU_{B\to A} = pU_{A\to B}. \tag{3.11}
$$

That is

$$
\begin{aligned}
&1-(1-U^*_{B\to A|B})^m - mp(1-U^*_{B\to A|B})^{m-1}(U^*_{B\to A|B}-U^*_{B\to A|A}) \\
&-p+p(1-U^*_{B\to A|B})^m \\
&= p - p(1-U_{A\to B|B})^m
\end{aligned}
\tag{3.12}
$$

From the above equations, we find that it is difficult to get the analytical solutions. However, we can get the following theorem.

3.3.1. THEOREM. *Under the dynamics of (3.1), a player can switch her strategy if and only if she meets a player with the different strategy, it holds that*

$$
\begin{cases}
\text{A will persist if } u_{B\to A|A} > \frac{1-(1-u_{A\to B|B})^m}{m}; \\
\\
\text{B will persist if } u_{A\to B|B} > \frac{1-(1-u_{B\to A|A})^m}{m}.
\end{cases}
\tag{3.13}
$$

Proof: If a player only switch when she meets players with the different strategy, it is the case of $u_{A\to B|A} = u_{B\to A|B} = 0$.

When $u_{B\to A|A} \to u^*_{B\to A|A}$, the probability that a player i adopts strategy A is p^*_i, where $0 < p^*_i \ll 1$. Assume that after a long time evolution, $p^*_i \approx p^* \approx p^*_{\Omega_i}$and then after substituting into Eq. (3.5), and neglecting the high-order terms in p^*_i, we get

$$
mp^*u^*_{B\to A|A} = p^* - p^*(1-u_{A\to B|B})^m. \tag{3.14}
$$

For fixed values of $u_{A\to B|B}$ and m, we have

$$u^*_{B\to A|A} = \frac{1-(1-u_{A\to B|B})^m}{m}. \tag{3.15}$$

Symmetrically,

$$u^*_{A\to B|B} = \frac{1-(1-u_{B\to A|A})^m}{m}. \tag{3.16}$$

□

These results are as the same as our conclusions in Theorem (2.3.1) of chapter 2. One can easily conclude that for a fixed $u_{A\to B|B}$, the larger m is, the easier it is for the two strategies to coexist. In a specific scenario, when players use all of their neighbours for updating in a homogenous interaction network in which m equals to the largest eigenvalue of the adjacency matrix, the above results confirm a relation between the coexistence of the two competing strategies and the spatial structure of the population.

From the above results, we find that it is easy for the coexistence of the two competing strategies, in the presence of small difference between $u_{A\to B|B}$ and $u_{B\to A|A}$. Given the value of $u_{A\to B|B}$, we know that when $u_{B\to A|A} = u_{A\to B|B}$, the frequency of A-players is 0.5. How about the changing of this frequency when $u_{B\to A|A}$ becomes smaller? For a given $u_{A\to B|B}$, it is of utmost importance to look for the critical value $u^*_{B\to A|A}$ below which $\hat{p}$ is always zero and above which $\hat{p}$ is always positive. This is because the existence of $u^*_{B\to A|A}$ clearly indicates that even a strategy is in a disadvantaged position in competing with the other, it can still survive in the population. This is expected to provide new insights into why cooperation can survive in nature even defection usually earns higher payoffs. In what follows, we calculate $u^*_{B\to A|A}$.

3.4 Evolutionary dynamics in different game models

Until now, we have shown that one essential factor is the interaction number for each player. However, we still believe that the two probabilities which indicate the shifting rates to each other are also important for describing a specific game. For example, if we consider a Prisoner's Dilemma game (PDG), the probability of defection to cooperation should be much smaller than the opposite. Whereas, in a Snowdrift game (SDG), the two probabilities should have smaller difference than that in a PDG. In other words, the two probabilities should not be independent of each other. Motivated by the above considerations, we now study the results in the game models: PDG, SDG, and the Stag-Hunt game (SHG).

Employing A to represent strategy C and B is D, the switching probability matrix reads

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \left(\begin{array}{cc} u_{C \to D|C} & u_{C \to D|D} \\ u_{D \to C|C} & u_{D \to D|D} \end{array} \right) \end{array}$$

Replaced by

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \left(\begin{array}{cc} u_1 & u_2 \\ u_3 & u_4 \end{array} \right) \end{array}$$

For the PDG, the payoff matrix reads

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \left(\begin{array}{cc} b-c & -c \\ b & 0 \end{array} \right) \end{array},$$

In PDGS, we know that playing as a defector can earn higher payoff regardless of the others' strategies. So, here $u_1 > u_3$,$u_1 > u_4$, $u_2 > u_3$ and $u_2 > u_4$.

For the SDG, the payoff matrix reads

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \left(\begin{array}{cc} b-\frac{c}{2} & b-c \\ b & 0 \end{array} \right) \end{array}.$$

We know that in SDGs, you would better to have the different strategy from your opponent. So, here we have $u_1 > u_2$, $u_1 > u_3$, $u_4 > u_2$ and $u_4 > u_3$.

For the SHG, the payoff matrix reads

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \left(\begin{array}{cc} b & 0 \\ b-c & b-\frac{c}{2} \end{array} \right) \end{array}.$$

In SHGs, a player can get higher payoff when she choose the same strategy with her opponent. So, here, the relations between the four switching probabilities are $u_2 > u_1$, $u_2 > u_4$, $u_3 > u_1$ and $u_3 > u_4$

The payoff matrix is

$$\begin{array}{c|cc} & C & D \\ C & a_{11} & a_{12} \\ D & a_{21} & a_{22} \end{array}\ .$$

We use

$$\begin{cases} u_1 = 1/(1+e^{-\beta(a_{21}-a_{11})}) \\ u_2 = 1/(1+e^{-\beta(a_{22}-a_{12})}) \\ u_3 = 1/(1+e^{-\beta(a_{11}-a_{21})}) \\ u_4 = 1/(1+e^{-\beta(a_{12}-a_{22})}) \end{cases}, \tag{3.17}$$

respectively, where β is selection intensity. It should be noted that the above configuration of switching probabilities is not the only one. In different scenarios, we should use different methods.

In this case, we get

$$p_i(t+1) = p_i(t)[1 - U^i_{C\to D}(t)] + [1 - p_i(t)]U^i_{D\to C}(t); \tag{3.18}$$

$$U^i_{C\to D} = 1 - \sum_{k=0}^{m} \binom{m}{k} p^k_{\Omega_i}(1-p_{\Omega_i})^{m-k}(1-u_1)^k(1-u_2)^{m-k}; \tag{3.19}$$

$$U^i_{D\to C} = 1 - \sum_{k=0}^{m} \binom{m}{k} p^k_{\Omega_i}(1-p_{\Omega_i})^{m-k}(1-u_3)^k(1-u_4)^{m-k}. \tag{3.20}$$

In the following, some results by numerical method are shown. First, let us look at Figure 3.2 which is the results for PDG. As expected, the fraction of cooperation at steady state is negatively related with the cost c of of cooperation. The selection intensity β also play an important role. The general conclusion is that larger values of c and larger values of β inhibit the cooperation. The most important results of our model is that larger values of m promote the two strategies to get coexistence.

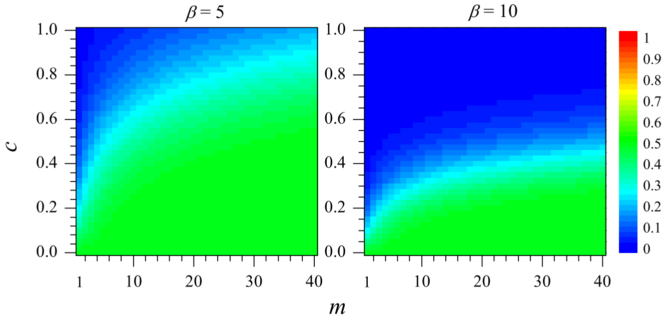


Figure 3.2: Numerical solutions of the frequency of C-players for the contact-based model in the PDG. Larger m can better facilitate the coexistence of the two strategies. In RR networks, each player chooses m neighbours for updating while in SF networks, one chooses all of her neighbours for updating. Moreover, larger β can promote the dominance of D.

Figure 3.2 illustrates the results for SDG. From Fig. 3.2, one can get the same conclusion that larger values of m support the coexistence of different strategies. In this chapter, we focus on the effects of the number of agents consulted for strategy updating and we can safely conclude that larger values of m promote the coexistence of the competing strategies in PDG, SDG and SHG.

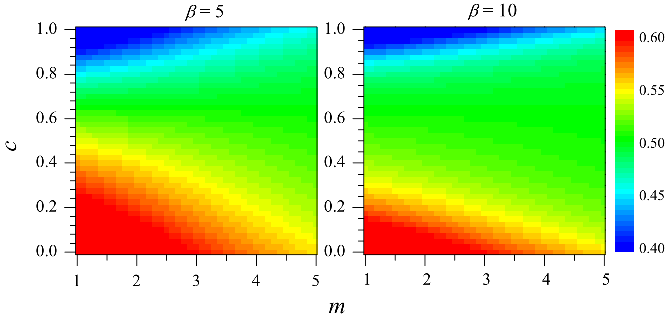


Figure 3.3: Numerical solutions of the frequency of C-players for the contact-based model in the SDG. Larger m can better facilitate the coexistence of the two strategies. m is the number of neighbours selected for updating.

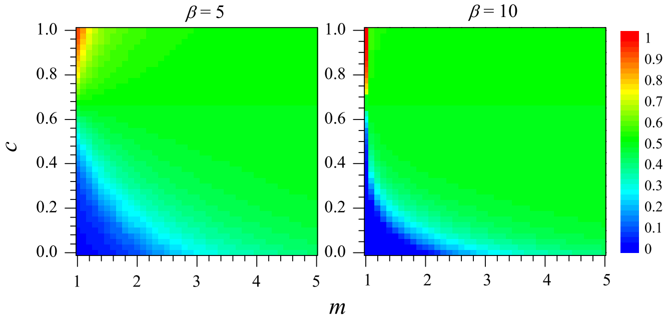


Figure 3.4: Numerical solutions of the frequency of C-players for the contact-based model in the SHG. The settings are the same as in Fig 3.2. Solid lines correspond to the theoretical prediction and symbols are for simulation results.

However, what is the essential difference between the two models in chapters 2 and 3? To answer this question, we should consider how to get the switching probabilities in the two models. In chapter 2, we need two switching probabilities, $u_{A\to B}$ and $u_{B\to A}$. The two switching probabilities describe a scenario where each player can change her strategy if and only if she meets another one with a different strategy. If we suppose the payoff matrix M should be known by each player, the difference of M_{12} and M_{21} play a key role on the switching probabilities. In other words, players do not need to know the whole figure of the game they are playing with others. They only know the difference between the payoffs according the two competing strategies. In chapter 3, when we implement our model, we need to find a method to derive the four probabilities from the payoff matrix. Our way is natural realistic. In the begging, we should consider a potential assumption that a player should know the whole payoff matrix of the game. That is why a player wants to switch even when she meet another same strategy. For instance, if a player is a cooperative player and she is playing a PDG with another one, she wants to be a defector when she knows her opponent is a cooperator. That is rational because that she wants to earn higher payoff. The potential assumption is that the whole matrix is known for everyone. That is the essential difference between the two chapters. In Chapter 2, players do not need the whole landscape of the game. They only need the payoff difference between the two different strategies. To be specific, they only need the information of the difference of M_{12} and M_{21} of the matrix M. However, in

Chapter 3, players need to know the whole payoff matrix because they can change their strategies for a higher payoff. They should know that whether they can get a higher payoff if they switch. In a nutshell, in Chapter 2, a player switches because she does not want earn lower payoff than her opponent while in Chapter 3, a player switches because she wants to earn a higher payoff than before. It is not easy to say which one is more realistic but the most important conclusion is that the number of agents consulted for strategy updating plays a key role on coexistence of competing strategies.

In the above results, the shifting rates are given by the given payoff matrix, now let us consider the effects of shifting rates without considering the payoff matrix. In fact, we do not know the exact values of the four switching probabilities, however we know the relative differences among them. To be specific, in a PDG, the probability that a cooperator switch to a defector should be larger than the opposite. Here, we can make some simple assumptions of the four switching probabilities and see what will happen.

One can easily conclude that for fixed values of the four probabilities, the larger m is, the easier it is for the two strategies to coexist. The differences of cooperation levels in the three typical game models depend on the specific implementations.

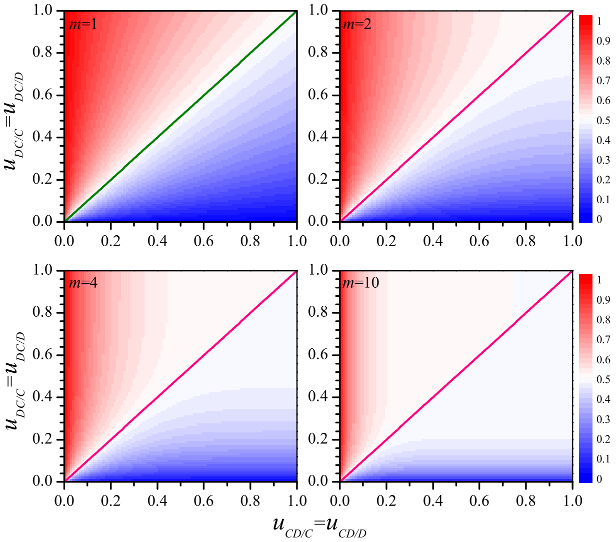


Figure 3.5: Numerical solutions of the frequency of C-players for the contact-based model in the PDG. $u_1 > u_3, u_4$, $u_2 > u_3, u_4$. Larger m can better facilitate the coexistence of the two strategies. The horizontal axis means $u_1 = u_2$, while the vertical axis means $u_3 = u_4$. In RR networks, each player chooses m neighbours for updating while in SF networks, one chooses all of her neighbours for updating. Moreover, larger β can promote the dominance of D.

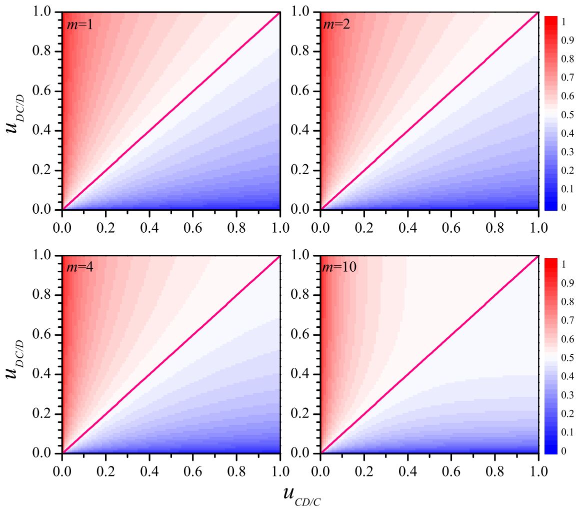


Figure 3.6: Numerical solutions of the frequency of C-players for the contact-based model in the SDG. $u_1 > u_2, u_3, u_4 > u_2, u_3$. Here, the horizontal axis means u_1, while the vertical axis means u_4. $u_2 = u_3 = 0$. Solid lines correspond to the theoretical results where $u_{C\to D|C} = u_{D\to C|D}$. Larger m can better facilitate the coexistence of the two strategies. m is the number of neighbours selected for updating.

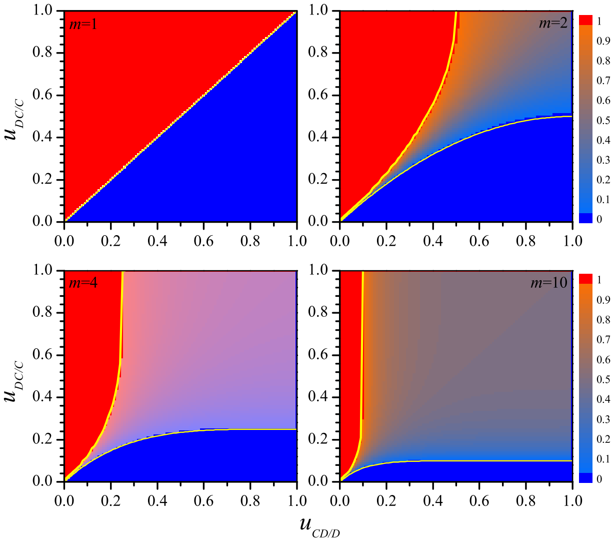


Figure 3.7: Numerical solutions of the frequency of C-players for the contact-based model in the SHG. $u_2 > u_1, u_4$, $u_3 > u_1, u_4$. Here, the horizontal axis means u_2, while the vertical axis means u_3. $u_1 = u_4 = 0$. The settings are the same as in Fig 3.2. Solid lines correspond to the theoretical prediction and symbols are for simulation results.

3.5 Conclusion remarks

In a nutshell, in this chapter we have shifted our attention from the general strategy updating rules, where players need to know their neighbors' exact payoff information, to novel contact-based strategy dynamics. We have used the players' switching probabilities as a key step to establish the frequencies of strategies in the long run through both theoretical analysis and numerical computations.

We use two different ways to generate the switching probabilities. In the beginning, we derive switching probabilities from payoff matrices, and then we provide some assumptions on the switching probabilities for specific game models. Our results confirm that the evolutionary outcome is intrinsically related to the number of neighbours randomly selected for updating and the relationship of the switching probabilities between competing strategies. When each player chooses only one neighbour for updating, the simple comparison between the two switching probabilities decides the complete dominance of a single winning strategy. In sharp comparison, when each player chooses more neighbours simultaneously in each round, the two strategies can easily coexist with predictable frequency proportions.

Our results have disclosed new explanations in idealized models about the survival of dominated strategies in structured populations and may motivate new threads in studying strategy evolutions in animal groups, social communities and large-scale complex networks in general.

Moreover, we should emphasize that the two models in chapters 2 and 3 are essentially different. The motivation of the two contact-based models is that the incomplete information in games. Models for evolutionary games always assume that players can get all information from their neighbours. That means you know the payoff matrix and the payoffs and strategies of your neighbours. If the players can not get complete information, how do they update? There are two scenarios: if a players meets another player with a different strategy (chapter 2), she may imitate her or if she knows the strategy of a opponent, she may change her own strategy (chapter 3). The two models need different information and in fact they depend on different parts of the payoff matrix when we want to derive switching probabilities from payoff matrices.

Chapter 4

Strategy Updating with Multiple Time Scales in Evolutionary Games

This chapter investigate the roles of diversity of time scales in the evolution of public goods games. When applying evolutionary game theory to the analysis of evolutionary dynamics of large populations, a standard assumption is that natural selection acts on individuals in the same time scale, e.g. players use the same frequency to update their strategies. In this study, we remove this restricting assumption by dividing the population into fast and slow groups according to the players' strategy updating frequencies, and investigate how different strategy compositions of one group influence the evolutionary outcome of the other's fixation probabilities. Analytical analysis and numerical calculations are performed to study the evolutionary dynamics of strategies in typical classes of two-player games (Prisoner's Dilemma game, Snowdrift game and Stag-hunt game). We show that heterogeneity in strategy-update time scales leads to dramatic changes in the dynamics of strategies. We give an approximation formula for the fixation probability of mutant types in finite populations and study the outcome of evolution under weak selection. We find that heterogeneity in time scales enriches the evolutionary dynamics and under simplifying conditions, the more complicated possible outcomes can be surprisingly effectively predicted when knowing the population composition and payoff parameters.

4.1 Introduction

How cooperation among non-relatives can persist in the face of cheating remains a fundamental, profound and broad-ranging unsolved question in evolutionary biology (Hardin 1968, Axelrod 1984, Heckathorn 1996). Evolutionary game theory is the mathematical framework that has provided the deepest insight into this issue (Grim 1995, Gintis 2000, Doebeli and Hauert 2005). Several approaches have been used in the analysis of strategy evolution and one popular approach is the integration of the microscopic patterns of interactions among individuals into the evolutionary setting (West et al. 2007, Nowak 2006). Furthermore, coevolution of interaction patterns and cooperative behavior has also been identified as a key factor that may enhance or hinder altruism (Henry et al. 2011, Perc and Szolnoki 2010, Gross and Blasius 2008b). In spite of exciting progresses that have been reported in the past, there are still situations of great practical relevance that remain less explored, and one of them is the role played by time scales when individuals interact and update their strategies.

Time scales may be associated to different temporal dynamics in evolutionary games. The most important two are how often the players interact and how frequent they update their strategies (Sánchez and Cuesta 2005, Iñiguez et al. 2011, Chowdhury et al. 2003, Roca et al. 2006b, Rong et al. 2010, Wu et al. 2009). In fact, the majority of the literature does not distinguish the two that much and tend to discuss them at the same time by assuming that each round of interaction is always followed by a round of updating, in which individuals can change their current strategy according to different rules.

In this study, motivated by the fact that individuals may have different capabilities to update their strategy frequently (Bastolla et al. 2005, Claussen and Traulsen 2008, Frank and Sarkar 2010, Lehman and Tilman 2000, Perc and Szolnoki 2008), we focus on the heterogeneity in how often an individual updates its strategy after repeatedly interacting with the peers. Our goal is thus to understand better how updating frequency may affect strategic competition and thereby promote or inhibit altruistic behaviors. As a first cut, we assume the overall population can be divided into two groups, and the individuals in one update faster than those in the other. When the updating dynamics of the fast and slow groups can be completely decoupled, we give closed-form approximations for the fixation probabilities of the groups and such predictions are validated by simulations for Prisoner's dilemma, snowdrift and stag-hunt games. Further analysis are carried out for populations under weak selection as well. All the theoretical computation and simulation results reveal that heterogeneity in strategy-update time scales indeed leads to much richer evolutionary outcomes. The different strategy composition of one group always influences the evolution of the other group; the extent to which the influence is exerted depends on the game payoffs and the relative sizes of the groups.

The main body of this chapter is organized as follows. Section 4.2 introduces the basic game model and analyzes the evolutionary dynamics of the strategies under different updating time scales. Section 4.3 discusses the implications of the analytical results for the three typical types of two-player games. Section 4.4 investigates the model under weak selection. Finally, we make concluding remarks.

4.2 Evolutionary dynamics with different strategy-update frequences

4.2.1 Setup of the evolutionary dynamics

Consider a finite well-mixed population of N individuals that are playing a two-player game, where each player can make an option from two strategies, A (e.g. cooperation) and B (e.g. defection). An A-player interacting with another A-player receives the payoff a, and otherwise when interacting with a B-player, obtains b. Similarly, a B-player receives c when playing with an A-player and d with another B-players. The payoffs are summarized in the following payoff matrix

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a & b \\ c & d \end{pmatrix} \end{array}. \tag{4.1}$$

The game is played round after round, and we use $\pi_A(t)$ and $\pi_B(t)$, $t = 0, 1, 2, \ldots$, to denote the average payoffs of A and B players in round t respectively. Now and then at the end of a game round, an individual is chosen randomly to update its strategy. To be more concrete, when just finishing round t, the chosen updating individual chooses randomly another individual from the population to compare their strategies; if the strategies are the same, the updating individual keeps its strategy and otherwise if the strategies are different, say the updating individual plays A

and its chosen comparing individual plays B, the updating individual switches its strategy according to the probability given by the Fermi function

$$p(t) = \frac{1}{1 + e^{-\omega(\pi_B(t) - \pi_A(t))}}, \tag{4.2}$$

where the constant ω is called the *intensity of selection* since $\omega \to \infty$ leads to strong selection where the probability for selecting fitter individual is 1 and when $\omega \ll 1$, the update reduces to the Moran process under weak selection (Nowak et al. 2004). Obviously, one only needs to swap the positions of π_A and π_B on the righthand side of (2) to compute $p(t)$ if at t the updating individual plays B and its comparing individual plays A.

It is the aim of this chapter to study how the players' strategy-updating frequencies affect the evolutionary dynamics of the overall population. Towards this end, we assume that the whole population consists of two sub-populations and an updating individual only chooses a comparing individual from its own group: one is called the *fast group* since every round one individual is chosen randomly from this group to update, and the other is called the *slow group* since a member from this group is chosen to update every $s \geq 1$ rounds. Therefore, when $s = 1$, the overall population is homogeneous in the strategy-update frequencies and when $s \to \infty$ the update processes of the fast and slow groups are completely decoupled. We call s the *time scale* of the strategy updating actions and thus when $s > 1$, the overall population is heterogeneous in the time scales of individuals' updates. To keep the analysis tractable and emphasize the most relevant features of the results, we focus on investigating analytically the case when $s \to \infty$; in the supplementary material of this paper, we show through simulations that when s takes other values, the main conclusions of the chapter still hold. Here, $s \to \infty$ means that the two groups are decoupled in strategy updating, however players have influences on payoffs of other in the other group.

4.2.2 Analytical analysis for fixation probabilities

We first compute explicitly the average payoffs π_A and π_B. Consider the situation when in the current round there are M fast individuals, or equivalently $N - M$ slow ones, in the population. Let j be the number of A-players in the fast group, and i be that in the slow group. Then

$$\begin{cases} \pi_A = \frac{j+i-1}{N-1}a + \frac{N-j-i}{N-1}b, \\ \pi_B = \frac{j+i}{N-1}c + \frac{N-j-i-1}{N-1}d. \end{cases} \tag{4.3}$$

So for the fast players, in the next round, the number of A-players will change according to the following probabilities

$$\begin{cases} T_j^+(t) = \frac{j}{M}\frac{M-j}{M}\frac{1}{1+e^{-\omega(\pi_A-\pi_B)}}, \\ T_j^-(t) = \frac{j}{M}\frac{M-j}{M}\frac{1}{1+e^{\omega(\pi_A-\pi_B)}}, \end{cases} \tag{4.4}$$

where T_j^+ denotes the probability to increase by one and T_j^- to decrease by one. One can check that the sum of T_j^+ and T_j^- is always $\frac{j}{M}\frac{M-j}{M}$, which is exactly the probability that the updating individual and its comparing individual have different strategies. The ratio

$$\chi_j = \frac{T_j^-}{T_j^+} = e^{-\omega(\pi_A-\pi_B)} \tag{4.5}$$

determines the *fixation probability* ϕ_j (Nowak et al. 2004) which in this problem setup is the probability that all the fast players use A strategy in the end; more precisely

$$\phi_j = \frac{\sum_{k=1}^{j-1}(\prod_{m=1}^{k}\chi_m)}{\sum_{k=1}^{M-1}(\prod_{m=1}^{k}\chi_m)}. \tag{4.6}$$

It follows from Eq. (4.3) that

$$\pi_A - \pi_B = \frac{2u(j+i)}{N-1} + \frac{2v}{N-1}, \tag{4.7}$$

where

$$\begin{cases} u = \frac{a-b-c+d}{2} \\ v = \frac{-a+bN-dN+d}{2}. \end{cases} \tag{4.8}$$

Combining Eqs. (4.5)-(4.7), we obtain

$$\phi_j = \frac{\sum_{k=1}^{j-1} e^{\frac{-\omega}{N-1}[k(k+2i+1)u+2kv]}}{\sum_{k=1}^{M-1} e^{\frac{-\omega}{N-1}[k(k+2i+1)u+2kv]}}. \tag{4.9}$$

By applying the computational technique in (Traulsen et al. 2006), it can be shown that when $u \neq 0$, the fixation probability is approximated by the following theorem.

4.2.1. THEOREM. *Under the above process, it holds that*

$$\phi_j = \frac{Erf(\xi_j) - Erf(\xi_0)}{Erf(\xi_M) - Erf(\xi_0)}, \tag{4.10}$$

where Erf(x)=$\frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy$ *is the Gauss error function (Jeffrey 1979)*

$$\xi_j = \sqrt{\frac{\omega}{u(N-1)}}\left[(j+i)u + v\right], \tag{4.11}$$

and that when $u = 0$*, the fixation probability is approximated by*

$$\phi_j = \frac{e^{\frac{-2\omega vj}{N-1}} - 1}{e^{\frac{-2\omega vM}{N-1}} - 1}. \tag{4.12}$$

Proof:

$$\begin{aligned}\pi_A - \pi_B &= \frac{1}{N-1}[(i+j-1)a + (N-j-i)b - (i+j)c - (N-i-j-1)d] \\ &= \frac{1}{N-1}[(i+j)(a-b-c+d) - a + Nb - Nd + d] \\ &= \frac{(i+j)}{N-1}(a-b-c+d) + \frac{1}{N-1}(-a+bN-dN+d)\end{aligned} \tag{4.13}$$

We employ $u = \frac{a-b-c+d}{2}$ and $\nu = \frac{-a+bN-dN+d}{2}$, thus

$$\pi_A - \pi_B = \frac{2u(i+j)}{N-1} + \frac{2\nu}{N-1}. \tag{4.14}$$

Because

$$\phi_j = \frac{\sum_{k=1}^{j-1}(\prod_{m=1}^{k} \chi_m)}{\sum_{k=1}^{M-1}(\prod_{m=1}^{k} \chi_m)} = \frac{\sum_{k=1}^{j-1} e^{-\frac{\omega}{N-1}[k(k+2i+1)\mu+2k\nu]}}{\sum_{k=1}^{M-1} e^{-\frac{\omega}{N-1}[k(k+2i+1)\mu+2k\nu]}} \tag{4.15}$$

and $x_m = e^{-\omega(\pi_A - \pi_B)}$, we get

$$\phi_j \approx \frac{\int_0^j e^{-\frac{\omega}{N-1}\mu(m+i)^2-2\beta m\nu} dm}{\int_0^m e^{-\frac{\omega}{N-1}\mu(m+i)^2-2\beta m} dm}. \tag{4.16}$$

By employing $\beta = \frac{\omega}{N-1}$,

$$\int_0^j e^{-\frac{\omega}{N-1}\mu(m+i)^2-2\beta m\nu} dm = \int_0^j e^{-\beta\mu(m+i)^2-2\beta m\nu} dm. \tag{4.17}$$

If $\mu \neq 0$, let $t = \sqrt{\beta\mu}(m+i)$, then

$$\frac{1}{\sqrt{\beta\mu}} \int_{\sqrt{\beta\mu}i}^{\sqrt{\beta\mu}(k+i)} e^{-t^2-2\sqrt{\frac{\beta}{\mu}}\nu t+2\beta\nu i} dt \tag{4.18}$$

$$= \frac{e^{\frac{\beta}{\mu}}\nu^2 + 2\beta\nu i}{\sqrt{\beta\mu}} \int_{\sqrt{\beta\mu}i}^{\sqrt{\beta\mu}(k+i)} e^{-(t+\sqrt{\frac{\beta}{\mu}}v)^2} dt. \tag{4.19}$$

By employing $y = t + \sqrt{\frac{\beta}{\mu}}v$,

$$\int_0^j e^{-\frac{\omega}{N-1}\mu(m+i)^2-2\beta m\nu} dm \tag{4.20}$$

$$= \frac{e^{\frac{\beta}{\mu}}\nu^2 + 2\beta\nu i}{\sqrt{\beta\mu}} \int_{\sqrt{\beta\mu}(\mu i+\nu)}^{\sqrt{\beta\mu}[(k+i)\mu+\nu]} e^{-y^2} dt \tag{4.21}$$

$$= \frac{e^{\frac{\beta}{\mu}}\nu^2 + 2\beta\nu i}{\sqrt{\beta\mu}} \left(\int_0^{\sqrt{\beta\mu}[(k+i)\mu+\nu]} e^{-y^2} dy - \int_0^{\sqrt{\beta\mu}(i\mu+\nu)} e^{-y^2} dy \right). \tag{4.22}$$

By employing $\beta = \frac{\omega}{N-1}$,

$$\int_0^j e^{-\frac{\omega}{N-1}\mu(m+i)^2-2\beta m\nu} dm \tag{4.23}$$

$$= \mathrm{Erf}\left(\sqrt{\frac{\omega}{\mu(N-1)}}(k+i)\mu + \nu \right) - \mathrm{Erf}\left(\sqrt{\frac{\omega}{\mu(N-1)}} i\mu + \nu \right). \tag{4.24}$$

Here,

$$\mathrm{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy. \tag{4.25}$$

Similarly, we get

$$\int_0^k e^{-\frac{\omega}{N-1}\mu(m+i)^2-2\beta m\nu}dm \tag{4.26}$$

$$= \text{Erf}(\varepsilon_k) - \text{Erf}(\varepsilon_0), \tag{4.27}$$

where

$$\varepsilon_n = \sqrt{\frac{\omega}{\mu(N-1)}}[(n+i)\mu+\nu]. \tag{4.28}$$

Hence,

$$\phi_j = \frac{\text{Erf}(\varepsilon_j) - \text{Erf}(\varepsilon_0)}{\text{Erf}(\varepsilon_m) - \text{Erf}(\varepsilon_0)}. \tag{4.29}$$

When $\mu = 0$,

$$\int_0^k e^{-\frac{\omega}{N-1}\mu(m+i)^2-2\beta m\nu}dm \tag{4.30}$$

$$= \int_0^k e^{-2\beta m\nu}dm \tag{4.31}$$

$$= -\frac{1}{2\beta\nu}(e^{-2\beta m\nu} - 1). \tag{4.32}$$

That is

$$\phi_j \approx \frac{e^{-\frac{2\omega\nu j}{N-1}} - 1}{e^{-\frac{2\omega\nu m}{N-1}} - 1}. \tag{4.33}$$

□

Note that the approximations Eqs. (4.10) and (4.12) are applicable to any intensity of selection ω and any $j \in \{1, \ldots, M-1\}$.

Now we turn our attention to the slow players. Because of the assumption that $s \to \infty$, when a slow player is chosen to update, the strategic choices of the fast players have already evolved into one of the two absorbing all-A or all-B states. Therefore, following similar steps of deriving the fixation probabilities for fast players, we obtain the fixation probabilities for the slow players, namely when $u \neq 0$,

$$\phi_j = \frac{\text{Erf}(\xi_j) - \text{Erf}(\xi_0)}{\text{Erf}(\xi_{N-\bar{M}}) - \text{Erf}(\xi_0)}, \tag{4.34}$$

where $\bar{M} = 0$ if the fast group evolves into the all-B state and $\bar{M} = M$ otherwise, Erf(x) is again the Gauss error function and

$$\xi_j = \sqrt{\frac{\omega}{u(N-1)}}\left[(j+\bar{M})u + v\right]; \tag{4.35}$$

when $u = 0$,

$$\phi_j = \frac{e^{\frac{-2\omega v j}{N-1}} - 1}{e^{\frac{-2\omega v (N-M)}{N-1}} - 1}. \tag{4.36}$$

In view of the fixation probabilities for the fast group (4.10) and (4.12) and for the slow group (4.34) and (4.36), we can actually write down the fixation probabilities for the overall population of N individuals of both fast and slow players. Let n denote the size of the current group of interest, j and i the numbers of A-players in the current group and the other group respectively, and ξ_j, u and v the same as in Eqs. (4.10) and (4.8) respectively. Then the unified expression for the fixation probabilities is that when $u \neq 0$,

$$\phi_j = \frac{\text{Erf}(\xi_j) - \text{Erf}(\xi_0)}{\text{Erf}(\xi_n) - \text{Erf}(\xi_0)}, \tag{4.37}$$

and when $u = 0$,

$$\phi_j = \frac{e^{\frac{-2\omega v j}{N-1}} - 1}{e^{\frac{-2\omega v n}{N-1}} - 1}. \tag{4.38}$$

Note that i takes its value from $\{0, 1, \ldots, N-M\}$ when the current group is the fast group while i is either 0 or M when the slow group is of interest.

With the calculated fixation probabilities at hand, we are ready to investigate the evolutionary outcomes for three different types of games in which strategy B dominates, A and B coexist, or coordination of A and B is preferred.

4.3 Simulation for typical two-player games

From the analytical analysis in the previous section, it is clear that because of the effect of the time scale, the fixation probabilities depend on not only the initial number of A-players, but also the relative sizes of the fast and slow groups. In this section, to further demonstrate this point and more importantly, to gain insight into how population-level cooperation is influenced, we carry out simulation studies on three typical types of two-player games that are classified according to the structures of the payoff matrix specified in Eq. (1), namely dominance of B ($c > a$ and $d > b$), coexistence of A and B ($a < c$ and $b > d$) and a coordination game ($a > c$ and $b < d$). In all the simulations, we take $N = 80$ and $M = 40$.

4.3.1 Game with dominating B

Consider the game in which strategy B always dominates, so a B-player always obtains a higher payoff than an A-player no matter what the fraction of B-players is in the population. Thus, it must be true that $c > a$ and $d > b$. A well-known example is the Prisoner's Dilemma game (PDG) with $c > a > d > b$, in which a defector is always promised with the highest fitness when facing a cooperator, an exploited cooperator is worse off than a defector playing with another defector, and thus defection is the unique Nash equilibrium (Schelling 1980). In our simulations, we take $a = 3$, $b = 1$, $c = 5$, $d = 2$, and then A corresponds to cooperation and B defection in a PDG.

We show the results in Fig. 4.1(a) for $\omega = 0.25$ and Fig. 4.1(b) for $\omega = 0.05$, which are in perfect agreement with the analytical prediction. It is clear that when the number of A-players in the slow group varies, the fixation probability of A-players in the fast group changes correspondingly, although the changes are not significant. So heterogeneity in time scales affects the outcome of the simulated PDG, but not significantly.

4.3.2 Game with coexisting A and B

Consider the game in which B is the best reply to A ($c > a$), and at the same time A is the best reply to B ($b > d$). A typical example is the Hawk-Dove game or the Snowdrift game (SDG). For infinite populations, the replicator dynamics predict the stable coexistence of A and B. For simplicity, we take the payoffs in the SDG to be $a = 3$, $b = 2$, $c = 5$, $d = 1$, and then A is to cooperate and B to defect.

Compared with the simulation results for PDG, a substantially different phenomenon takes place for SDG in the evolutionary outcomes that shows the strong influence of time scales. Fig. 4.2 shows that more B-players in the slow group lead to much bigger fixation probabilities of A strategy in the fast group. The reason is rooted in the fact that the more B-players in the slow group, the higher payoffs of A-players in the fast group, and hence higher chance for having more A-players. Again this matches the analytical prediction.

4.3.3 Coordination game

Finally, let us discuss coordination games in which $a > c$ and $b < d$, and then A is the best reply to A and at the same time B is the best reply to B. The replicator equation of such systems exhibit bistability: if the fraction of A-players is sufficiently high in the beginning, A-players will reach fixation; otherwise, B-players will dominate. The stronger the intensity of selection, the less likely it is that a single A-player can take over a B population. Here, we focus on the Stag-Hunt game (SHG) as an example of a coordination game. We take $a = 5$, $b = 1$, $c = 3$, $d = 2$ here, where A presents cooperation and B denotes defection.

The evolutionary outcomes are shown in Fig. 4.3. Again one sees perfect matching between theoretical prediction and simulation. It can be observed that time scale heterogeneity clearly changes the evolutionary outcomes. For instance, more A-players in the slow group lead to large increment of A's fixation probability. The reason is that the more A-players in the slow group, the higher payoffs of A-players in the fast group, and thus more intensely the strategy A is promoted. The observation that increasing the composition of one strategy in the slow group benefits the evolution of the same strategy in the fast group merits special attention since it is the opposite of the results in SDG.

Although our analytical results in the previous section and the simulation results in this section only study the idealized case when $s \to \infty$, in the supplementary materials we demonstrate that when s takes its values of 1, 2, 5, 10, 50 and 100, similar conclusions on how the strategy composition of the slow group affects the evolutionary outcome of the fast group are still applicable. This underscores the importance of the insight gained from our analytical prediction and in fact, now one can always predict with confidence whether a strategy in the fast group will be promoted or inhibited when facing a slow group of different strategy compositions. One may question, however, whether the conclusions are still applicable for populations under weak selection (payoff trivially influences the fitness of individuals) when $w \ll 1$, since then the approximation in our previous calculation becomes less effective. To address this concern, in the next section, we study the case when $\omega \ll 1$.

4.4 Fixation probability under weak selection

4.4.1 Strategy evolution for fast players

Here, we use the fixation probability of neutral mutants ($1/M$) as a benchmark to verify whether the selection favors A replacing B in the subpopulation of fast players. If $\phi_1 > 1/M$, it means the selection favors A replacing B.

4.4.1. THEOREM. *If* $\frac{a-b-c+d}{3}[\alpha + 3(1-\alpha)\beta] + (b-d) > 0$ *holds, the selection in subpopulation of fast players favors* A *replacing* B.

Proof: For $\omega \ll 1$, our model reduces to the Moran process under weak selection. Then, we address the weak selection approximation as follows. When $\omega \ll 1$, we get

$$\chi_j = \frac{T_j^-}{T_j^+} \approx 1 - \omega(\pi_A - \pi_B) \tag{4.39}$$

which after being substituted to Eq. (4.6) leads to

$$\begin{aligned}\phi_1 &\approx \frac{1}{1 + \sum_{k=1}^{M-1}[1 - \omega(\frac{u}{N-1}k^2 + (\frac{u}{N-1} + \frac{2v}{N-1} + \frac{2ui}{N-1})k)]} \\ &= \frac{1}{M} + \frac{\omega(M-1)}{6M(N-1)}[N(a-b-c+d)(3(1-\alpha)\beta \\ &\quad + \alpha) + 3(b-d)N - 2a - b - c + 4d],\end{aligned} \tag{4.40}$$

where $\alpha = M/N, \beta = i/(N-M), u = \frac{a-b-c+d}{2}$ and $v = \frac{-a+b+N-d+N}{2}$. If $\frac{a-b-c+d}{3}[\alpha + 3(1-\alpha)\beta] + (b-d) > 0$, we get $\phi_1 > 1/M$. □

Note that when $\alpha = 1$, namely the population is homogeneous in their time scales, the above results agree with those for a variety of Moran processes under weak selection (Ohtsuki, Bordalo and Nowak 2007).

4.4.2 Strategy evolution for slow players

After the fast players have evolved into their absorbing states, the slow players start their evolution processes. We have to carry out our computation for two separate absorbing states of fast players, all-A and all-B, seperately.

Scenario I: fast players converge to all-A

4.4.2. THEOREM. *If* $(a-b-c+d)(2\alpha+1)+3(b-d)>0$ *holds, the selection in subpopulation of slow players favors* A *replacing* B *when the fast subpopulation converges to all-*A *state.*

Proof: Let j be the number of A-players among slow players. Then the payoffs of strategy A and B for slow players are

$$\begin{cases} \pi_A = \frac{M+j-1}{N-1}a + \frac{N-M-j}{N-1}b, \\ \pi_B = \frac{M+j}{N-1}c + \frac{N-M-j-1}{N-1}d. \end{cases} \tag{4.41}$$

So the probability to have $j+1$ or $j-1$ A-players in the slow players in the next game round when having j A-players in the current round are

$$\begin{cases} T_j^{+} = \frac{j}{N-M}\frac{N-M-j}{M}\frac{1}{1+e^{-\omega(\pi_A-\pi_B)}}, \\ T_j^{-} = \frac{j}{N-M}\frac{N-M-j}{M}\frac{1}{1+e^{+\omega(\pi_A-\pi_B)}}. \end{cases} \tag{4.42}$$

Similarly,

$$\begin{aligned} \phi_j &\approx \frac{1+\sum_{k=1}^{j}[1-\omega(\frac{u}{N-1}k^2+(\frac{u}{N-1}+\frac{2v}{N-1}+\frac{2u}{N-1}M)k)]}{1+\sum_{k=1}^{N-M-1}[1-\omega(\frac{u}{N-1}k^2+(\frac{u}{N-1}+\frac{2v}{N-1}+\frac{2u}{N-1}M)k)]} \\ &= \frac{j}{N-M} + \frac{\omega(N-M-1)}{6(N-M)(N-1)}[N(a-b-c+d) \\ &\quad (2\alpha+1)+(a-b-c+d)j+3(b-d)N-3a+3d], \end{aligned} \tag{4.43}$$

where α, u and v are the same as in Eq. (4.19).

Under weak selection, if strategy A performs better than neutral selection (i.e. $\phi_1 > 1/(N-M)$), the following condition should be satisfied

$$(a-b-c+d)(2\alpha+1)+3(b-d)>0. \tag{4.44}$$

Thus, natural selection favors a single mutant A to eventually replace a population of B-players. $\square$

Scenario II: fast players converge to all-B

4.4.3. THEOREM. *If* $(a-b-c+d)(1-\alpha)+3(b-d)>0$ *holds, the selection in subpopulation of slow players favors* A *replacing* B *when the fast subpopulation converges to all-*A *state.*

Proof: In this case, the payoffs for playing A or B in slow players are

$$\begin{cases} \pi_A = \frac{j-1}{N-1}a + \frac{N-M-j}{N-1}b, \\ \pi_B = \frac{j}{N-1}c + \frac{N-M-j-1}{N-1}d. \end{cases} \tag{4.45}$$

The fixation probability under weak selection reads

$$\begin{aligned} \Phi_j &\approx \frac{1+\sum_{k=1}^{j-1}[1-\omega(\frac{u}{N-1}k^2+(\frac{u}{N-1}+\frac{2v}{N-1})k)]}{1+\sum_{k=1}^{N-M-1}[1-\omega(\frac{u}{N-1}k^2+(\frac{u}{N-1}+\frac{2v}{N-1})k)]} \\ &= \frac{j}{N-M} + \frac{\omega(N-M-1)}{6(N-M)(N-1)}[N(a-b-c+d) \\ &\quad (1-\alpha)+(a-b-c+d)j+3(b-d)N-3a+3d]. \end{aligned} \tag{4.46}$$

Here, α, u and v are the same as in Eq. (4.19).

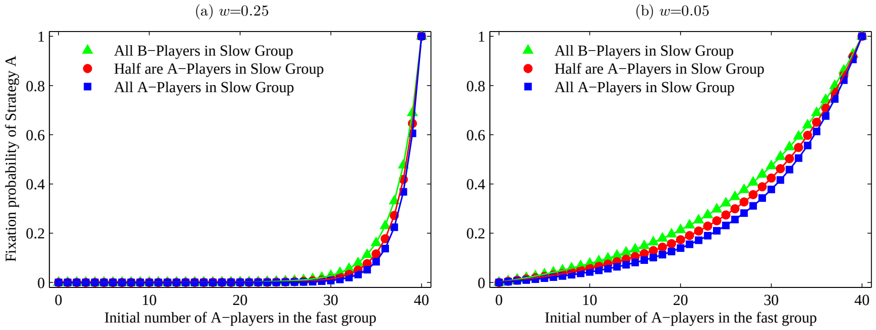
Under weak selection, strategy A performs better than neutral selection, if

$$(a-b-c+d)(1-\alpha)+3(b-d)>0. \tag{4.47}$$

In this case, natural selection favors a single mutant A to eventually replace a population of B-players. $\square$

So no matter whether the fast players converge to the absorbing state of all-A or all-B, the evolutionary outcome of the slow players is always affected by the relative sizes of fast and slow players which is further scaled by the payoffs.

Until now, we have shown that when the two subpopulations are decoupled, the timescales in updating play a important role for the evolution of strategies. To verify our theoretical results, we did computer simulations when the two subpopulations are partly decoupled. Here we show the results when the initial numbers of A-players are 1 (green lines), 20 (red lines), 39 (blue lines) and the values of s are 1,


(a) w=0.25

(b) w=0.05

Figure 4.1: Fixation probabilities for the game of dominance of strategy B with different selection intensity: (a) $\omega = 0.25$, and (b) $\omega = 0.05$. The payoff parameter values of Prisoner's Dilemma game are: $a = 3$, $b = 1$, $c = 5$, $d = 2$. The following settings are the same in Figs. 4.2 and 4.3. Computer simulation results (symbols) coincide perfectly with the approximation results (solid lines). The approximation results are from equation (4.10). Each simulation result corresponds to the average frequency of fixation of A-players from 100 independent realizations. Here, the results show that diversity of time scales on updating has only limited effects on the fixation of probabilities.

2, 5, 10, 50, 100. Each datum corresponds to the fraction of fixation of strategy A in 100 independent realizations. We find that the results are very similar for different values of s, so we do not show legends in the following figures. Our simulation results confirmed that our main conclusions in the main text are valid. The values of s do not have significant influences on the fixation probabilities. The fixation probabilities in any real situations are in the range of the two lines corresponding to all A-players and all B-players. Simulation results:

4.5 Conclusion

The origin of cooperation has been one of the hot spots in evolutionary biology for decades with natural selection in its kernel. In fact, selection frequencies may vibrate in a population. Our theoretical model is largely different from previous studies by introducing individual heterogeneity in their strategy-update time scales. Herein,

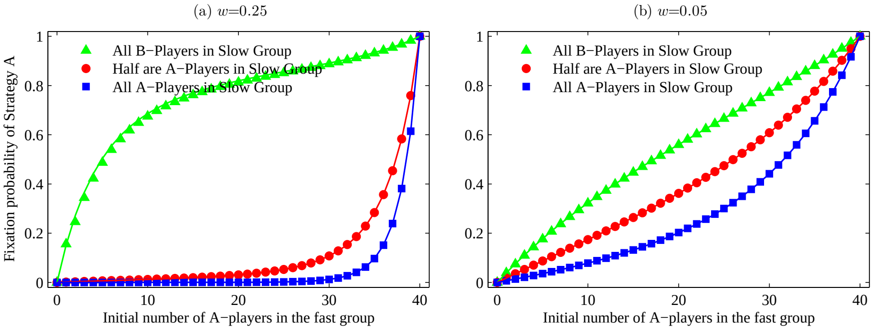


Figure 4.2: Evolutionary outcomes for games with stable coexistence of A and B. (a) $\omega = 0.25$, and (b) $\omega = 0.05$. The payoff parameter values of Snowdrift game are: $a = 3$, $b = 2$, $c = 5$, $d = 1$. Here, the results show that diversity of time scales on updating has significant effects on the fixation of probabilities. Specifically, more opposite strategies in the other group promote a strategy to get fixation in its own group.

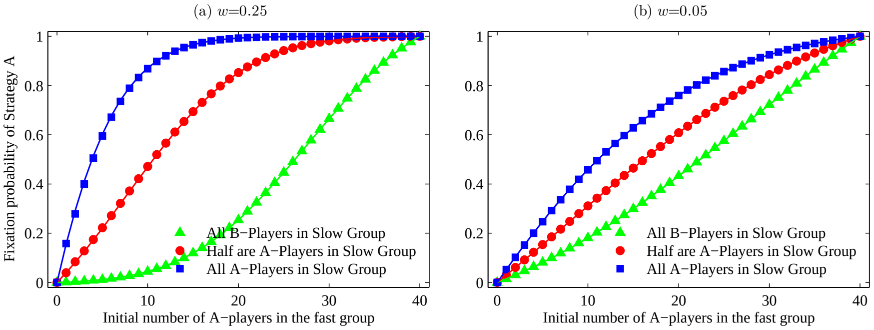


Figure 4.3: Evolutionary outcomes for coordination games. (a) $\omega = 0.25$, and (b) $\omega = 0.05$. The payoff parameter values of Stag-hunt game are: $a = 5$, $b = 1$, $c = 3$, $d = 2$. Here, the results show that diversity of time scales on updating has significant effects on the fixation of probabilities. But, unlike the results shown in Fig. 4.2, more opposite strategies in the other group inhibit a strategy to get fixation in its own group.

by introducing a crucial parameter s as the ratio between time scales of fast and low players, we are enabled to provide closed-form approximation for the evolutionary

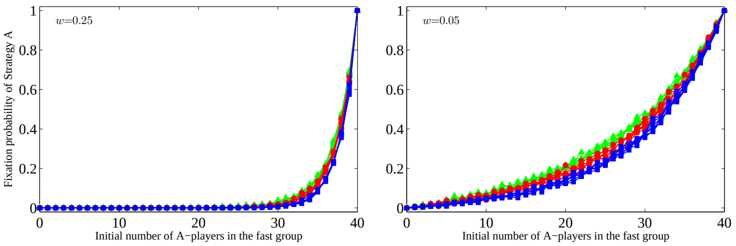


Figure 4.4: Simulation results of games with dominance of B.

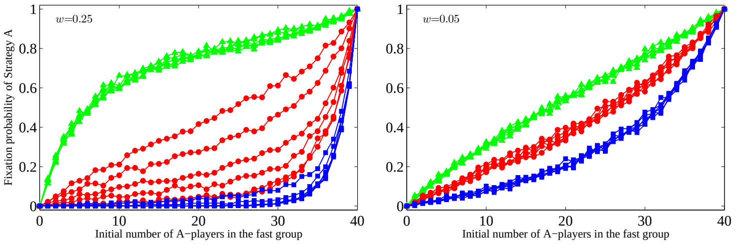


Figure 4.5: Simulation results of games with coexistence of A and B .

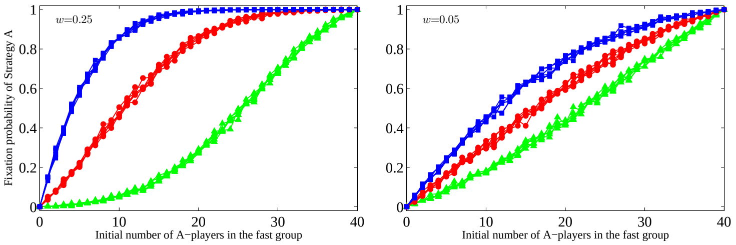


Figure 4.6: Simulation results of coordination games.

outcome of fast and slow groups when $s \to \infty$. One crucial step is that under the simplifying condition, the fast players always enter their absorbing states before the slow players start to update. This condition can be easily removed when one is only interested in simulation study and in fact our simulation results have indicated that the conclusion in the paper still holds when the fast and slow strategy-updating dynamics are coupled.

We have derived a sequence of approximation formulas that determine the fixation probabilities under a range of initial conditions. The difference in time scales leads to much richer evolutionary dynamics for typical two-player games, which underlines the importance and generality of our findings. We find that time scale diversity has different influences on different game models. Specifically, in the Prisoner's dilemma game where only defection is the dominant strategy, the fixation probabilities of strategies in one group change only slightly when the composition of strategies of the other group changes. In the snowdrift game which allows for stable coexistence of cooperators and defectors in well-mixed populations, the corresponding changes are much more significant. In the stag-hunt game in which each strategy is the best reply to itself, the corresponding changes are again significant, but the promoted strategy is the opposite of that in the snowdrift game. In addition to the approximation results, we study the evolutionary dynamics with different time scales under weak selection.

The result reported lends itself to multiple extensions. Up to now we have limited our study to the situation where only two types of players are considered, while multiple types are often more common in practical settings. Thus, investigating what happens in the presence of increasing diversity or even dynamically varying strategy-update frequencies is an intriguing topic to be studied in the future. Our model certainly does not explain all aspects of time scales in the context of evolutionary game theory and alternative definitions of time scales deserve further attention to look for plausible explanations for the individual heterogeneity and ultimately the persistence of cooperation.

Chapter 5

Effects of Insurance in Threshold Public Goods Games

In the previous mentioned chapters, we have introduced models with switching probabilities and diversity of time scales in strategy updating process. Our main motivation is to investigate the individual coexistence is common in nature. Importantly, the adopted models are general, and not only apply with specific scenarios. Then, in this chapter and the next chapter, we will introduce some results related to specific situations related with coexistence of individual diversity.

The occurrence and maintenance of cooperative behaviors in public goods systems have attracted great research attention across multiple disciplines. A threshold public goods game requires a minimum amount of contributions to be collected from a group of individuals for provision to occur. Here we extend the common binary-strategy combination of cooperation and defection by adding a third strategy, called insured cooperation, which corresponds to buying an insurance covering the potential loss resulted from the unsuccessful public goods game. Particularly, only the contributing agents can opt to be insured, which is an effort decreasing the amount of the potential loss occurring. Theoretical computations suggest that when agents face the potential aggregate risk in threshold public goods games, more contributions occur with increasing compensation from insurance. Moreover, permitting the adoption of insurance significantly enhances individual contributions and facilitates provision, especially when the required threshold is high. This work also relates the strategy competition outcomes to different allocation rules once the resulted contributions exceed the threshold point in populations nested within a dilemma.

5.1 Introduction

The origin and stability of cooperation is a hot subject in social and behavioural sciences (Axelrod 1984, Axelrod and Dion 1988). A complicated conundrum exists as defectors have an advantage over cooperators, whenever cooperation is costly and consequently, defection pays off. Therefore social dilemmas are situations in which the optimal decision of an individual contrasts with the optimal decision for the group (Hamilton 1963, Nowak et al. 1996, Sigmund and Nowak 1999). In the investigation of this plight the most prevailing framework is game theory together with its extensions involving evolutionary context (Fehr and Fischbacher 2003, Sigmund et al. 2010a, Conradt 2011, Nowak and Sigmund 2002, Zhang, Chen, Zhang, Wang and Chu 2010a).

Throughout evolution, crucial human activities like hunting for food, conserving common forestry or fisheries resources, and warfare, constitute public goods. In situations like these, each group member gains benefits from the goods, including those who pay no cost of providing the goods. This arouses the question of why characters regularly participate in costly cooperative activities like warfare and risky hunting. Perhaps one of the most frequently used multiple-agent-two-strategy models to describe the confusion of how cooperation arises is the public goods game (PGG) (Brandt et al. 2006, Semmann et al. 2003, Hauert et al. 2002b, Santos et al. 2008, Szolnoki and Perc 2010a). It focuses on the gains arising in multi-person interactive decision situations when probably a part of the population decide to cooperate (Croson and Marks 1998, Marks and Croson 1999, Marks and Croson 1998, Cadsby and Maynes 1999).

Quite a few solutions or mechanisms have been put forward to explain the perplexing problem of cooperation evolution. The kin selection theory focuses on cooperation among individuals that are genetically related, whereas theory of direct reciprocity emphasizes the selfish incentives for cooperation in bilateral long-term interactions (Nowak 2006, Ohtsuki and Nowak 2007, Pacheco et al. 2008). The theories of indirect reciprocity and signalling indicate how cooperation in larger groups can emerge when cooperators can build a reputation (Nowak and Sigmund 2005, Brandt and Sigmund 2005). Besides, punishment also plays a crucial role in the resolution of cooperative dilemma (Clutton-Brock and Parker 1995, Gurerk et al. 2006, Hauert et al. 2007, Sigmund 2007, Gächter et al. 2008). The integration of the microscopic patterns of interactions among the individuals composing a large population into the evolutionary setting, affords a way out for cooperators to survive in paradigmatic scenarios. A common framework is that each node in a graph carries one player and, edges determine who plays with whom (Barabási and Albert 1999, Nowak and May 1992, Zhang, Zhang, Xie and Wang 2011, Zhang, Zhang, Xie and Wang 2010).

Although the public goods game is deemed as one of the most common games in the study of the cooperation evolution, there are still some social dilemmas for which a different game would be a more appropriate model. In many cases of a collective action, the achieving of the group goal depends on the amount of common goods contributions. It is a common observation that many public goods contributed by collective actions are provided if contributions reach or exceed the required threshold of contributions; otherwise, no goods is provided (Milinski et al. 2008, Wang et al. 2009, Wu et al. 2013). Thus, a threshold public goods game requires a minimum amount of contributions to be raised from a group of individuals for provision to occur (Cadsby et al. 2007, Cadsby et al. 2008, Croson and Marks 2000). Researchers have examined how several factors, including incomplete information and identifiability of individual contributions, inhibit or foster successful public goods provision (Souza et al. 2009, Szolnoki and Perc 2010b, Archetti and Scheuring 2011, Chen, Szolnoki, Perc and Wang 2012, Santos and Pacheco 2011, Chen, Szolnoki and Perc 2012, Pacheco et al. 2009b, Boza and Számadó 2010).

Our previous work (Zhang et al. 2013) has introduced insurance against punishment and studied the roles of speculation adopted by defectors in public goods systems. Along this line, our aim here is to devise a scenario of evolutionary competition between three competing strategies, and study the roles of insurance for cooperators in the promotion of public cooperation. We are interested in the capacity of agents to contribute and produce the public goods when they are confronted with ambiguous risks or losses, meanwhile, facing the choice of being insured. In this threshold public goods model, agents can buy an insurance that sequentially covers the potential loss. We consider these aspects in an insurance deal, since the premium should not only be high enough to compensate the insurer for bearing the individual's risk, it should at the same time be low enough so that an individual is willing to insure her risk for this premium. Besides, if the threshold is not reached, contributions are not returned to the providers.

We add the insured cooperation as the third strategy to extend the individual strategy profiles originally consisting of cooperation and defection. These sets of hypotheses are generated from the motivation of our designing insurance in threshold public goods game. In the first place, everyday experience tells us that agents differ in personal features, such as the often-observed different economic status, or consciousness and demand for insurance in real world. When facing some potential loss, players may show heterogeneity in risk preferences. There is one more paramount point, cooperators will lose all their contributions when the group contribution falls short of the threshold. Intuitively, it is reasonable that the purchasers of insurance are cooperators since they are the altruistic contributors of the public goods activities and bear the risk of losing all their contributions. Therefore it is fair that they are provided the option of transferring their future loss to some insurance policy. In doing so, they could get some (part or full) compensation for their altruistic behaviors. In other words, the proposed insurance mechanism is provided as a means for encouraging those contributors and an effort deceasing the size of any loss occurring. Conversely, defectors can rest easy with no contribution for the generation of common goods. In this sense, it is meaningful to provide insurance choice only for contributors to avoid or decrease the unfavorable loss in this game setting. Finally, it is plausible that the insurance provider may be a profit management, and it will prefer cooperators over defectors as the object of insurance. The reason is that success accomplishment of public goods will help the insurance company save more benefits, otherwise it has to cover the loss for the insurers. Especially, the success provision of public goods is closely related to the number of cooperators. In this new framework, the two-strategy public goods game can be convincingly reframed as a cooperative dilemma among cooperators, insured cooperators and free riders.

The rest of the chapter proceeds as follows. In section 2 we describe the threshold public goods games with three strategies in the static context. Next, we present and discuss the main dynamic outcomes of the system, whereas conclusive remarks are given in the final section.

5.2 The model

In a typical threshold public good game (TPGG), each player in a group receives an endowment and individually decides how much of it to be contributed to a public goods system. If the group contribution exceeds a certain threshold, then the public goods is successfully provided and each player receives an equal reward, irrespective of her strategy. If the threshold is not reached, contributions are not returned to the players. Rational players intend to selfishly free ride on others' contributions, as contributors always benefit others at a cost to their benefits. Therefore, this rationale leads to social dilemmas and the predictable abandonment of the public goods.

To illustrate, suppose that in a finite population of size N ($N > 1$), individuals are provided with identical endowment c, and each must privately decide how much (between all and none here) of her endowment to contribute. After multiplying the accumulated contributions by r, each individual receives an identical benefit, if the required threshold T^* is reached by the group as a whole. Note that $rc < T^* < rcN$ so that it is impossible for the threshold to be reached based solely upon the contribution of one player, but it is possible for it to be attained based upon the contributions of more than one player (Cadsby and Maynes 1999).

As mentioned, when facing with potential loss, some cooperators prefer buying an insurance covering the possible loss and we call them the insured cooperators. Other cooperators may disregard this insurance and readily bear the potential loss, and they will be referred to as (common) cooperators. For the public goods game played by N players, both of the insured cooperators and common cooperators are contributors and their numbers are denoted by N_i (insured cooperators) and N_c (cooperators) respectively. Thus the population is composed of $N_i + N_c$ contributors and N_d free-riders.

Next, if the threshold is already achieved, how to define the payoff function of the participants gained from more contributions and provision is a crucial step. For the sake of generality, herein we consider two types of payoff functions that are plausible and conform to real situations for the study of cooperation, described in the following two scenarios respectively.

Scenario I: If the group contribution exceeds the required threshold, all the participants will henceforth share the fixed return T^*/N from the accomplished public goods game. And that is, contributions above the threshold point of provision are wasted. There exists a set of living examples conforming to this model setting, such as voting for building a public garden or dam. The neighborhood residents are asked to individually fill in a questionnaire, or vote, or petition the government to get the project approved. For example, whether the public project will be approved and built, depends on the amount of supporters and the required minimum numbers needed for successful action. The residents might not know how many signatures are needed to get the project built. In the example above, the project gets approved only if enough voters achieve the threshold, and excess signatures play a meaningless role in affecting the results.

Scenario II: If enough contributions are made to reach the stated threshold level of contributions, contributions above the provision point are not wasted, but result in further group benefit and thus more contributions are still meaningful. Herein we assume that the public goods is provided in an amount increasing with the aggregate level of contributions even though the specified threshold has already been met. The evenly distributed benefit is further assumed to be of the linear form $rc(N_c+N_i)/N$, where more contributors will provide larger benefits to the group. A large amount of meaningful and visual examples lend support to the above model setting. The more contributions are raised, the higher probability that a project will be successfully constructed. Returning to the earlier example, the neighborhood residents decide to build the public dam by voluntary contributions. The rates of successful provision and observed efficiency of the project are directly and positively related to the amount of contributions. Clearly, an effective dam requires a minimum of contributions to resist the invasion of flood. While if the required threshold is reached, more contributions exceeding the threshold still remain a significant role for a much more effective dam.

However, real-world dilemmas are typically not models with an obvious or clearly defined classification, and thus we combine these two scenarios with a variable ω as follows

$$U = \omega T^* + (1-\omega) rc(N_c + N_i). \tag{5.1}$$

By changing the parameter ω, our model allows to transverse smoothly from scenario I (i.e. $\omega = 1$) to scenario II (i.e. $\omega = 0$). In between the two extremes, we have a mixed situation of these two scenarios in the threshold public goods system.

Further, the payoff to an individual depends only upon her strategy and the combination of the strategies of her opponents. Each player chooses to contribute all or nothing. The proposed TPGG with three strategies has identical allocation rules with the common PGG if the public goods achieves the threshold: each player receives an equal amount of reward from the successful game, minus her own cost related to her strategies. As already stressed, the contributors within a TPGG group are composed by common cooperators (whose number is N_c) and insured cooperators (whose number is N_i). Let us now linger on the game dynamics of the investigated population.

And, we look at the situation the threshold of common goods is attained by

$$rc(N_c + N_i) \geq T^*, \tag{5.2}$$

where r denotes the amplification effect on the common pool, and T^* is the required threshold for the public goods provision to succeed.

Each player derives exclusively from the contributions provided by cooperators and insured cooperators, minus her cost to the common pool. For a group of size N probably consisting of the three characters (i.e. cooperator C, defector D and insured cooperator I), the payoffs of these three roles are specified as follows:

$$\begin{cases} P_c = \frac{U}{N} - c \\ P_d = \frac{U}{N} \\ P_i = \frac{U}{N} - c - \lambda \end{cases} . \tag{5.3}$$

The enhancement factor $r > 1$ means that if all cooperate, they are better off than if all defect. For a public goods game to deserve its name, $r < N$ should be satisfied, where each individual is better off defecting than cooperating. In this game, each unit of investment is multiplied by r and the resulting goods is distributed among all participants irrespective of their strategies. The first term in the expression represents the benefit that the agent obtains from the public goods, while the second term denotes her cost. For a cooperator, the cost is the investment c to the public goods. For an insured cooperator, the cost is the contribution c to the common pool and her payment λ to the insurance. Still, defectors withhold their share and exploit other players.

If the contributions are not sufficient to provide the public goods,

$$rc(N_c + N_i) < T^*, \tag{5.4}$$

the contributors lose their contributions and the goods is not provided finally. Thus, the net payoffs of the three strategies are determined by

$$\begin{cases} P_c = -c \\ P_d = 0 \\ P_i = \varepsilon - c - \lambda \end{cases} . \tag{5.5}$$

Compared with formula (5.3), each player is better off if the goods is provided than if it is not. For insured cooperators, they will be compensated by the insurance against the risk of 'wasting' their contributions on this unrealized project. Thus, the payoff advantages of defectors over insured cooperators depend on the involved parameters: the cooperative contribution c, the compensation ε ($\varepsilon > 0$) provided by the insurance, and the insurance cost λ. So, it is difficult to say whether those who do not contribute are better off than those who do contribute.

For simplicity and without loss of generality, we set the cooperative cost c from a contributor (either a cooperator or an insured cooperator) to 1. For $r > 0$, we can rewrite $rc(N_c + N_i) \geq T^*$ as $N - N_d \geq (T^*/r)$, and thus introduce $H = \text{ceil}[N - (T^*/r)]$. Notably, this ceiling function of H returns the smallest integer greater than or equal to $N - (T^*/r)$. Substituting the function H for T^* thus yields a simple judgment: $N_d < H$ leads to the success provision of the TPGG, and $N_d \geq H$ means the failure of the game. In the following study, we employ the threshold value H as the maximum number of defectors above which public goods game ends in failure. In this model, the resulting dynamics will be closely related to a variety parameters, as illustrated in Fig. 5.1 which provides some examples of the proposed TPGG.

5.3 Evolutionary dynamic outcomes

Here we posit a very large, well-mixed population of players. From time to time, sample groups of N such players are chosen at random and could join in a threshold public goods game. Notably, the probability that two players in large populations ever encounter again can be neglected. The probability that there are m defectors among the $N-1$ other agents in the sample population of size N in which a given player finds herself, is determined by

$$\binom{N-1}{m} x_d^m (1-x_d)^{N-1-m}. \tag{5.6}$$

This probability is independent of whether the agent is a contributor or a defector. x_d denotes the fraction of defectors in the population. The only determinant in the well-mixed population is the payoff that the agent herself receives. Consequently, the expected payoff for a defector in such a group is

$$P_d = \sum_{m=0}^{H-2} \frac{\omega T^* + (1-\omega) r (N-1-m)}{N} \binom{N-1}{m} x_d^m (1-x_d)^{N-1-m}. \tag{5.7}$$

The payoff of a cooperator is given by

$$\begin{aligned} P_c = \sum_{m=0}^{H-1} [\frac{\omega T^* + (1-\omega) r (N-m)}{N} - 1] \binom{N-1}{m} x_d^m (1-x_d)^{N-1-m} \\ + \sum_{m=H}^{N-1} (-1) \binom{N-1}{m} x_d^m (1-x_d)^{N-1-m}. \end{aligned} \tag{5.8}$$

The payoff of an insured cooperator will thus be

$$\begin{aligned} P_i = \sum_{m=0}^{H-1} [\frac{\omega T^* + (1-\omega) r (N-m)}{N} - 1 - \lambda] \binom{N-1}{m} x_d^m (1-x_d)^{N-1-m} \\ + \sum_{m=H}^{N-1} (\varepsilon - 1 - \lambda) \binom{N-1}{m} x_d^m (1-x_d)^{N-1-m}. \end{aligned} \tag{5.9}$$

Further, the advantage of one strategy over another depends on the payoff difference between them, below we will discuss the strategy competition results in detail.

Competition between strategy C and I:

5.3.1. THEOREM. *there are two interior roots on the edge of ID when $\phi_1(x_{d,1}) + \varepsilon - \lambda - 1 > 0$ and $\varepsilon - (\lambda + 1) < 0$, one interior root on the edge of ID when $\phi_1(x_{d,1}) + \varepsilon - \lambda - 1 = 0$ and $\varepsilon - (\lambda + 1) < 0$ or when $\phi_1(x_{d,1}) + \varepsilon - \lambda - 1 > 0$ and $\varepsilon - (\lambda + 1) > 0$, and* *no interior root on the edge of* ID *when* $\phi_1(x_{d,1}) + \varepsilon - \lambda - 1 < 0$*, where* $\phi_1(x_d) = \frac{[r(1-\omega)-N\varepsilon](N-H)}{N}\int_0^{1-x_d} t^{N-H-1}(1-t)^{H-1}dt + \frac{T^*}{N}\binom{N-1}{H-1}x_d^{H-1}(1-x_d)^{N-H}$ *and* $x_{d,1} = \frac{T^*(H-1)}{[r(1-\omega)-N\varepsilon](N-H)+T^*(N-1)}$.

Proof:

$$P_c - P_i = \lambda - \varepsilon\sum_{m=H}^{N-1}\binom{N-1}{m}x_d^m(1-x_d)^{N-1-m}. \tag{5.10}$$

Then we get $\lim_{x_d\to 0}(P_c - P_i) \approx \lambda > 0$, and $\lim_{x_d\to 1}(P_c - P_i) \approx (\lambda - \varepsilon) < 0$. Competition between strategy I and D:

$$\begin{aligned}P_i - P_d = \varepsilon - \lambda - 1 + \sum_{m=0}^{H-1}[\frac{r(1-\omega)}{N} - \varepsilon]\binom{N-1}{m}x_d^m(1-x_d)^{N-1-m}\\ +\frac{T^*}{N}\binom{N-1}{H-1}x_d^{H-1}(1-x_d)^{N-H}\\ = \varepsilon - \lambda - 1 + \frac{[r(1-\omega)-N\varepsilon](N-H)}{N}\int_0^{1-x_d}t^{N-H-1}(1-t)^{H-1}dt\\ +\frac{T^*}{N}\binom{N-1}{H-1}x_d^{H-1}(1-x_d)^{N-H}.\end{aligned} \tag{5.11}$$

By introducing $\phi_1(x_d)$, we can rewrite Eq. (5.11) as

$$P_i - P_d = \varepsilon - \lambda - 1 + \phi_1(x_d), \tag{5.12}$$

and hence,

$$\begin{aligned}\frac{d\phi_1(x_d)}{dx_d} = \frac{[r(1-\omega)-N\varepsilon](N-H)}{N}\binom{N-1}{H-1}[-x_d^{H-1}(1-x_d)^{N-H-1}]\\ +\frac{T^*}{N}\binom{N-1}{H-1}[(H-1)x_d^{H-2}(1-x_d)^{N-H} - (N-H)x_d^{H-1}(1-x_d)^{N-H-1}]\end{aligned} \tag{5.13}$$

Provided that $0 < x_d < 1$ holds, the above Eq. (5.13) keeps the same sign with $-[r(1-\omega)-N\varepsilon](N-H)x_d + T^*(H-1)(1-x_d) - T^*(N-H)x_d$. Resolving the equation $-[r(1-\omega)-N\varepsilon](N-H)x_d + T^*(H-1)(1-x_d) - T^*(N-H)x_d = 0$ yields

$$x_{d,1} = \frac{T^*(H-1)}{[r(1-\omega)-N\varepsilon](N-H) + T^*(N-1)}. \tag{5.14}$$

Consequently, both the maximum and minimum values of $\phi_1(x_d)$ exist, since $\phi_1(x_d)$ is continuous in $[0,1]$. Given that $\frac{d\phi_1(x_d)}{dx_d} = 0$ when $x_d = x_{d,1}$, $\frac{d\phi_1(x_d)}{dx_d} > 0$ if $x_d < x_{d,1}$ holds, and $\frac{d\phi_1(x_d)}{dx_d} < 0$ when $x_d > x_{d,1}$, $P_i - P_d$ reaches the maximum value at $x_{d,1}$. Then we can safely get $x_{d,1} = \frac{T^*(H-1)}{T^*(N-1)-N\varepsilon(N-H)}$ at $\omega = 1$, $x_{d,1} = \frac{T^*(H-1)}{T^*(N-1)+(r-N\varepsilon)(N-H)}$ at $\omega = 0$.

From Eq. (5.11) we get $\lim_{x_d \to 0}(P_i - P_d) \approx \frac{r(1-\omega)}{N} - \lambda - 1 < 0$, and $\lim_{x_d \to 1}(P_i - P_d) \approx \varepsilon - (\lambda + 1)$.

To sum up, there are two interior roots on the edge of ID when $\phi_1(x_{d,1}) + \varepsilon - \lambda - 1 > 0$ and $\varepsilon - (\lambda + 1) < 0$, one interior root on the edge of ID when $\phi_1(x_{d,1}) + \varepsilon - \lambda - 1 = 0$ and $\varepsilon - (\lambda + 1) < 0$ or when $\phi_1(x_{d,1}) + \varepsilon - \lambda - 1 > 0$ and $\varepsilon - (\lambda + 1) > 0$, and no interior root on the edge of ID when $\phi_1(x_{d,1}) + \varepsilon - \lambda - 1 < 0$. □

Competition between strategy C and D:

5.3.2. THEOREM. *there are two interior roots on the edge of CD when $\phi_2(x_{d,2}) > 1$, one interior root when $\phi_2(x_{d,2}) = 1$, and no interior root when $\phi_2(x_{d,2}) < 1$, where $\phi_2(x_d) = \frac{r(N-H)(1-\omega)}{N}\binom{N-1}{H-1}\int_0^{1-x_d} t^{N-H-1}(1-t)^{H-1}dt + \frac{T^*}{N}\binom{N-1}{H-1}x_d^{H-1}(1-x_d)^{N-H}$ and $x_{d,2} = \frac{T^*(H-1)}{r(1-\omega)(N-H)+T^*(N-1)}$.*

Proof: In analogy to the above methods, the sign of $P_c - P_d$ determines whether it pays to switch from defection to cooperation or vice versa, with $P_c - P_d = 0$ being the equilibrium condition. Fig. 5.2 illustrates three examples with respect to T^*, to help depicting the complicated situations of $P_c - P_d$.

$$P_c - P_d = -1 + \sum_{m=0}^{H-1} \frac{r(1-\omega)}{N}\binom{N-1}{m} x_d^m (1-x_d)^{N-1-m} + \frac{T^*}{N}\binom{N-1}{H-1} x_d^{H-1}(1-x_d)^{N-H}. \tag{5.15}$$

By employing

$$\phi_2(x_d) = \frac{r(N-H)(1-\omega)}{N}\binom{N-1}{H-1}\int_0^{1-x_d} t^{N-H-1}(1-t)^{H-1}dt + \frac{T^*}{N}\binom{N-1}{H-1} x_d^{H-1}(1-x_d)^{N-H}, \tag{5.16}$$

Eq. (5.15) can be reduced to

$$P_c - P_d = -1 + \phi_2(x_d). \tag{5.17}$$

Next,

$$\begin{aligned}\frac{d\phi_2(x_d)}{dx_d} &= -\frac{r(1-\omega)(N-H)}{N}\binom{N-1}{H-1}[-x_d^{H-1}(1-x_d)^{N-H-1}] \\ +\frac{T^*}{N}\binom{N-1}{H-1}&[(H-1)x_d^{H-2}(1-x_d)^{N-H}-(N-H)x_d^{H-1}(1-x_d)^{N-H-1}] \\ &= \binom{N-1}{H-1}x_d^{H-2}(1-x_d)^{N-H-1}[-\frac{r(1-\omega)(N-H)}{N}x_d \\ &\quad +\frac{T^*}{N}(H-1)(1-x_d)-\frac{T^*}{N}(N-H)x_d] \end{aligned} \tag{5.18}$$

$0 < x_d < 1$ helps the Eq. (5.18) keep the same sign with $-r(1-\omega)(N-H)x_d + T^*(H-1)(1-x_d) - T^*(N-H)x_d$. Then,

$$-r(1-\omega)(N-H)x_d + T^*(H-1)(1-x_d) - T^*(N-H)x_d = 0 \tag{5.19}$$

gives rise to

$$x_{d,2} = \frac{T^*(H-1)}{r(1-\omega)(N-H)+T^*(N-1)}. \tag{5.20}$$

$P_c - P_d = -1$ when $x_d = 0$, and $P_c - P_d = -1$ when $x_d = 1$. Similarly, $\phi_2(x_d)$ is a continuous function in the interval of $[0,1]$, and thus both the maximum and minimum values of $\phi_2(x_d)$ can be found. Considering that $\frac{d\phi_2(x_d)}{dx_d} > 0$ if $x_d < x_{d,2}$, and $\frac{d\phi_2(x_d)}{dx_d} < 0$ if $x_d > x_{d,2}$, $P_c - P_d$ reaches its maximum value at $x_{d,2}$. In this case, $\omega = 1$ leads to $x_{d,2} = \frac{H-1}{N-1}$, and $\omega = 0$ results in $x_{d,2} = \frac{T^*(H-1)}{r(N-H)+T^*(N-1)}$. It thus follows that: there are two interior roots on the edge of CD when $\phi_2(x_{d,2}) > 1$, one interior root when $\phi_2(x_{d,2}) = 1$, and no interior root when $\phi_2(x_{d,2}) < 1$. □

In the continuous time model, the evolution of the fractions of the three strategies are given by

$$\dot{x}_k = x_k(P_k - \bar{P}), \tag{5.21}$$

where k can be c, d, i, and $\bar{P} = x_cP_c + x_dP_d + x_iP_i$. Now consider some typical possible cases of different parameters and the resulting game dynamics one by one, pointed out by Fig. 5.3.

Case 1 ($\varepsilon - \lambda - 1 + \phi_1(x_{d,1}) < 0, \phi_2(x_{d,2}) - 1 < 0$): In this case, full defection equilibrium (D) is the only stable and a global attractor. For an insured cooperator, her contribution for common goods and the cost for insurance cannot be totally reimbursed and thus she suffers negative payoffs if the threshold is not reached. Each individual has an incentive to free ride for the higher payoffs, and thus the dominant strategy equilibrium in case 1 is the defection.

Case 2 ($\varepsilon - \lambda - 1 + \phi_1(x_{d,1}) > 0$, $\varepsilon - \lambda - 1 > 0$, and $\phi_2(x_{d,2}) - 1 < 0$): Herein, there is a border equilibrium consisting of insured cooperation and defection. And this equilibrium is stable and a global attractor. In comparison with case 1, the compensation ε from insurance is increased and the resulted $\varepsilon < \lambda + 1$ will foster the survival of insured cooperators gaining higher payoffs than defectors. Thenceforth, larger compensation provided by insurance will stimulate more contributors to jointly produce the threshold public goods when they face the ambiguous risks and losses.

Case 3 ($\varepsilon - \lambda - 1 + \phi_1(x_{d,1}) < 0, \phi_2(x_{d,2}) - 1 > 0$): In this case, there are two border equilibrium points consisting of cooperation and defection. The one close to the full cooperation is a stable equilibrium and the other near full defection is unstable. In comparison with case 1, the increasing threshold T^* leads to two stable equilibria here: full defection and the coexistence of cooperation and defection. Which equilibrium the system will evolve to depends on the initial states of the population.

Case 4 ($\varepsilon - \lambda - 1 + \phi_1(x_{d,1}) > 0, \phi_2(x_{d,2}) - 1 > 0$): In this case, there are two stable border equilibria: one consisting of cooperation and defection, and the other consisting of insured cooperation and defection. In comparison with case 2, the increment of required threshold T^* results in the two equilibria on the edge of CD here. Similar to case 3, lager ω will propel the equilibrium point on the edge of CD to approach to the point of pure defection. We offer an accessible explanation of this observation: larger ω implies a bigger competitive advantage of defectors over cooperators based on payoffs, which is essential for the stability of the competing strategies.

Summarizing the four cases above, we can conclude that the insurance guarantee for contributive behaviors encourages contributions and provision, but in a manner which interacts with both the required threshold and the reimbursed compensation from insurance. Results presented above show that larger required threshold T^* helps contributors gain more advantages in payoffs than free riders, therefore improving the provision for public goods. In addition, increasing the compensation ε

from insurance also dramatically alters the dynamic outcomes of the game. Defectors reap the benefit of the common goods without any contribution into it, which inhibits the spread of contributive behaviors. Although defectors always do better than cooperators in the public goods games with binary contributions, insurance proposed here can offer the possibility for contributors of receiving higher payoffs than defectors, and so contributors will increase. The insurance reduces or removes the risk that contributions made towards the public goods will be lost if the threshold is not attained. Supported by sufficiently high compensation ε, contributors can avoid extinction by the potential payoff advantages over defectors, or even the possible dominance of the population. It is also worth emphasizing that the allocation rules (adjusted by ω here) of the public goods after the contributions reach the threshold point, also act as a focal point for survival of cooperation. Smaller ω enhances the payoff advantages of contributors over defectors and hence cooperation thrives in our model. Hence, the insurance guarantee encourages contributions and provision in threshold public goods games, and suggests a positive role in unriddling the bewilderment of the 'Tragedy of the Commons'.

5.4 Conclusions

In the threshold public goods game, public goods are provided if the joint contributions meet or exceed a predetermined threshold level of provisions; otherwise, no public goods is provided. With the existence of the potential risks, we are interested in the capacity of agents to contribute and to produce the public goods when they can opt to be insured at some cost. Therefore individuals joining the game are provided with three strategy options: cooperation, defection and insured cooperation. Here, the public goods is provided in a threshold fashion with a predetermined threshold T^*: if the accumulated contributions reach or exceed T^* then the public goods is provided, otherwise it is not. In addition, the public goods are allocated according to two different rules if the contributions exceeds the threshold: fixed value or a linear form of contributors. In this model our attention is paid to relating individual contributions in threshold public goods game to riskiness and risk aversion mechanisms.

Theoretical computations show that the evolutionary dynamics are intrinsically regulated by the game parameters specified by the proposed insurance choice. We demonstrate that compensation from insurance is of crucial importance for stabilizing cooperation among competing strategies. Larger compensation will tempt more agents to contribute, thus inhibiting the spread of free riding behavior. Further, increasing the threshold can also elicit more contributions to the threshold public goods game. And, the allocation rules of the public goods after the contributions catch up with the threshold point, also notably affect the final results.

Researchers are often intrigued by employing public goods games to simulate collective dilemmas existing in the real world. In this endeavor, incorporating features of the real-world dilemma into the game also deserves attention. Our work is therefore a potential remedy to collective cooperation problem nested within a dilemma when cooperators are provided with some insurance, implying that the insurance for competing strategies deserves more attention in theoretical and empirical studies. The work reported here also lends itself to multiple extensions. An immediate one, for example, would be improving the theoretical validity of the study here by introducing insurance in experimental research. One feasible experimental research is to conceive of a threshold public goods game with more complicated forms or functions (usually nonlinear) of insurance. Moreover, it would be interesting to see whether insurance provided for both cooperators and defectors in the populations can foster cooperation. For instance, the volunteers in experiments can face multiple actions (e.g., cooperation, defection, insured cooperation and insured defection). In this way, we can gain a thorough understanding about the roles of insurance in the real-life collective dilemmas: such as the construction of some public projects and in other cases where a public good needs to be provided. A closer look at the nature of insurance in situations that are called collective dilemmas can foster the advancement of our understanding of cooperative and selfish behaviors. Hence, learning how insured agents can forgo individual interests for collective interest is useful for understanding social behaviors and developing social policy.

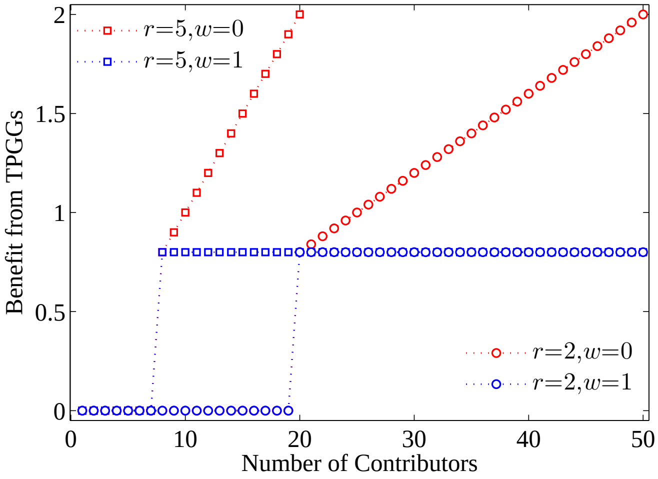
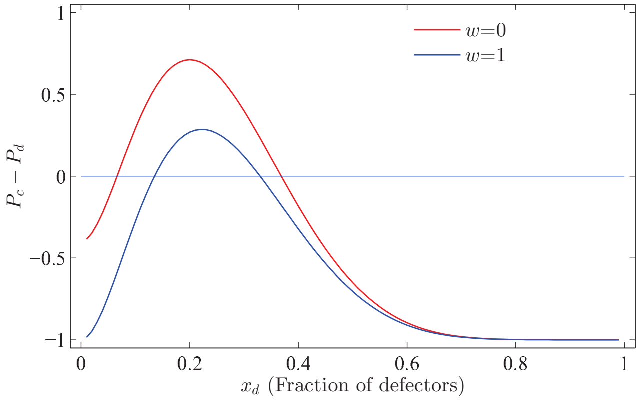
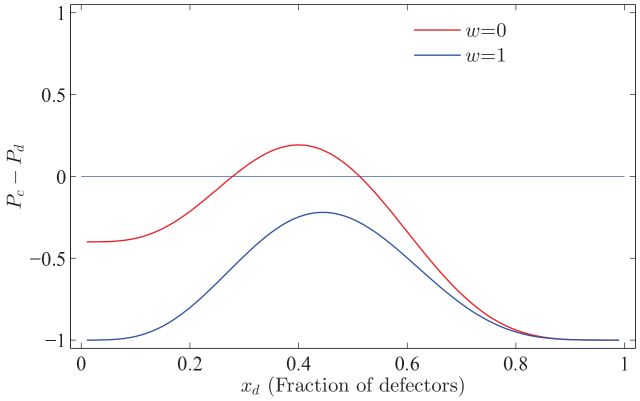


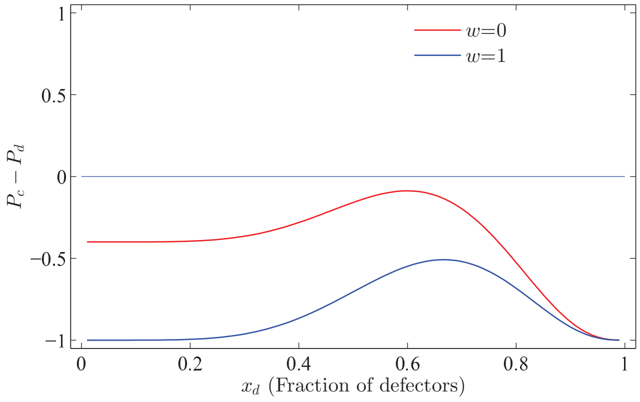
Figure 5.1: Diagrams illustrating four examples of TPGG, whose dynamics outcomes are closely related to the model parameter involved. Parameters here: $N = 50, T^* = 40$. The x-axis is indexed by the number of contributors (including cooperators and insured cooperators), and the y-axis represents the individual benefits from TPGG. Results show that, when $r = 5$, individuals can gain positive benefits from TPGG if there are at least 8 contributors. When $r = 2$, at least 20 contributors in one TPGG are needed to bring each participant with positive benefits. As mentioned, varying the parameter ω can transverse the model smoothly from scenario I (i.e. $\omega = 1$) to scenario II (i.e. $\omega = 0$) about the payoff functions in the TPGG after the threshold point has already been reached. In between the two extremes, we obtain a mixed situation of the payoff distribution rules in the threshold public goods system.



(a) $N = 10$, $r = 6$, $T^* = 40$.

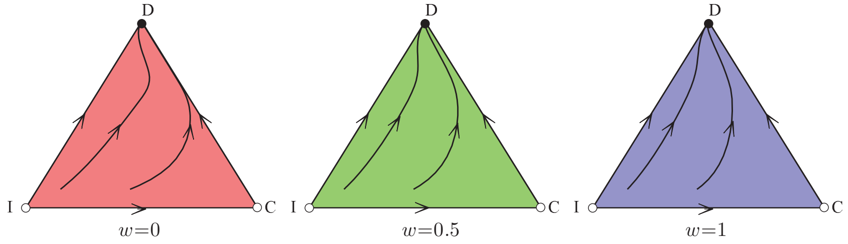


(b) $N = 10$, $r = 6$, $T^* = 30$.

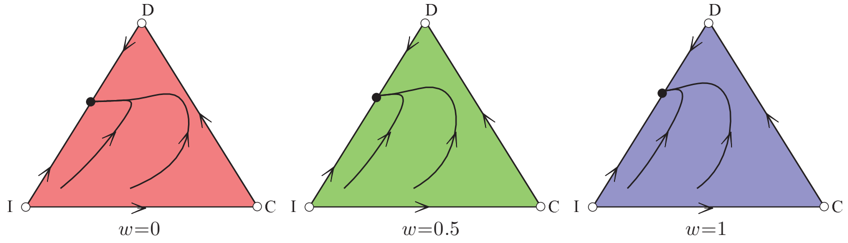


(c) $N = 10$, $r = 6$, $T^* = 15$.

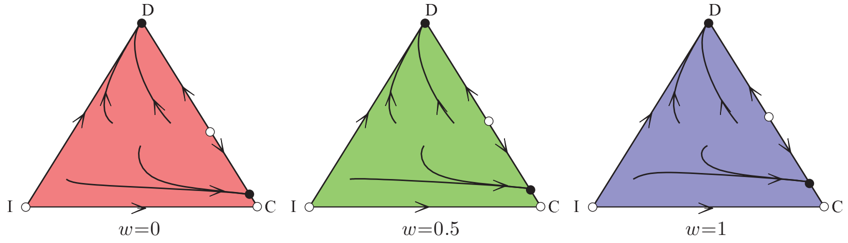
Figure 5.2: (Color online) Examples illustrating the payoff difference $P_c - P_d$ between cooperators P_c and defectors P_d, which is closely related to the required threshold T^*. Lines connecting the symbols are just to guide the eye. The mentioned examples suggest that the possible roots of the $P_c - P_d$ will be: none, a unique or two roots situated in the interval (0,1).



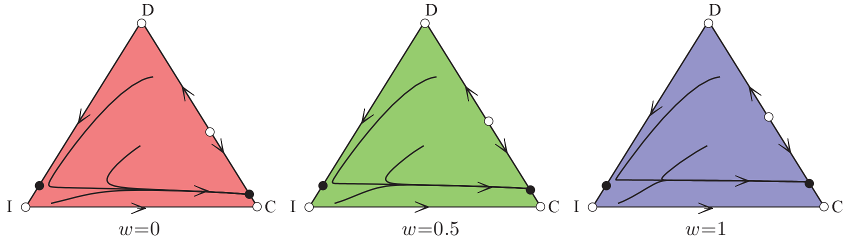
(a) Case 1: $N = 10$, $r = 6$, $\lambda = 0.6$, $\varepsilon = 1.2$, $T^* = 15$.



(b) Case 2: $N = 10$, $r = 6$, $\lambda = 0.6$, $\varepsilon = 4$, $T^* = 15$.



(c) Case 3: $N = 10$, $r = 6$, $\lambda = 0.6$, $\varepsilon = 1.2$, $T^* = 40$.



(d) Case 4: $N = 10$, $r = 6$, $\lambda = 0.6$, $\varepsilon = 4$, $T^* = 40$.

Figure 5.3: (Color online) The dynamic outcomes under different cases. The corners C (cooperation), D (defection), and I (insured cooperation) are equilibrium pints. Open dots are unstable equilibrium points and closed dots are stable equilibrium points. In case 1, full D is the only stable equilibrium while in the other three cases, other strategies may be the dominative ones. Therefore, we can conclude that our model promotes contribution by adding the third strategy: insured cooperation.

Chapter 6

Effects of Opting Out and Insurance in Public Goods Games

Self-interest frequently causes individuals engaged in joint enterprises to choose actions that are counterproductive. Free-riders can invade a society of cooperators, causing a tragedy of the commons. In our previous works, we have investigated the influences of buying a policy that sequentially covers all punishment costs on the evolution of cooperation in public goods games with potential punishment on defectors, illustrating that insurance against punishment does not destabilize cooperation under realistic assumptions. There is compelling evidence that voluntary participation are effective mechanisms in ascertaining the evolution and stability of cooperation. As an extension form of evolutionary public goods game, competition among cooperative, defective with probabilistic punishment, speculation insured by some policy, and loner strategies is investigated here. By means of an evolutionary game theoretical approach, results suggest that our model displays complex dynamic behaviors. Depending on the initial condition, the state converges either to a domination of cooperators, or to a rock-scissors-paper type heteroclinic cycle of three strategies. Our model is, therefore, expected to shed light on the role of voluntary participation and speculation in solving the befuddling problem about the emergence of cooperative behaviors.

6.1 Introduction

Situations in which the private interest can be at odds with the public interest constitute an important class of societal problems. Evolutionary game theory is an interdisciplinary mathematical tool which seems to be able to embody several relevant features of the problem and, as such, is used in much cooperation-oriented research. In particular, the oft-cited public goods game (Hardin 1968, Axelrod and Dion 1988, Heckathorn 1996, Fehr and Gächter 2002, Brandt et al. 2006, Hauert et al. 2002b) is a paradigm example for investigating the emergence of cooperation in spite of the fact that self-interest seems to dictate defective behavior.

In typical public goods games, the so-called social dilemmas can be considered as binary situations in which two strategies are available: either choose alternative cooperation (C) in order to serve the public interest, or choose alternative defection (D), which serves the immediate private interest. The individual contributions are multiplied by a factor r and then divided equally among all players. With r smaller than the group size, this is an example of a riddle from the evolutionary viewpoint: individuals who do not contribute, but exploit the public goods, fare better than those who pay the cost by contributing. Thus, natural selection favors defection and leads to a social dilemma, because when all defect the mean payoff is lower than that when all cooperate.

A variety of solutions for this dilemma have been discussed in the past studies. The theory of kin selection focuses on cooperation among individuals that are genetically closely related, whereas theories of direct reciprocity focus on the selfish incentives for cooperation in bilateral long-term interactions (Imhof and Nowak 2010, Nowak 2006, Ohtsuki and Nowak 2007, Pacheco et al. 2008). The theories of indirect reciprocity and costly signalling indicate how cooperation in larger groups can emerge when the cooperators can build a reputation (Nowak and Sigmund 2005, Berger 2011, Brandt and Sigmund 2005). Current research has also highlighted two factors boosting cooperation in public goods interactions, namely, punishment of defectors (Gurerk et al. 2006, Hauert et al. 2007, Sigmund 2007, Brandt et al. 2003, Helbing et al. 2010, Gächter et al. 2008, Henrich 2006) and the option to abstain from the joint enterprise. Voluntary participation (Hauert et al. 2002b, Hauert et al. 2002a) allows individuals to adopt a risk-aversion strategy, termed loner. A loner refuses to participate in unpromising public enterprises and instead relies on a small but fixed payoff.

A strong body of theoretical and empirical evidence points to the importance of punishment as a major factor for sustaining cooperation in public goods games (Sigmund et al. 2010b, Fehr and Gachter 2005, Fowler et al. 2005). In addition, our previous work (Zhang et al. 2013) has studied a simple model to investigate the question whether stable cooperation can break down in the presence of speculation, a kind of risk-aversion strategy. Results indicate scenarios where speculation either leads to the reduction of the basin of attraction of the cooperative equilibrium or even the loss of stability of this equilibrium, if the costs of the insurance are lower than the expected fines faced by a defector. We reach the conclusion that an insurance of this type is not viable and under realistic assumptions speculation does not destabilize cooperation.

However, to our knowledge, past research paid little attention to the joint roles of punishment, voluntary participation and speculation in affecting the public goods provision. Actually, agents often have multiple choices in decision making due to the individual personality, especially when facing the potential punishment if defecting. Agents probably perform different behaviors due to the often observed different consciousness of risk prevention in real world. For example, resolute defectors will persist in their defection strategy, though taking the risk of being punished with a probability. Speculators incline to buy an insurance policy covering the costs of punishment when caught defecting. While timid loners will conservatively obtain an autarkic income independent of the other players' decision. These mentioned choices can better represents the possible attempts to raise money for public goods in complicated real-life situations. Thus, in most biological scenarios, the heterogeneity of competing individuals is an irrefutable fact and multiple choices is undeniably a part of biological reality. With this formulation, as an extension of our previous work proposing speculation (Zhang et al. 2013), we add the fourth strategy, called loner, which can refuse to participate and get some small but fixed income. As mentioned, it is based on the assumption that players can voluntarily decide whether to participate in the joint enterprise or not.

The four behavioral types in the population are: (a) the cooperators ready to join the group and to contribute their effort, (b) the defectors who join, but do not contribute, moreover, defectors are caught with a certain probability and a fine is imposed on them when caught. Here we are less interested in the specific establishment of an effective system of punishment, but the two additional options (speculation and loner) found in several systems. To address this question, we consider a public goods game with an external-agency punishment system indicated above.

(c) the speculators who purchase an insurance policy covering the costs of punishment when caught defecting. It means that by paying a fixed cost for their insurance policy, speculators can defect without paying any fine from punishment. (d) the loners unwilling to join the public goods game, but prefer to rely on a small but fixed payoff.

By means of a theoretical approach, we investigate the joint evolution of cooperation, defection, speculation and loner, focusing on the question whether such model will subsequently allow the stable establishment of sizable levels of cooperation.

6.2 The model setting

Our investigation is based on the public goods game (PGG), a paradigm to study the evolution of costly cooperation among selfish individuals, since it highlights the potential differences between individual interests and the social optimum. In the standard, obligatory public-goods model, the social dilemma can be considered as binary situations in which two strategies are available: either choose alternative C (cooperation) in order to serve the public interest, or choose alternative D (defection), which serves the immediate private interest.

To model this scenario with four strategies by evolutionary game theory, we assume a large population consisting of cooperators, defectors, speculators, and loners. To be precise, each participant receives an equal benefit rcx_c which is proportional to the fraction of cooperators (x_c) among the players. The costs associated with behaviors differ among strategies. Cooperators pay a fixed cost c as the contribution for the public goods game. Defectors contribute nothing, but will be possibly caught and then confronted with punishment. Their expected fine is α, which reflects the product of the probability of being detected and the fine in cost of detective. Speculators neither contribute for common goods nor pay a fine when caught, instead they pay an amount λ corresponding to the insurance policy. Loners obtain a fixed pay-off σ from a solitary pursuit without contribution.

We consider a very large, well-mixed population of players. From time to time, sample groups of N such players are chosen randomly and offered to join in a public goods game. Notably, the probability that two players in large populations ever encounter again can be neglected.

Within such a group, if N_c denotes the number of cooperators and N_l is the number of loners among the public goods players, the net payoffs of the four strategies are respectively given by

$$\begin{cases} P_c = \frac{rcN_c}{N-N_l} - c \\ P_d = \frac{rcN_c}{N-N_l} - \alpha \\ P_s = \frac{rcN_c}{N-N_l} - \lambda \\ P_l = \sigma \end{cases}, \tag{6.1}$$

where r denotes the amplification effect on the common pool. In this game, each unit of investment is multiplied by r and the product is distributed among all participants (except loners) irrespective of their strategies. The first term in the expression represents the benefit that the agent obtains from the public goods, while the second term denotes cost. For cooperators, the cost is the investment c to the public goods, and for speculators, the cost is the payment λ to the insurance. Selfish individuals will therefore always avoid the cost of altruism, i.e. a collective of selfish players will never cooperate.

In order to compute the payoff values for cooperators, defectors and speculators, we first derive the probability that n of the N sampled individuals are actually willing to join the public goods game. In the case $n = 1$ (no co-player shows up) we assume that the player has no other option than to play as a loner, and obtains payoff σ. This happens with probability x_l^{N-1}. Here, x_l is the fractions of loners. For a given player (C, D or S) willing to join the public goods game, the probability of finding, among the $N-1$ other players in the sample, $n-1$ co-players joining the group ($n > 1$), is given by

$$\binom{N-1}{n-1}(1-x_l)^{n-1}(x_l)^{N-n} \tag{6.2}$$

The probability that m of these players are cooperators is

$$\binom{n-1}{m}(\frac{x_c}{x_c+x_d+x_s})^m(\frac{x_d+x_s}{x_c+x_d+x_s})^{n-1-m} \tag{6.3}$$

where x_c, x_d, x_s respectively denote the fractions of cooperators, defectors and speculators in the population.

For simplicity and without loss of generality, we set the cost c of cooperation equal to 1. In the above case, the payoff for a defector is $rm/n - \alpha$, while the payoffs for a cooperator and a speculator are respectively specified by $r(m+1)/n-1$ and $rm/n-\lambda$. Hence, the expected payoff for a defector in such a group is:

$$\begin{aligned}&(\frac{rm}{n}-\alpha)\sum_{m=0}^{n-1}\binom{n-1}{m}(\frac{x_c}{1-x_l})^m(1-\frac{x_c}{1-x_l})^{n-m-1}\\&=\frac{r}{n}\cdot(n-1)\frac{x_c}{1-x_l}-\alpha\end{aligned}$$

The payoff of a cooperator in a group of n players is:

$$\begin{aligned}&[\frac{r(m+1)}{n}-1]\sum_{m=0}^{n-1}\binom{n-1}{m}(\frac{x_c}{1-x_l})^m(1-\frac{x_c}{1-x_l})^{n-m-1}\\&=\frac{r}{n}\cdot(n-1)\frac{x_c}{1-x_l}+\frac{r}{n}-1\end{aligned}$$

The payoff of a speculator in a group of n players is:

$$\begin{aligned}&(\frac{rm}{s}-\lambda)\sum_{m=0}^{N-1}\binom{n-1}{m}(\frac{x_c}{1-x_l})^m(1-\frac{x_c}{1-x_l})^{n-m-1}\\&=\frac{r}{n}\cdot(n-1)\frac{x_c}{1-x_l}-\lambda\end{aligned}$$

The payoff of a loner is the constant value of σ.

Then, the expected payoff for a defector in the population is,

$$\begin{aligned}P_d&=\sigma x_l^{N-1}+\sum_{n=2}^{N}[\frac{r}{n}\cdot(n-1)\frac{x_c}{1-x_l}-\alpha]\binom{N-1}{n-1}\\&\qquad(1-x_l)^{n-1}(x_l)^{N-n}\\&=\sigma x_l^{N-1}+\frac{rx_c}{1-x_l}[1-\frac{1-x_l^N}{N(1-x_l)}]-\alpha(1-x_l^{N-1})\end{aligned}\tag{6.4}$$

In the continuous time model, the evolution of the fractions of the four strategies proceeds according to

$$\dot{x}_i=x_i(P_i-\bar{P}),\tag{6.5}$$

where i can be c,d,s,l, and $\bar{P}=x_cP_c+x_dP_d+x_sP_s+x_l\sigma$.

6.3 Evolutionary dynamics outcomes

We firstly focus on the replicator dynamics starting from a three-strategy state in the population, then we pay attention to analyzing the output when all the four strategies initially exist in the population.

For the replicator dynamics of three-strategy evolution, we comprehensively consider four scenarios depicted in Fig. 1 till Fig. 4 as follows. The advantage of one strategy over another depends on the payoff difference between them, hence

$$\begin{aligned}
P_d - P_c &= \sum_{n=2}^{N}[1 - \frac{r}{n} - \alpha]\binom{N-1}{n-1}(1-x_l)^{n-1}(x_l)^{N-n} \\
&= 1 - \alpha + (r - 1 + \alpha)x_l^{N-1} - \frac{r}{N}\frac{1-x_l^N}{1-x_l}; && (6.6) \\
P_d - P_s &= \sum_{n=2}^{N}[\lambda - \alpha]\binom{N-1}{n-1}(1-x_l)^{n-1}(x_l)^{N-n} \\
&= (\lambda - \alpha)(1 - x_l^{N-1}); && (6.7) \\
P_s - P_c &= 1 - \lambda + (r - 1 + \lambda)x_l^{N-1} - \frac{r}{N}\frac{1-x_l^N}{1-x_l}. && (6.8)
\end{aligned}$$

□

In the above calculations, $N > 1$, $1 < r < N$ and $\alpha > 0$. The sign of $P_i - P_j$ in fact determines whether it pays to switch from cooperation to defection or vice versa, $P_i - P_j = 0$ being the equilibrium condition, where i, j can be strategy C, D, S, and L.

We now proceed to the study of evolutionary dynamics when $\lambda \neq \alpha$ where four strategies coexist in the population, being referred to an interior point. We make the following three assumptions and will get the results that: at least one strategy will become extinct with the evolution of the system initialized from an interior point.

6.3.1. THEOREM. *If $\lambda \neq \alpha$, at least one strategy will become extinct with the evolution of the system initialized from an interior point. Here, an interior point means that the fraction of every strategy is larger than zero.*

Proof: We now analyze the system in different situations.
(1) when $\lambda \neq \alpha$, supposing $\lambda > \alpha$ (i.e. $P_d > P_s$), when $x_l \neq 0$. We suppose that there

is a closed set, meaning that the subsequent evolving state of each initial state in this set also belongs to this set. So $x_c > 0$, $x_d > 0$, $x_s > 0$ and $x_l > 0$ in this closed set.

(1.1) We first suppose only one point $(x_c^*, x_d^*, x_s^*, x_l^*)$ in this closed set, and satisfying $x_c^* > 0, x_d^* > 0, x_s^* > 0, x_c^* > 0$, and $\dot{x}_c^* = \dot{x}_d^* = \dot{x}_s^* = \dot{x}_l^* = 0$, thus

$$\begin{cases} \dot{x}_d^* = x_d^*(p_d^* - \bar{p}^*) \\ \dot{x}_s^* = x_s^*(p_s^* - \bar{p}^*) \end{cases} \tag{6.9}$$

Herein, the result $\dot{x}_d^* = \dot{x}_s^* = 0$ needs $\dot{p}_d^* = \bar{p}^* = \dot{p}_s^*$, which contradicts with $\dot{p}_d^* - \dot{p}_s^* > 0$. Therefore we can safely get the conclusion that there is no interior stable point.

(1.2) We next assume that the interior domain is a limit cycle. In this case, the four strategy players will gain the same average payoffs driven by the replicator equation, where $\bar{p}_c = \bar{p}_d = \bar{p}_s = \bar{p}_l$. However, $\bar{p}_d = \bar{p}_s$ contradicts with $p_d > p_s$, indicating that the closed set is not a limit cycle.

(1.3) We then verify whether the interior domain is a chaos, where also $x_c > 0, x_d > 0, x_s > 0, x_l > 0$. By introducing the fraction of defections in a population consisting of defectors and speculators, $f = \frac{x_d}{x_d + x_s}$, thus

$$\dot{f} = (\frac{x_d}{x_d + x_s})' = \frac{\dot{x}_d x_s - x_d \dot{x}_s}{(x_d + x_s)^2} = \frac{x_d x_s (p_d - p_s)}{(x_d + x_s)^2} > 0. \tag{6.10}$$

Then, $\lim_{t \to \infty}(\frac{x_d}{x_d + x_s}) = 1$ and $x_s \to 0$.

The above mentioned results suggest that, when $\lambda > \alpha$ there is no such a closed set, in which the evolving state of each initial state which consist of these four strategies in this set also belongs to this set.

(2) When $\lambda < \alpha$ and according to the results in (1), there is no internal domain.

(3) When $\lambda = \alpha$ and thus $p_d = p_s$, the four-strategy system was reduced to the simplex $T = (C, D, L)$ or $T = (C, S, L)$. We will discuss this situation in the following.

Summing up the above dynamics, we can safely get the following conclusions: $\lambda = \alpha$ reduce the system to a three-strategy game, and $\lambda \neq \alpha$ will lead to the distinction of at least one strategy.

6.3.1 Scenario 1: the corners of the simplex $T = (C, D, L)$

6.3.2. THEOREM. *If $r > 2 - 2\alpha$ holds, there there exists a threshold value of x_l in the interval $(0, 1)$, above which $P_d - P_c < 0$.*

Proof: Here, we employ the function $G(x_l) = (1 - x_l)(P_d - P_c)$ which has the same roots as $P_d - P_c$. For $x_l \in (0, 1)$,

$$\begin{aligned} G(x_l) &= (1 - x_l)(P_d - P_c) \\ &= (1 - \frac{r}{N} - \alpha) - (1 - \alpha)x_l + (r - 1 + \alpha)x_l^{N-1} + (\frac{r}{N} + 1 - \alpha - r)x_l^N \end{aligned} \tag{6.11}$$

$$G'(x_l) = (\alpha - 1) + (N - 1)(r - 1 + \alpha)x_l^{N-2} + N(\frac{r}{N} + 1 - \alpha - r)x_l^{N-1} \tag{6.12}$$

Note that $G(1) = G'(1) = 0$,

$$G''(1) = (N - 1)(N - 2)(r - 1 + \alpha)x_l^{N-3} + N(N - 1)(\frac{r}{N} + 1 - \alpha - r)x_l^{N-2} \tag{6.13}$$

$$G''(1) \quad = \quad (N - 1)(2 - 2\alpha - r) \tag{6.14}$$

We have

$$\begin{aligned} G(x_l) &\simeq G(1) + G'(1)(z - 1) + \frac{1}{2}G''(1)(z - 1)^2 \\ &= \frac{1}{2}(N - 1)(2 - 2\alpha - r)(1 - x_l)^2. \end{aligned} \tag{6.15}$$

For $r > 2 - 2\alpha$, $\lim_{x_l \to 1^-} G(x_l) < 0$,

$$G''(x_l) = x_l^{N-3}(N - 1)[(N - 2)(r - 1 - \alpha) + x_l(r + N - N\alpha - Nr). \tag{6.16}$$

Since $G''(x_l)$ changes sign at most once in the interval $(0, 1)$, we claim that there exists a threshold value of x_l in the interval $(0, 1)$, above which $P_d - P_c < 0$.

From the above analysis, we get

$$\begin{cases} G(x_l) = (1 - x_l)(P_d - P_c) \\ G(0) = 1 - \frac{r}{N} - \alpha \\ G(1) = 0 \end{cases} \tag{6.17}$$

As illustrated in Fig. 1, the game dynamics takes on three qualitatively different cases, which will be discussed one by one.

Case 1.1($1 - r/N - \alpha > 0$, i.e. $G(0) > 0$):

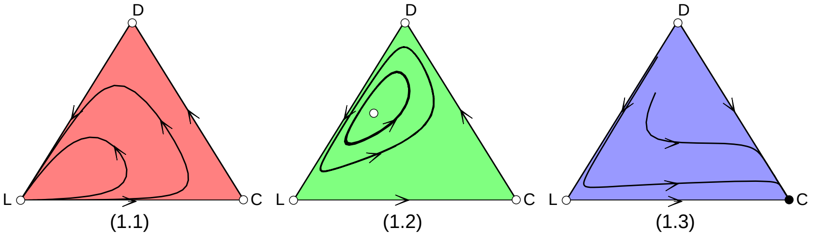
$$\lim_{x_l \to 1^-} G(x_l) = \frac{1}{2}(N-1)(2-2\alpha-r)(1-x_l)^2. \tag{6.18}$$

When $r < 2 - 2\alpha$, $G(x_l) > 0$, $x_l \in (0,1)$, the three corners represent a rock-scissors-paper type heteroclinic cycle, and there is no stable equilibrium of the game dynamics in this case.

Case 1.2 ($1 - r/N - \alpha > 0$,$r > 2 - 2\alpha$, $G(1^-) > 0$): the three corners represent a heteroclinic cycle. It is a center surrounded by closed orbits. Similar to case 1.1, there is no stable equilibrium of the game dynamics in this case.

Case 1.3 ($1 - r/N - \alpha < 0$, i.e. $r > 2 - 2\alpha$): In this case, for all x_s, pure speculation (S) and pure defection (D) are both unstable equilibria of the game dynamics. The cooperation equilibrium (C) is stable and in fact a global attractor.

Summarizing the three cases in this scenario corresponding to the simplex $T = (C, D, L)$, we can conclude that the three corners represent a rock-scissors-paper type heteroclinic cycle if $1 - r/N - \alpha > 0$ (cases 1.1 and 1.2) while pure cooperation is a global attractor if $1 - r/N - \alpha < 0$ (case 1.3). Hence, the outcome of the game dynamics depends on the model parameters. □



(1.1) (1.2) (1.3)

Figure 6.1: The evolution dynamics results of $T = (C, D, L)$, where in the absence of speculation. (1.1): $r < 2 - 2\alpha$. (1.2): $r > 2 - 2\alpha$; and (1.3): $1 - r/N - \alpha < 0$. Parameters: $N = 5, \delta = 0.3$, and $r = 1.6, \alpha = 0.1$ for (1.1); $r = 3, \alpha = 0.1$ for (1.2); $r = 3, \alpha = 0.5$ for (1.3). Open dots are unstable equilibrium points and closed dots are stable equilibrium points. It suggests that three corners represent a rock-scissors-paper type heteroclinic cycle if $1 - r/N - \alpha > 0$ (cases 1.1 and 1.2) while pure cooperation is a global attractor if $1 - r/N - \alpha < 0$ (case 1.3).

6.3.3. PROPOSITION. *When $T = (C, D, L)$, under the replicator dynamics of (6.5), it holds*

that
if $1 - r/N - \alpha > 0$ *and* $r < 2 - 2\alpha$*, there is no inner fixed point in* T*;*
if $1 - r/N - \alpha > 0$ *and* $r > 2 - 2\alpha$*, there is one inner fixed point in* T*;*
if $1 - r/N - \alpha < 0$*, full* C *is only stable fixed point in* T*.*

Proof: When $r > 2 - 2\alpha$, there exists a fixed point $x_l \in (0, 1)$ that $P_d = P_c$. From Eq. (6.4), we can get the only x_c and $x_d = 1 - x_l - x_c$, hence there is one inner fixed point in T. If $1 - r/N - \alpha > 0$ and $r < 2 - 2\alpha$, $P_d > P_c$ for all $x_l \in (0, 1)$, so there is no fixed point in T. If $1 - r/N - \alpha < 0$, we have $r > 2 - 2\alpha$, ($N > 2$). Then it must be true that $P_c > P_d$, so full C is only stable fixed point in T. □

6.3.2 Scenario 2: the corners of the simplex $T = (C, D, S)$

$$\begin{cases} P_d - P_c = 1 - \alpha - \frac{r}{N} \\ P_d - P_s = \lambda - \alpha \\ P_c - P_s = \lambda + \frac{r}{N} - 1 \end{cases} \tag{6.19}$$

Case 2.1 ($\lambda - \alpha > 0, 1 - \alpha - r/N > 0$ and $1 - \lambda - r/N > 0$): Here, pure cooperation and pure speculation are both unstable equilibria of the game dynamics. Full defection equilibrium (D) is stable and in fact a global attractor.

Case 2.2 ($\lambda - \alpha > 0, 1 - \alpha - r/N > 0$ and $1 - \lambda - r/N < 0$): In this case, pure cooperation and pure speculation are both unstable equilibria of the game dynamics. Pure defection equilibrium (D) is stable and a global attractor. The difference between case 2.1 and case 2.2 is that when there are only cooperators and speculators in the population, pure cooperation is the attractor in case 2.2 while pure speculation is the attractor in case 2.1.

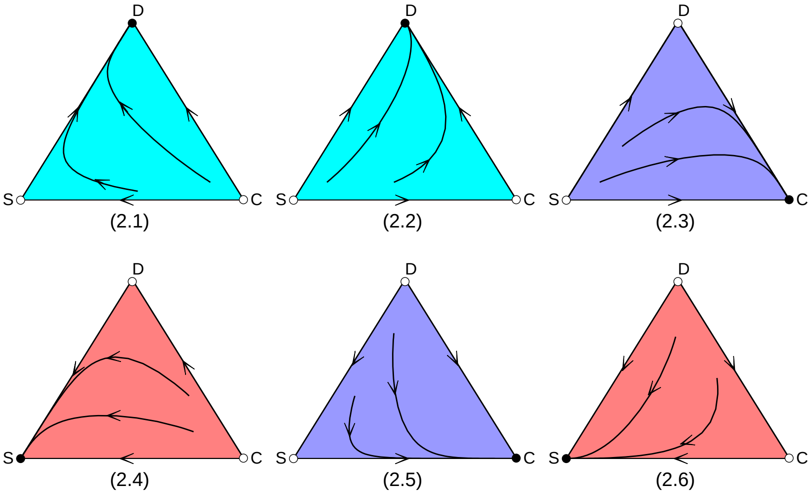
Case 2.3 ($\lambda - \alpha > 0$, $1 - \alpha - r/N < 0$, and $1 - \lambda - r/N < 0$): Herein, pure defection and pure speculation are both unstable equilibria of the game dynamics. Pure cooperation is a stable and global attractor.

Case 2.4 ($\lambda - \alpha < 0, 1 - \alpha - r/N > 0$, and $1 - \lambda - r/N > 0$): In this case, pure speculation is the only stable and global attractor.

Case 2.5 ($\lambda - \alpha < 0, 1 - \alpha - r/N < 0$, and $1 - \lambda - r/N < 0$): Pure cooperation is thus the only stable and global attractor.

Case 2.6 ($\lambda - \alpha < 0, 1 - \alpha - r/N < 0$, and $1 - \lambda - r/N > 0$): Pure speculation is the only stable and global attractor. The difference between case 2.6 and 2.4 is that when the population consists of only cooperators and defectors, pure cooperation is the attractor in case 2.6 while pure defection is the attractor in case 2.4.

Summarizing the six cases in scenario 2 corresponding to the simplex $T = (C, D, S)$, we can see that there is always a global attractor in the system. And similar with scenario 1, the outcome of the game dynamics depends on model parameters.



(2.1) (2.2) (2.3)

(2.4) (2.5) (2.6)

Figure 6.2: The evolution dynamics results of $T = (C, D, S)$, where in the absence of defection. We consider six cases, which are discussed in cases 2.1 till 2.3 in the upper panel of Fig. 6.2. Fig. 6.2 focuses on the situation $\lambda - \alpha > 0$ implying that the fine for defectors is higher than the costs of cooperation. Lower panels of Fig. 6.2 considers the opposite case $\lambda - \alpha < 0$, where defection is the dominating strategy. Results show that there is always a global attractor in the system, and the outcome of the game dynamics depends on model parameters. Parameters: $N = 5, r = 3, \delta = 0.3$, and $\alpha = 0.1, \lambda = 0.2$ for (2.1); $\alpha = 0.1, \lambda = 0.8$ for (2.2); $\alpha = 0.5, \lambda = 0.8$ for (2.3); $\alpha = 0.1, \lambda = 0.2$ for (2.4); $\alpha = 0.8, \lambda = 0.5$ for (2.5); $\alpha = 0.8, \lambda = 0.1$ for (2.6).

6.3.4. PROPOSITION. *When $T = (C, D, S)$, under the replicator dynamics of (6.5), it holds that*

if $\lambda - \alpha > 0$ and $1 - \alpha - r/N > 0$: full D is only stable fixed point in T;

if $1-\alpha-r/N<0$ *and* $1-\lambda-r/N<0$*: full* C *is only stable fixed point in* T*;*
if $\lambda-\alpha<0$ *and* $1-\lambda-r/N>$*: full* S *is only stable fixed point in* T*;*

Proof: When $x_l=0$, if $1-\alpha-r/N>0$, $P_d>P_c$; if $\lambda-\alpha>0$, $P_d>P_s$, therefore if $x_d>0$, $P_d>\bar{P}$. That means full D ($x_d=1$) is only stable fixed point in T.
When $x_l=0$, if $1-\alpha-r/N<$, $P_c>P_d$; if $1-\lambda-r/N<0$, $P_c>P_s$, therefore if $x_c>0$, $P_c>\bar{P}$. That means full C ($x_c=1$) is only stable fixed point in T.
When $x_l=0$, if $\lambda-\alpha<0$, $P_s>P_d$; if $1-\lambda-r/N>0$, $P_s>P_c$, therefore if $x_s>0$, $P_s>\bar{P}$. That means full S ($x_s=1$) is only stable fixed point in T. □

6.3.3 Scenario 3: the corners of the simplex $T=(C,L,S)$

It is easily observed that $x_l=0$ leads to $P_c-P_s=\lambda-1<0$. Thus, the three corners represent a rock-scissors-paper type heteroclinic cycle. There is no stable equilibrium in this case.

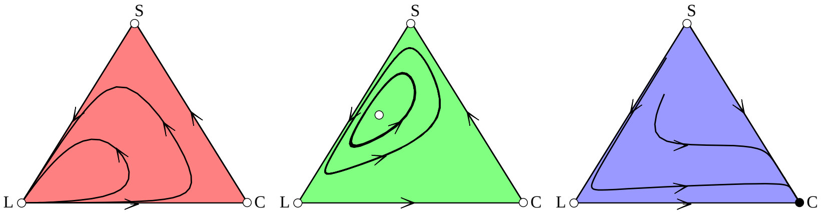
6.3.5. PROPOSITION. *When* $T=(C,S,L)$*, under the replicator dynamics of (6.5), it holds that*
if $1-r/N-\lambda>0$ *and* $r<2-2\lambda$*, there is no inner fixed point in* T*; if* $1-r/N-\lambda>0$ *and* $r>2-2\lambda$*, there is one inner fixed point in* T*; if* $1-r/N-\lambda<0$*, full* C *is only stable fixed point in* T*.*

Proof: By using λ takes the place of α, we can get the similar results with proposition 6.3.3. □

6.3.4 Scenario 4: the corners of the simplex $T=(D,L,S)$

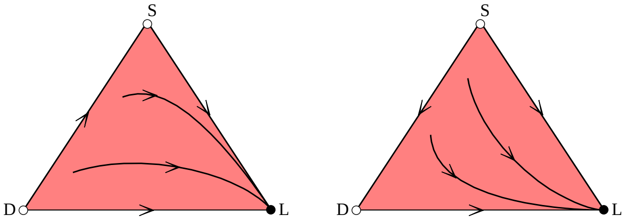
Case 4.1 ($\lambda-\alpha<0$): In this case, pure loners is the only stable and in fact the only global attractor.
Case 4.2 ($\lambda-\alpha<0$): Still, pure loners remains the only stable and in fact the only global attractor. The difference between case 4.1 and 4.2 is that when there are only speculators and defectors in the population, pure speculation is the attractor in case 4.1 while pure defection is the attractor in case 4.2.

Summarizing the two cases in scenario 4 corresponding to the simplex $T = (C, D, S)$, we can conclude that pure loners is the only global attractor in the system.



(3.1) (3.2) (3.3)

Figure 6.3: The evolution dynamics results of $T = (C, S, L)$, where in the absence of speculation. (3.1): $r < 2 - 2\lambda$. (3.2): $r > 2 - 2\lambda$; and (3.3): $1 - r/N - \lambda < 0$. Parameters: $N = 5, \delta = 0.3$, and $r = 1.6, \lambda = 0.1$ for (3.1); $r = 3, \lambda = 0.1$ for (3.2); $r = 3, \lambda = 0.5$ for (3.3). It suggests that three corners represent a rock-scissors-paper type heteroclinic cycle if $1 - r/N - \lambda > 0$ (cases 3.1 and 3.2) while pure cooperation is a global attractor if $1 - r/N - \lambda < 0$ (case 3.3).



(4.1) (4.2)

Figure 6.4: The evolution dynamics results of $T = (D, L, S)$ where in the absence of cooperation.(4.1) resulting game dynamics in the absence of speculation, where pure loners is the only global attractor in the system. Parameters: $N = 5, r = 3, \delta = 0.3$, and $\alpha = 0.4, \lambda = 0.1$ for (3); $\alpha = 0.4, \lambda = 0.1$ for (4.1); $\alpha = 0.1, \lambda = 0.4$ for (4.2).

6.3.6. PROPOSITION. *When $T = (S, D, L)$, under the replicator dynamics of (6.5), it holds that*

full L is only stable fixed point in T;

Proof: When $x_c = 0$, $P_l - P_d = (\alpha + \sigma)(1 - N_l^{N-1}) > 0$ and $P_l - P_s = (\lambda + \sigma)(1 -$

$N_l^{N-1}) > 0$, therefore full L ($x_l = 1$) is only stable fixed point in T. □

6.4 Conclusions

Public goods pose a riddle from the evolutionary viewpoint. The extensive research here is mainly focused on the exploration of strategic options. In the standard, obligatory public-goods model, the two simplest strategies choose always defection and cooperation, and the corresponding players are called defectors (shortly D) and cooperators (C), respectively. There is a growing evidence that the threat of punishment can induce self-interested players to prefer actions that sustain the public goods, and turn away from free riding. When facing potential punishment as a defector, speculation and optional participation are also feasible choices driven by individual diversity in a wide range of real-world situations.

Our previous work find scenarios where speculation either leads to the reduction of the basin of attraction of the cooperative equilibrium or even the loss of stability of this equilibrium, if the costs of the insurance are lower than the expected fines faced by a defector. As an extension of our study proposing speculation strategy(Zhang et al. 2013), here we base our analysis of the evolutionary game on replicator dynamics for four strategies: C (cooperators), D (defectors), S (speculators) and L (nonparticipants). For simplicity we assumed that punishment of a given effectiveness is externally imposed upon the defectors in a public goods game. We do not consider the question how the punishment system was established or who carries the costs of punishment.

Here, we show that the evolutionary fate of the system depends on special assumptions about model parameters. When starting from the three-strategy state, the observed domination of some strategy or a rock-paper-scissors type of cycle suggests that the additional strategic options can radically alter the evolution of cooperation. Specifically, larger multiplication factor r and punishment α on defectors can facilitate cooperation to be a dominant strategy in the absence of speculation (scenario 1). Here, we show that the option to abstain from the joint enterprise offers an escape from the social trap. This leads to the decline of exploiters and allows the reemergence of cooperators. Further, public goods cooperation can also be favored to be an equilibrium by moderate values of punishment α and cost of insurance λ in the absence of loner (scenarios 2). It is also intriguing that cooperation fails to dominate the population in the competition with speculation and loner strategy, even though in the absence of defection (scenarios 3). When the initial state consists of the four strategies, at least one strategy will evolve to vanish within the evolution.

Summarizing, we show that this conclusion depends on the particular assumptions of the proposed model here. An interesting future direction would be to address whether the presence of more strategy options altogether affect the dynamics of behaviors in the field of human cooperation.

Chapter 7

General Conclusions

Summarize the past, anticipate the future.

This final chapter summarizes the main results that have been presented in this thesis and provides recommendations for future research.

7.1 Conclusions

Cooperation is a cornerstone of social organization and commonplace in human societies. Altruism refers to a costly behavior that benefits others. However, mutual cooperation is often found in nature even when selfish behavior is apparently rational for individuals. Why and under what circumstances, presumptively selfish agents cooperate is a question of longstanding interest to multidisciplinary research. In investigating this cooperation dilemma problem the standard framework utilized is evolutionary game theory. Evolutionary game theory is an interdisciplinary mathematical tool which seems to be able to embody several relevant features of the problem and, as such, is used in much cooperation-oriented research. Vast theoretical mechanisms for emergence and maintenance of cooperation in social dilemma games have been reported thus far.

Rooted in biology and reaching out to complex networks and control engineering, this thesis thoroughly investigates the competition and coexistence of competing strategies in gaming population in the framework of evolutionary games.

Chapter 2 develop a general model for the updating of states in a network that allows us to derive conditions for the steady-state coexistence of strategies. The analysis reveals that coexistence crucially depends on the number of agents consulted for updating. We conclude that updating rules are as important for evolution on a network as network structure and the nature of the interaction.

Chapter 3 extends the model to a more general one for the updating of states in a network. We introduce individual player's switching probabilities between competing partners, not only competing strategists. It allows us to derive conditions for the steady-state coexistence of competing strategies. New theoretical models and results developed in this thesis are useful for probing into how strategies are being taken in structured populations. It provides an original and novel approach for studying evolution dynamics, while also pave feasible ways for possible robotic study in future. The results imply that strategy updating deserves more attention in empirical and theoretical studies.

Chapter 4 presents the dynamic outcomes of gaming populations when the diversity of time scales is introduced in the strategy update process. We have break with the traditional assumption concerns that nature selection acts on the population at the same time scale, i.e. players have the same frequency in updating their strategies. We eliminate this restriction by dividing the population into two groups (fast ones and slow ones), and investigate the evolutionary dynamics in finite populations with time scales on updating and study the influences of different composition of the two groups on fixation probabilities. Numerical and analytical calculations are performed to study the evolution dynamics of strategies in the special classes of two-player games (Prisoner's dilemma game, Snowdrift game and Stag-hunt game). Results show that the decoupling of time scales on updating leads to dramatic changes in the dynamics of strategies. We give a proximation formula of fixation probability of mutant types in finite populations and investigate the outcome of evolution under weak selection. This work is a preliminary study on time scales on updating and more attention is required on this topic in future.

Chapter 5 shows the strategy dynamics in threshold public goods games in which players can buy insurance for their contribution. A threshold public goods game requires a minimum amount of contributions to be collected from a group of individuals for provision to occur. If the threshold is not achieved, the loss can be covered. Our analytical results show that when agents face the potential aggregate risk in threshold public goods games, more contributions occur with increasing compensation from insurance. Moreover, insurance significantly enhances individual contributions and facilitates provision, especially when the required threshold is high.

Chapter 6 bases our analysis of the evolutionary game on replicator dynamics for four strategies: C (cooperators), D (defectors), S (speculators) and L (nonparticipants). For simplicity we assume that punishment of a given effectiveness is externally imposed upon the defectors in a public goods game. We do not consider the question how the punishment system is established or who carries the costs of punishment. Results suggest that the evolutionary fate of the system depends on special assumptions of model parameters. The corresponding results highlight the kinds of model parameters for which evolution favours cooperation, and those in which it does not. Moreover, the observed domination of some strategy or a rock-paper-scissors type of cycle suggests that the additional strategic options can radically alter the evolution of cooperation, and, the coexistence of competing strategies are possible under some conditions.

7.2 Further research topics

All the above phenomena indicate that there are possibly many ways or factors to influence and enhance the competition or coexistence of strategy behaviors among selfish populations. Further investigations would be required to clarify the distinguished role of multiple strategies, appearance of different subpopulation structures, and inhomogeneities of games in these networked populations. Here we only identify three possible directions for subsequential research, servings as a modest spur to induce others to come forward with their valuable contributions in future.

Game Competition among multiple strategies. An interesting future direction would be to address whether the presence of more strategy options altogether affect the dynamics of behaviors in the field of human cooperation. This thesis has dealt with four strategies available for the gaming populations: cooperation, defection, loner and speculation. More candidate strategies will be taken into consideration in network models in future research. For example, the insured cooperation, insured defection strategy, which means strategy players can get some insurance when the game. This is more plausible when the success of the game depends on some threshold point, or the defectors will suffer some punishment in the evolutionary game playing. In addition, this thesis has focused on the synchronization or consensus problem, under the condition that all the agents update their states synchronously. To mimic the real social systems and probe the origin of altruistic behaviors in nature, this motivates extending the study in this thesis to the case with competition among multiple strategies . That is to say, each agent is provided with a multiple-strategy profile, and can choose any one from it when playing games with others. However, accordingly the evolutionary dynamics will really a complicated because it will be closely related with multiple game parameters led by the various strategies. It is a challenging problem if one considers a combination of different communication constraints.

Competing among players situating on different topologies. Many tools to foster cooperation and solve the social dilemma in an efficient way have been designed and tested. It is well known that the evolution of cooperative behavior is dependant upon certain environmental conditions. One such condition that has been extensively studied is the use of a spatially structured population. The key concept of spatially structured populations is: agents are assigned to the vertices of a network, which can be a regular lattice or has a more complex structure. The edges denote links between players in terms of game dynamical interactions. Then, agents are constrained to interact only with their adjacent neighbors to play evolutionary games in which more successful strategies spread on the system. This thesis has studied some structured population situating ont the complex networks. To simplify the research work, we only focus on the case that the whole population situates on a single complex network. The multiple structures among the gaming population can be further investigated when the population can be divided into several groups with their respective networks. For this idea, the correlationship between multiple networks is the key problem needed to be solved. This kinds of investigation can help us get more hints about the potential relationship between evolutionary dynamic process and the characteristics (e.g. population structure, individual heterogeneity) of gaming population.

Competing among players playing different games. Almost most past studies, including the work mentioned in this thesis, focus on the simple case where only one game model (e.g. prisoner's dilemma game, snowdrift game, or public goods game, and so on) is employed as the metaphor to describe the conflicting tension between the short-term benefits of defection and the long-term benefits of cooperation. Everyday experience told us that large heterogeneity between agents is common in real social systems. Individuals are probably involved in complex interacting networks, where they join different games within different gaming groups. For example, they may are asked to participate in a public goods game in a collective group, such as with their colleagues or neighbors for a public goods project. Meanwhile, they may join a prisoner's dilemma game with only one partner in some gaming situations. To focus on the main research object in each work, this thesis has only considered the mentioned simple case where only one game is employed in the large gaming population. Actually, studying the competing among players playing different games is also a challenging project in this field, because the evolutionary process will be complicated or difficult to gain the results. For example, we should wrestle with the challenging calculation of payoffs gained in different games. In the future, we can pay more attention to the multiple games mentioned here.

An Application issue-Competing among different gaming 'players'. The current work in this thesis studies the competing strategies among the population by the means of theoretical analysis and numerical simulation. The robotics (e.g. E-pucks in our group) has also attracted plenty of attention from researchers in many fields, and many works have been proposed and published, suggesting that robotics to study the collective dynamics is a promising direction. Along this thriving research line, we could first try to investigate evolutionary problems among the robotic populations, to verify the theoretical analysis results. It is of interest to study a mixed population consisting of robotics and real agents, and focus on the behaviors of these different 'players'.

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Summary

The emergence and existence of cooperation in real societies continues to intrigue researchers from multiple areas, and is basking in a great boom as a multi-disciplinary field of research. A dilemma based on universal Darwinism derives from the fact that defectors gain an advantage over cooperators, whenever cooperation is costly so consequently, not cooperating pays off. However, cooperative behaviors abound in nature. To delve into this problem the widespread framework utilized is game theory along with its extensions involving evolutionary process. The selfish behavior here is manifested by a defective strategy, aspiring to obtain the greatest benefit from the gaming interactions with others. Vast theoretical or experimental mechanisms for emergence and maintenance of cooperation in social dilemma games have been reported thus far.

As mentioned earlier, the cooperative dilemma problem draws attentions of researchers from various disciplines, and one of the reasons may probably lies on the fact that it is really an interdisciplinary and cutting-edge topic. And, cooperative dilemmas in which the self-interest may at odds with the collective interest actually constitute a significant form of vast societal problems, where the maximum payoffs led by the mutual or collective cooperation are in great need. Up to now a great deal of research is aimed at pointing out the way to breakthrough it, by virtue of the viewpoints or methods from several different branches of the sciences, such as biology, physics, sociology, computer science, economics, etc. Each discipline has dealt with this problem differently, and meanwhile each may have much to learn

from the other.

Inspired by the research focus in the two groups ('Theoretical Biology Group' and 'Network Analysis and Control Group' in Groningen University) where I carried out my PhD study, I establish the theoretical study of this topic from the biological or sociological perspective, at the same time, with the help of the simulations in the framework of complex networks under more realistic assumptions. The central topic of this thesis is the competing and coexistence of different agents, since not only one type of strategist actually exists in conflicting situations often encountered in natural and social sciences. Attracted and also bewildered by this social diversity, the main research focus we have in mind is to theoretically and experimentally investigate the conflict and coordination of multiple competing behaviors, as summarized in this thesis.

First and foremost, the research project is related with our proposed switching probabilities between players. What is noteworthy is that the evolution dynamics of involving strategies are the core of the employed evolutionary game theory. A commonly used approach to strategic interaction is combing game theory and population dynamics in a replicator equation or imitation dynamics. This framework or structure is frequently employed in most existing literatures, whereas requires the specific values of payoffs as an indispensable factor. However, in many situations of decision-making under conflicting interests, the information about strategies or payoffs are not easy to acquire, especially for the capacity-constrained players. In a real case of games for benefits, there is normally no easy way for players to know of the ongoing performance of their partners.

In our work, the general situation is modeled here as a repeated game played in a sequence of periods, and strategy update are led by the so-called individual player's switching probabilities, as discussed in Chapter 2. We restrict our analysis to the case where switching only occurs between competing strategies. Results help us to find a causal link between the coexistence of competing strategies and the number of agents consulted for updating. This new theoretical model and the results are also beneficial for investigating how strategies are being taken in structured populations. Considering the complicated strategy decision process and the involved factors, strategy updating still deserves more attention in empirical and theoretical studies.

However, do strategy revisions merely occur between different strategists? In many competitive games, strategy revisions also emerge when two same strategists encounter. This is in line with some real-world mimicry, where decisions or actions of each player may be less directly linked to or affected by the actions of the other players. In this case, strategy revisions also occur in social contexts where players interact and happen to adopt the same strategy concurrently. To answer these questions, we design a general paradigm to study the strategy switching probabilities between competing players, and the related results are summarized in Chapter 3. The results reveal that the evolutionary fate of the coexisting strategies can be calculated analytically, and provide novel hints for the resolution of cooperative dilemma problems in a competitive context.

Then, it is understandable that the speed or frequency of strategy update may vary in each individual in real social societies. Previous studies have focused mainly on the assumption of homogeneous times scales in strategy updating of the populations. Because participants had varying reaction times or frequencies in strategy updating, we reported the modulation of player behavior attributable to different time scales in strategy updating, as summarized in Chapter 4. Our approach can be interpreted as individual heterogeneity regards to time scales.

To dissociate between slow and fast opponents, we divide the population into two groups endowed with respective time scales. And, we extend our analysis to three representative dilemma models (Prisoner's dilemma game, Snowdrift game, and Stag-hunt game). In sum, we have gained a sequence of approximation formulas that determine the fixation probabilities under variations of the initial conditions. Results suggest that the different time scales result in much richer evolutionary dynamics, and some inspirations can be gained to control the fall or rise of cooperative behaviors. To more convincingly show the individual diversity, our work lends itself to several extensions, such as an immediate and feasible one, one more type of players that updates with median time scales can be taken into account.

Next, with a view to the significance and diversity and strategy choices, we extend the general two-strategy profile by adding a third strategy, called insured cooperation, which corresponds to buying an insurance covering the potential loss resulted from the unsuccessful public goods game. We focus our study on the threshold public goods games. Particularly, only the contributing agents can opt to be insured, which is an effort decreasing the amount of the potential loss occurring. Our results in Chapter 5 show that permitting the adoption of insurance significantly enhances individual contributions and facilitates provision, especially when the required threshold is high.

Finally, a large majority of the current evolutionary game studies that shed light on the mechanism behind many cooperative phenomena in gaming systems concentrated on pair-wise interactions between individuals. It is tempting to introduce and investigate the individual diversity in terms of strategy in a competitive context. Thus it is conceivable that participants face the following options: cooperation, defection, speculation and being a loner. The traces of all of these forms of actions could be spotted in everyday life. In the analysis shown in Chapter 6, we identify specific characteristics of the game parameters in public goods games that conceptually mark the transitions among various steady states of the system.

Summarizing, modeling the characteristics of individual diversity, involves exploring the heterogeneous factors of individuals, from strategy choice to time scales of strategy updating and more practical forms that exist in societies. At the core are the issues of how reasonable assumptions are required to propose based on these realistic considerations, and the degree that individual decisions are influenced by regard for others. Hope our work can offer inspirations and references for the sustainable cooperative behaviors, avoiding the free riding phenomenon in some situations, and promoting high efficient transactions and cooperates in society finally.

Samenvatting

Wetenschappers zijn al jaren ge?ntrigeerd door samenwerking. In onze maatschappij, maar ook overal in het dierenrijk, kunnen tal van voorbeelden worden gevonden waarbij individuen met elkaar samenwerken. Onderzoek aan samenwerking word gekenmerkt door vele disciplines: biologie, sociologie, natuurkunde, computer wetenschappen en economie. Ieder van deze disciplines bestudeert het fenomeen van samenwerking met een eigen aanpak. Nieuwe inzichten ontstaan vaak wanneer deze vakgebieden met elkaar in aanraking komen. Redenerend vanuit de evolutieleer is het vaak moelijk te verklaren waarom individuen samenwerken. Waarom zou een individu een kostbare samenwerking aangaan, terwijl hij het risico loopt dat de interactie partner louter profiteerd van deze samenwerking en niets bijdraagt? Individuen die niet samenwerken kunnen op deze manier misbruik maken van individuen die zich co?peratief opstellen. Veel maatschappelijke problemen zijn inderdaad het gevolg van dit soort problemen, waarbij individuen die niet bijdragen aan de groep kunnen profiteren van co?peratieve individuen in de groep. Desalniettemin zijn er veel voorbeelden waarbij individuen samenwerken zonder problemen.

Samenwerking wordt vaak bestudeert door middel van spel theorie, hierbij spelen verschillende individuen een spel, waarbij ze kunnen kiezen om samen te werken of niet. De opbrengsten van een dergelijk spel zijn vaak het hoogst wanneer beide individuen samenwerken, maar ieder individu kan meer verdienen dan zijn interactie partner door ervoor te kiezen niet samen te werken. De vraag is hoe met een dergelijk spel samenwerking toch behouden kan blijven in de populatie. Ge?nspireerd door het werk in de twee groepen ('Theoretical Biology Group' en 'Network Analysis and Control Group' aan de Rijksuniversiteit van Groningen) waarbij ik mijn promotie onderzoek gedaan heb, heb ik modellen ontwikkeld voor het bestuderen van samenwerking. Ik maak gebruik van principes uit de spel theorie en bestudeer de interacties tussen individuen, die deel uit maken van een complex netwerk. Het centrale doel van dit proefschrift is om te begrijpen hoe verschillende strategi?en competeren en co?xisteren in een populatie door middel van theoretische en emperisch aanpak.

In het eerste deel van dit proefschrift maken we een basis model. We stellen voor dat individuen elkaars gedrag kopi?ren met een bepaalde kans. In vele modellen wordt uitgegaan dat individuen veel informatie hebben over elkaars strategi?n en success, deze informatie be?nvloed welke stategi?n worden gekopi?erd (bijvoorbeeld a.d.h.v. evolutionaire of culturele processen). In werkelijkheid hebben individuen echter zeer gelimiteerde informatie over hun interactie partners. Het is daarom aannemelijk dat in de meeste gevallen het onduidelijk is welke strategie de optimale is. In ons model kopi?ren individuen elkaars strategie met een bepaalde kans, zonder er van uit te gaan dat individuen meer informatie over een dergelijke strategie hebben. We kijken vervolgens hoe verschillens strategi?n met elkaar competeren en co?xisteren in gestructeerde populaties (hoofdstuk 2). Voor een nog algemenere implementatie van ons model, gaan we ervan uit dat niet alleen individuen met verschillende strategi?n elkaar kunnen kopi?ren, maar ook individuen met dezelfde strategie. Voor deze versie van het model kunnen we de resultaten analytisch berekenen (hoofdstuk 3). De modellen geven inzichten over hoe samenwerking behouden kan blijven in de populatie.

Een andere aanname die vaak wordt gemaakt in spel theorie is dat alle individuen even snel van strategie kunnen wisselen, m.a.w. de aanpassingssnelheid van individuen is identiek. In werkelijkheid is het echter aannemelijk dat individuen verschillen in hoe snel ze zich aanpassen aan hun interactie partners. In hoofstduk 4 bestuderen we hoe heterogeniteit in de aanpassingssnelheid de dynamiek van het model be?nvloedt. Uiteindelijk, focussen we ons op twee aanpassingssnelheden: individuen die zich langzaam aanpassen en individuen die zich snel aanpassen. We bestuderen hoe individuen zich gedragen in verschillende spel theoretische dilemmas: 'prisoner's dilemma', 'snowdrift game' en 'stag-hunt game'. Afhankelijk van hun strategie, krijgen individuen een opbrengst uit hun interactie met andere individuen. Individuen kopi?ren elkaars gedrag met een bepaalde kans. De opbrengst uit de interacties be?vloedt tevens de kans waarop individuen sterven en reproduceren. We laten zien dat de verschillende aanpassingssnelheden resulteren in een rijke dynamiek van het model, zeker in vergelijking met een implementatie waarbij alle individuen even snel van strategie kunnen wisselen. Bovendien geven deze modellen inzichten over de manieren waarop samenwerking bevorderd kan worden.

In de laatste hoofstukken van dit proefschrift bestuderen we een grotere diversiteit aan strategi?n. In plaats van twee strategi?n - samenwerkende en niet samenwerkende individuen - voegen we een derde strategie toe aan ons model. Deze strategie betaald een bepaalde hoeveelheid vaste kosten, om zich zo weerbaar te maken tegen individuen die niet samenwerken. Je zou het kunnen zien als het afsluiten van een verzekering tegen potentiele verliezen. In hoofdstuk 5 bekijken we hoe een 'verzekerde' strategie samenwerking tussen individuen kan bevorderen. In aanwezigheid van verschillende strategi?n kan de populatie naar verschillende evenwichten gaan. In hoofstuk 6 bestuderen we de model parameters die van invloed zijn op de transities tussen deze evenwichten en de inzichten die het geeft voor het bevorderen en behouden van samenwerking.

De modellen in dit proefschriften laten zien hoe verschillende eigenschappen van belang kunnen zijn voor de samenwerking tussen individuen: de strategi?n die individuen gebruiken, de kans waarmee ze hun strategie veranderen, de snelheid waarmee dat gebeurt en de interactie partners die individuen hebben. De belangrijke vraag blijft echter, welke eigenschappen in werkelijkheid het meest doorslaggevend zijn. Uiteindelijk hopen we dat onze modellen bijdragen aan het verkrijgen van verdere inzichten, die de samenwerking tussen mensen in onze samenleving kunnen bevorderen.