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Abstract

As an alternative to the parsimonious Nelson-Siegel and Svensson yield models, the paper presents a yield model that is both parsimonious and arbitrage-free. The model has four factors and one, or three parameters. One factor is interpreted as inflation and the remaining three factors are real portfolio values. Without explicit specification of the factor dynamics, the model can be estimated by nonlinear least squares, which can be linearized when yields are observed daily. When applied to nominal Treasury yields the model outperforms the other models in terms of root mean squared error. Although the inflation part of the model is extremely simple, the estimates of one-month real interest rates perform intriguingly well. The curvature factor is shown both theoretically and empirically to have predictive potential for future slopes of the yield curve.

Key words: Term structure of interest rates; Arbitrage free yields; Nelson-Siegel yield model; Svensson yield model; Inflation

JEL classification: E43

1 Introduction

The popular Nelson and Siegel (1987) yield model uses flexible Laguerre functions to fit yield curves with a single parameter and three factors.[1] It is simple and it captures many of the shapes of yield curves that are observed over time. Among the various extensions that add flexibility, the Svensson (1995) yield model is particularly popular. It uses four factors and two parameters. Although these models are not arbitrage-free, they are used by central banks to construct zero-coupon yield curves.[2] As the Nelson-Siegel model is also widely used by practitioners, De Pooter (2007) ranks it among the most popular term-structure estimation methods.

The Nelson-Siegel model and its extensions can also be used to give accurate term structure forecasts (Diebold and Li, 2006; De Pooter, 2007). Here a trade-off is found between in-sample fit and out-of-sample forecasting performance. More flexible models improve the in-sample fit but may perform poorly in terms of forecasting. This is found for arbitrage-free extensions as well. Christensen, Diebold, and Rudebusch (2011) formulate arbitrage-free extensions by adding deterministic terms with additional parameters. They find that parsimonious versions exhibit significantly better out-of-sample forecast performance.

Here I consider an alternative to the Nelson and Siegel (1987) and Svensson (1995) models. It is a yield model that is parsimonious as well, and it is arbitrage-free from the start. It does not need additional terms with additional parameters, which would make the model less parsimonious. It does not need a fifth factor such as the arbitrage-free extension of the Svensson model (Christensen, Diebold, and Rudebusch, 2009). It is simply a four-factor arbitrage-free yield model with one, or three parameters. It can be estimated by least squares. When compared to the in-sample fit of the Nelson-Siegel model, its arbitrage-free

[1]Alternative flexible parameterizations of the yield curve include the use of Legendre polynomials (Almeida and Vicente (2008) and natural cubic splines Bowsher and Meeks (2008).

[2]The Bank of International Settlements for International Settlements (2005) reports that time 9 out of 13 banks use either the Nelson-Siegel or the Svensson model to construct zero-coupon yield curves. Currently, the Central European Bank uses the Svensson model as described in the technical note at `https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_area_yield_curves/html/technical_notes.pdf`.

extension with independent factors, and the Svensson model, which are all estimated by least squares, the new model performs best in terms of root mean squared error.

Furthermore, in terms of interpretability the new model goes beyond the level, slope and curvature interpretability of Nelson-Siegel extensions. That is to say, I distinguish between an inflation factor and real factors that describe real bond prices. Although, the specification of the inflation component is extremely simple and only nominal yields are used as data, the measurements of the one-month real interest rate look less volatile and less affected by the trend in inflation when compared to the one-month real rate reported by Haubrich, Pennacchi, and Ritchken (2012). Furthermore, whereas the decay parameters in the Nelson-Siegel extension lack interpretation, the parameters in the new model relate to volatility of inflation and long-term growth rates of portfolios.

A Nelson-Siegel extension with two slope factors and one curvature factor can be considered as an approximation of the arbitrage-free new model at the short end of the yield curve. Thus curvature is related to a factor in the new model. It is interpreted in terms of real interest rate futures positions. The level of that factor is shown, both theoretically and empirically, to have forecasting potential for future slopes of the yield curve. However, actual out-of-sample forecasting would require a full, arbitrage-free specification of the factor dynamics, which is not studied in this paper.[3]

The paper is organized as follows. Section 2 presents the yield model and discusses its estimation. The derivation as an arbitrage-free yield model is described in Section 3. The model is interpreted in Section 4. An empirical application to Treasury yields is given in Section 5 and Section 6 concludes.

[3]In a separate paper arbitrage-free factor dynamics will be specified, aimed at measuring risk premia and forecasting future yields.

2 The yield model

Let the yield of a zero-coupon bond at time t with maturity τ be specified as

$$y_t(\tau) = Y_{\pi t} - \frac{\sigma_\pi^2 \tau^2}{6} - \frac{\log\left[1 + (1 - e^{-\tau})\, Y_{St} + \{e^{-\tau}(1+\tau) - 1\}\, Y_{Ft} + \tau Y_{Lt}\right]}{\tau}, \qquad (1)$$

where four factors are given by $Y_{\pi t}$, Y_{St}, Y_{Ft} and Y_{Lt}, and σ_π^2 is a single parameter. The model is arbitrage free.[4] Its formulation is sufficiently parsimonious to allow for least squares estimation without further specification of the factor dynamics. The interpretation of the factors in terms of inflation (π), where σ_π is the volatility of inflation, and portfolios with high real expected returns (S and F) or low expected real returns (L) will be discussed further in Section 4. A less restricted version of the model will be used as well. It adds two long-term growth rates as parameters.

The model can be compared with the widely used Nelson and Siegel (1987) and Svensson (1995) models.[5] The former is a three-factor model with one parameter and the latter is a four-factor model with two parameters. Filipović (1999) showed the Nelson-Siegel model is not arbitrage free. The same critique applies to the Svensson model. The arbitrage-free extensions of (Christensen et al., 2009, 2011) use additional factor-volatility parameters in maturity-dependent yield-adjustment terms; the Svensson model requires an additional fifth factor as well.

As model (1) is sufficiently parsimonious it can be estimated by least squares. It can be compared to the least squares fit of the Nelson-Siegel model and its arbitrage-free extension, and the Svensson model. Just as it holds true for the parameters in these models, the parameter σ_π can be estimated by nonlinear least squares. However, given σ_π, the factors should be fitted by nonlinear least squares as well, whereas linear optimization is possible for the Nelson-Siegel and Svenson models. In practice, when a sequence of yield curves have to be fitted and the time steps are small, the factor values for the first yield curve, ($t = 1$), can be fitted by nonlinear least squares, while the other yield curves,

[4]The inflation component of the model requires a bounded horizon $\tau \leq \tau^*$.
[5]See Section 4 for a definition of these models.

$t = 2, 3 \ldots$, can be fitted linearly by using linearization and Ridge regression, as shown in Appendix A.1.

3 Derivation of the yield curve

Let the real and nominal stochastic discount factors be given by Λ_t^{R} and Λ_t^{N}. They are linked by the relation $\Lambda_t^{\mathrm{R}} = \Lambda_t^{\mathrm{N}} Q_t$, where Q_t is the price level process. Parsimony and tractability can be achieved by using a Fisher hypothesis where Λ_t^{R} and Q_t are independent. Arbitrage-free nominal bond prices at time t and maturing at time $T = t + \tau$ are thus given by

$$P_t^{\mathrm{N}}(T) = P_t^{\pi}(T) P_t^{\mathrm{R}}(T), \tag{2}$$

where $P_t^{\pi}(T) = \mathrm{E}_t(Q_T^{-1})/Q_t^{-1}$ and $P_t^{\mathrm{R}}(T) = \mathrm{E}_t(\Lambda_T^{\mathrm{R}})/\Lambda_t^{\mathrm{R}}$. Notice the conditional expectation at time t, is based on real-world dynamics under the P measure.[6]

3.1 The inflation component

Let the price level process take the simple form $Q_t = e^{\int_{-\infty}^{t} \pi_s \,\mathrm{d}s}$, where π_t satisfies

$$\mathrm{d}\pi_t = \kappa_\pi(\bar{\pi} - \pi_t)\,\mathrm{d}t + \sigma_\pi \,\mathrm{d}W_t^{\pi}, \tag{3}$$

and $\kappa_\pi > 0$, so the instantaneous inflation rate π_t is mean reverting. Similar to the Vasicek (1977) model, this amounts to

$$P_t^{\pi}(T) = e^{-\bar{\pi}\tau} e^{-a_\pi(\tau)\tau \;-\; b_\pi(\tau)(\pi_t - \bar{\pi})\tau}, \tag{4}$$

where $b_\pi(\tau) = (\kappa_\pi \tau)^{-1}(1 - e^{-\kappa_\pi \tau})$ and $a_\pi(\tau) = \frac{\sigma_\pi^2}{2\kappa_\pi^2}\left\{b_\pi(\tau) - 1 + \frac{\kappa_\pi \tau}{2} b_\pi^2(\tau)\right\}$.

In particular, I use an infinitesimally small κ_π, which accords with the usual finding, as noted by Christensen et al. (2011), that "one or more of the interest rate factors are close to being nonstationary processes under the P-measure". For $\kappa_\pi \to 0$, and a bounded

[6] This paper does not use risk-neutral dynamics under the Q measure.

horizon $\tau \leq \tau^*$, the following limits apply: $b_\pi(\tau) \to 1$ and $a_\pi(\tau) \to -\sigma_\pi^2 \tau^2/6$. As a result $P_t^\pi(T) \to e^{-\pi_t \tau + \sigma_\pi^2 \tau^3/6}$, which gives the inflation component of the yield curve given by

$$y_t^\pi(\tau) = \pi_t - \sigma_\pi^2 \tau^2/6. \tag{5}$$

3.2 The real component

To specify the short rate consider real portfolio values V_{it}^{R}, that should satisfy the martingale property $\mathrm{E}_t(V_{iT}^{\mathrm{R}} \Lambda_T^{\mathrm{R}}) = V_{it}^{\mathrm{R}} \Lambda_t^{\mathrm{R}}$, $i = 1, 2, 3$. Factors are given by detrended portfolio values $X_{it} = e^{-\bar{\mu}_i^{\mathrm{R}} t} V_{it}^{\mathrm{R}}$. The real short rate is specified as an affine function of these factors. Without loss of generality, it can be specified as

$$r_t^{\mathrm{R}} = c - X_{1t} - X_{2t} - X_{3t}. \tag{6}$$

As a result, without specifying the factor dynamics any further, the arbitrage-free real yield curve is given by

$$\tilde{y}_t^{\mathrm{R}}(\tau) = c - \tau^{-1} \log \left\{ 1 + \sum_{i=1}^{m} X_{it} k(\bar{\mu}_i^{\mathrm{R}} - c, \tau) \right\}, \tag{7}$$

$$k(\delta, \tau) = \int_0^\tau e^{-\delta s}\, \mathrm{d}s = \frac{1 - e^{-\delta \tau}}{\delta}. \tag{8}$$

A derivation is given in Appendix A.2. Such bond prices fit within the linearity-generating processes framework of Gabaix (2009) and they have been considered before in Bekker and Bouwman (2011).[7]

3.3 The transformed model

Based on the inflation yields (5) and the real yields (7), the nominal yields are given by $\tilde{y}_t^{\mathrm{N}}(\tau) = y_t^\pi(\tau) + \tilde{y}_t^{\mathrm{R}}(\tau)$, which amounts to a model different from model (1). The model needs a transformation. The reason is that when the nominal yield model $\tilde{y}_t^{\mathrm{N}}(\tau)$ is estimated by

[7] Contrary to the present paper, the estimation in the working paper Bekker and Bouwman (2011) depends on the specification of factor dynamics.

least squares on US data, a remarkable phenomenon can be observed. The fit is best when $\Delta_{\bar\mu} = \bar\mu_1^{\mathrm{R}} - \bar\mu_2^{\mathrm{R}}$ is very small, in which case X_{1t} and X_{2t} are strongly negatively correlated, having both very large negative and positive values over time. It seems that investments in the V_{1t} portfolio are hedged to a large extent by investments of the opposite sign in the V_{2t} portfolio, while the growth rates $\bar\mu_1^{\mathrm{R}}$ and $\bar\mu_2^{\mathrm{R}}$ are very close. As $\Delta_{\bar\mu}$ gets small, the absolute values of $X_{1t} - X_{2t}$ get large, but $X_{1t} + X_{2t}$ and $\Delta_{\bar\mu}(X_{1t} - X_{2t})/2$ behave well, just as X_{Lt}.

Therefore, transformed real factors (Y_{St}, Y_{Ft}, Y_{Lt}) are used that can be interpreted as limits of $(X_{t1} + X_{2t},\ \Delta_{\bar\mu}(X_{t1} - X_{t2}),\ X_{Lt})$ as $\Delta_{\bar\mu} \to 0$. The real yield model is thus transformed into

$$y_t^{\mathrm{R}}(\tau) = c - \tau^{-1} \log \left\{ 1 + \sum_{j=S,F,L} Y_{jt} h_j(\tau) \right\}, \tag{9}$$

where

$$\begin{aligned} h_S(\tau) &= k(\bar\mu_S^{\mathrm{R}} - c, \tau) = \lim_{\Delta_{\bar\mu}\to 0} \{k(\bar\mu_1^{\mathrm{R}} - c, \tau) + k(\bar\mu_2^{\mathrm{R}} - c, \tau)\}/2, \\ h_F(\tau) &= \frac{\partial k(\bar\mu_S^{\mathrm{R}} - c, \tau)}{\partial \bar\mu_S^{\mathrm{R}}} = \frac{\tau e^{-(\bar\mu_S^{\mathrm{R}} - c)\tau} - k(\bar\mu_S^{\mathrm{R}} - c, \tau)}{\bar\mu_S^{\mathrm{R}} - c} = \lim_{\Delta_{\bar\mu}\to 0} \frac{k(\bar\mu_1^{\mathrm{R}} - c, \tau) - k(\bar\mu_2^{\mathrm{R}} - c, \tau)}{\Delta_{\bar\mu}}, \\ h_L(\tau) &= k(\bar\mu_L^{\mathrm{R}} - c; \tau), \end{aligned}$$

and $\bar\mu_S^{\mathrm{R}} = \lim_{\Delta_{\bar\mu}\to 0}(\bar\mu_1^{\mathrm{R}} + \bar\mu_2^{\mathrm{R}})/2$, which is assumed to be larger than c, so that the convergence to $h_S(\tau)$ and $h_F(\tau)$ is uniform.

3.4 The restricted and unrestricted versions of the model

Based on the inflation yields (5) and the real yields (9), the unrestricted nominal yield model is given by

$$y_t^{\mathrm{N}}(\tau) = Y_{\pi t} - \frac{\sigma_\pi^2 \tau^2}{6} - \tau^{-1} \log \left\{ 1 + \sum_{j=S,F,L} Y_{jt} h_j(\tau) \right\}, \tag{10}$$

$$h_j(\tau) = k(\delta_j, \tau) = \frac{1 - e^{-\delta_j \tau}}{\delta_j},\ j = S, L, \quad h_F(\tau) = \frac{\tau e^{-\delta_S \tau} - k(\delta_S, \tau)}{\delta_S},$$

where $Y_{\pi t} = \pi_t + c$ and $\delta_{jt} = \bar{\mu}_j^{\mathrm{R}} - c$. It amounts to a model with three parameters, σ_π, δ_S and δ_L. The nominal short rate is given by $r_t^{\mathrm{N}} = Y_{\pi t} - Y_{St} - Y_{Lt}$, but inflation $\pi_t = Y_{\pi t} - c$ and the real short rate $r_t^{\mathrm{R}} = c - Y_{St} - Y_{Lt}$ are not identified by fitting nominal yields only, since c is not identified. The restricted model (1) is found for $\delta_S = 1$ and $\delta_L = 0$, where $h_S(\tau) = 1 - e^{-\tau}$, $h_F(\tau) = e^{-\tau}(1+\tau) - 1$ and $h_L(\tau) = \tau$.

4 Interpretation

Least squares estimation of the parameters $(\sigma_\pi, \delta_S, \delta_L)$ and factors $(Y_{\pi t}, Y_{St}, Y_{Ft}, Y_{Lt})$ based on observed nominal yields does not identify the inflation and real yield curves separately, since c is not identified. Reasonable outcomes for inflation and real yields are found for about $c = 10$pp. and estimates of the parameters $(\sigma_\pi, \delta_S, \delta_L)$ are close to $(90\text{bp.}, 100\text{pp.}, -1\text{pp.})$ per annum.

For $\bar{\mu}_S^{\mathrm{R}} > c > \bar{\mu}_L^{\mathrm{R}}$ and real factors (Y_{St}, Y_{Ft}, Y_{Lt}) that are uniformly bounded in probability, the real bond price for long maturities, $\tau \to \infty$, is given by

$$P_t^{\mathrm{R}}(T) = \left(\frac{e^{-\bar{\mu}_L^{\mathrm{R}} \tau}}{c - \bar{\mu}_L^{\mathrm{R}}} \right) \{Y_{Lt} + o_p(1)\};$$

in the restricted model the bond price is given by $P_t^{\mathrm{R}}(T) = e^{-\bar{\mu}_L^{\mathrm{R}} \tau} \tau \{Y_{Lt} + o_p(1)\}$. The long-end factor Y_{Lt} covaries perfectly with the real "long bond" that never matures. Therefore $\bar{\mu}_L^{\mathrm{R}}$ is the real long rate. It is nonstochastic, which agrees with Dybvig, Ingersoll, and Ross (1996) and Hubalek, Klein, and Teichmann (2002).[8] As bonds have positive values, we would like to see that Y_{Lt} is positive as well.

Due to the high growth rate $\bar{\mu}_S^{\mathrm{R}}$, the factor Y_{St} affects in particular the short end of the yield curve. In fact, the real short rate is given by $r_t^{\mathrm{R}} = c - Y_{St} - Y_{Lt}$, so Y_{St} is indeed very much related to the short end of the yield curve. The factor Y_{Ft} does not affect the long end or the short end. It can be related to a specific real forward rate or real interest rate future, which explains the index F. That is, let $F_t^{\mathrm{R}}(T)$ be a real

[8] The same does not hold true for the inflation yield curve, since it approximates an arbitrage-free curve only for a bounded time horizon $\tau \leq \tau^*$.

instantaneous-maturity forward rate, so that $F_t^{\mathrm{R}}(T)P_t^{\mathrm{R}}(T) = -\frac{\mathrm{d}P_t^{\mathrm{R}}(T)}{\mathrm{d}T}$, which is the real price of a contract at time t that pays the future real short rate r_T^{R} at time T. In particular $F_t^{\mathrm{R}}(0)P_t^{\mathrm{R}}(0) = r_t^{\mathrm{R}} = c - Y_{St} - Y_{Lt}$, which does not depend on Y_{Ft}. Let $T_F = t + \tau_F$, where $\tau_F = \log(1+\bar{\delta}_S/c)^{1/\bar{\delta}_S}$, then $\left.\frac{\mathrm{d}e^{-c\tau}(1-e^{-\bar{\delta}_S\tau})}{\mathrm{d}\tau}\right|_{\tau=\tau_F} = 0$. Consequently, the real price $F_t^{\mathrm{R}}(T_F)P_t^{\mathrm{R}}(T_F)$ of a contract that pays the real short rate $r_{T_F}^{\mathrm{R}}$ at time T_F does not depend on Y_{St}. It varies only with the futures factor Y_{Ft} and the long-bond factor Y_{Lt}.

4.1 Level, slope and curvature

The partial derivatives $\partial y^{\mathrm{R}}(\tau)/\partial Y_S$, $\mathrm{d}y^{\mathrm{R}}(\tau)/\mathrm{d}Y_F$ and $\mathrm{d}y^{\mathrm{R}}(\tau)/\mathrm{d}Y_L$, evaluated at average factor values can be found in Figure 1. The factor loadings behave as loadings for two slope

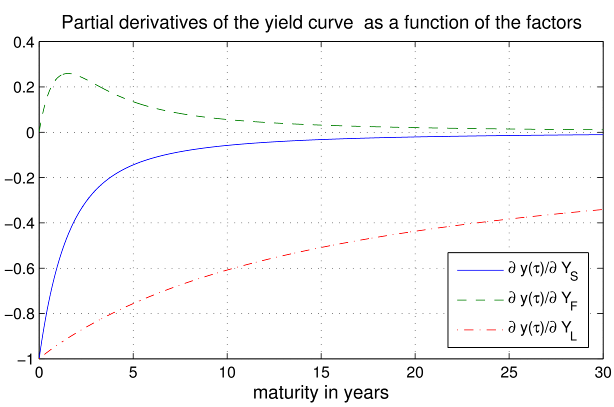

Figure 1: The derivatives $\partial y^{\mathrm{R}}(\tau)/\partial Y_S$, $\partial y^{\mathrm{R}}(\tau)/\partial Y_F$ and $\partial y^{\mathrm{R}}(\tau)/\partial Y_L$, evaluated at average factor values.

factors, Y_S and Y_L and a curvature factor Y_F. Due to the high growt rate $\bar{\mu}_S$, the factors Y_{St} and Y_{Ft} affect mostly the short end of the yield curve, while only Y_{Lt} affects the long end. Christensen et al. (2009) use a five-factor NS extension to formulate an arbitrage-free extension of the Svensson model. It is given by

$$y_t(\tau) = L_t + \sum_{i=1}^{2} S_{it}\left(\frac{1-e^{-\lambda_i\tau}}{\lambda_i\tau}\right) + \sum_{i=1}^{2} C_{it}\left(\frac{1-e^{-\lambda_i\tau}}{\lambda_i\tau} - e^{-\lambda_i\tau}\right), \tag{11}$$

where $(L_t, S_{1t}, S_{2t}, C_{1t}, C_{2t})$ are interpreted as a level, two slopes and two curvature factors. The Nelson-Siegel model has $S_{2t} = C_{2t} = 0$, and the Svensson model has $S_{2t} = 0$; the second slope is necessary for the arbitrage-free extension. For small maturities the real part of yield model (10) is given by

$$y_t^{\mathrm{R}}(\tau) = c - \sum_{i=S,L} Y_{it} \left(\frac{1 - e^{-\delta_i \tau}}{\delta_i \tau} \right) - \left(\frac{Y_{Ft}}{\delta_S} \right) \left(\frac{1 - e^{-\delta_S \tau}}{\delta_S \tau} - e^{-\delta_S \tau} \right) + o_p(1).$$

Together with the inflation level factor, the short end of the yield curve has indeed two slope factors, Y_{St} and Y_{Lt}, and one curvature factor Y_{Ft}.[9] In particular, the curvature is interesting. Concerning the interpretation of curvature, Diebold, Rudebusch, and Aruoba (2006) couldn't find any reliable macroeconomic links to curvature. Here we find a sensible theoretical interpretation, where curvature is linked to the futures factor Y_{Ft}. It has an interesting consequence for the factor dynamics as well.

4.2 Factor dynamics

As the cross-sectional model is relevant by iself, the present paper does not focus on a particular specification of the factor dynamics. Still, a very interesting result can be derived easily. Let the real portfolios V_{it} have diffusions $\mathrm{d}V_{it}^{\mathrm{R}} = V_{it}^{\mathrm{R}} \left(\mu_{it}^{\mathrm{R}} \,\mathrm{d}t + \sum_{j=1}^{3} \sigma_{ijt}^{\mathrm{R}} \,\mathrm{d}W_{jt}^{\mathrm{R}} \right)$, then the factors have diffusions $\mathrm{d}X_{it} = X_{it}(\mu_{it}^{\mathrm{R}} - \bar{\mu}_i^{\mathrm{R}}) \,\mathrm{d}t + X_{it} \sum_{j=1}^{3} \sigma_{ijt}^{\mathrm{R}} \,\mathrm{d}W_{jt}^{\mathrm{R}}$. The drift term of Y_{Lt} is given by $X_{3t}(\mu_{3t}^{\mathrm{R}} - \bar{\mu}_3^{\mathrm{R}}) = Y_{Lt}(\mu_{Lt} - \bar{\mu}_L)$. However, the drift term of Y_{St} has a different form. As the detrending components in Y_{St} are based on

$$X_{1t}\bar{\mu}_1^{\mathrm{R}} + X_{2t}\bar{\mu}_2^{\mathrm{R}} = \left(\frac{\bar{\mu}_1^{\mathrm{R}} + \bar{\mu}_2^{\mathrm{R}}}{2} \right) (X_{1t} + X_{2t}) + \Delta_{\bar{\mu}}(X_{1t} - X_{2t})/2,$$

the drift term of Y_{St} is given by $Y_{St}(\mu_{St} - \bar{\mu}_S) - Y_{Ft}$. That is to say, the drift term of the short-end slope factor Y_{St} increases as the futures factor Y_{Ft} decreases. All other things being equal, the growth rate of the short-end slope is higher for negative curvature, than it is for positive curvature. According to the model, curvature has predictive potential, where low curvature points in the direction of increasing slopes. This interpretation of curvature

[9]Björk and Christensen (1999) introduce a Nelson-Siegel extension with one level factor, two slope factors and a single curvature factor with the restriction that the second slope parameter is twice the value of the other slope parameter. However, it does not fit as well as the Svensson model.

holds for real-world dynamics: we conjecture curvature varies with changing interest rate futures positions.[10]

5 Application to US data

For the empirical application I use US government yields that are described in more detail in Appendix A.3. To estimate the parameters and to compute RMSE as a measure of fit, all yields from 4 January 1982 to 30 September 2008 are used, which amounts to $n = 6688$ days and $N = 65738$ yields. That way the period of low interest rates that follow the possibly anticipated QE announcement at 25 November 2008 are excluded. I assume the FED interventions affect the yield curve to such an extent that the Fisher hypothesis cannot be maintained. The same applies to the independence assumptions of the arbitrage-free extension of the Nelson-Siegel model.

Estimating the Nelson-Siegel model by NLS amounts to $\hat{\lambda}_1 = 0.5161$. For the arbitrage-free NS model, with independent factors, we find $\hat{\lambda}_1 = 0.4784$ and $\sigma_1^2 = -0.0047^2$, $\sigma_2^2 = -0.0635^2$ and $\sigma_3^2 = 0.0892^2$. So, least squares estimation produces negative variances. For the Svensson model we find $\hat{\lambda}_1 = 0.4738$ and $\hat{\lambda}_2 = 0.0684$. For the restricted model (1) we find $\hat{\sigma}_\pi^2 = 0.0094^2$, while $\delta_S = 1$ and $\delta_L = 0$. Finally for the unrestricted model (10) we have $\hat{\sigma}_\pi^2 = 0.0093^2$, $\hat{\delta}_S = 1.0015$ and $\hat{\delta}_L = -.0095$.

Results on root-mean squared errors (RMSE) can be found in Table 1, which also gives an estimate of the error variance σ_ε^2, which is corrected for the number of degrees of freedom, $\hat{\sigma}_\varepsilon^2 = (N - \ell)^{-1} \sum_{i=1}^N (y_i - \hat{y}_i)^2 m_i/(m_i - k)$, where m_i is the number of yields observed at the day yield y_i is observed, ℓ is the number of parameters, and k is the number of factors. The mean error and RMSE per maturity can be found in Table 2. Clearly, the four-factor models fit better than the three-factor Nelson-Siegel models. In particular the second curvature term of the Svensson model is helpful in reducing the idiosyncratic

[10] In arbitrage-free extensions of the Nelson-Siegel model (Christensen et al. 2009, 2011) curvature affects slope factors as well. However, these results hold under the Q measure; they are silent about the dynamics under the P measure. Christensen et al. (2011) mention: "This very indirect role of curvature accords with the empirical literature where it has been difficult to find sensible interpretations of curvature under the P measure (Diebold et al., 2006)".

	NS	AFNS	Svensson	Model (1)	Model (10)
RMSE	10.94	9.85	7.62	7.05	7.05
$\hat{\sigma}_\varepsilon$	11.84	10.75	8.68	8.35	8.34

Table 1: The root mean squared error (RMSE) and the standard error ($\hat{\sigma}_\varepsilon$) over all observed yields between 4 Jan 1982 and 30 Sept. 2008. All numbers are measured in basis points.

	NS		AFNS		Svensson		Model (1)		Model (10)	
Mat.	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
1/12	-5.96	14.96	-1.21	14.33	-1.10	10.36	-1.36	6.58	-1.42	6.56
3/12	-2.12	9.08	0.20	9.40	0.07	6.13	1.15	5.05	1.12	5.03
6/12	2.72	8.14	1.82	7.72	2.57	7.31	1.48	6.43	1.50	6.43
1	0.10	10.37	-3.78	11.10	-2.15	8.37	-4.91	7.45	-4.84	7.39
2	4.96	7.68	2.41	6.97	0.58	6.65	1.96	5.30	1.98	5.32
3	-1.00	4.99	0.20	4.68	-2.60	5.63	0.61	4.74	0.56	4.75
5	-3.89	8.03	0.69	7.35	-0.24	5.31	2.07	4.74	1.97	4.69
7	0.00	7.71	2.33	7.64	4.87	8.63	3.66	5.19	3.63	5.16
10	-4.75	10.45	-8.37	11.70	-2.51	8.38	-9.46	11.60	-9.34	11.51
20	24.31	25.97	16.81	19.26	2.34	7.25	8.75	12.63	9.02	12.82
30	-8.07	12.86	-4.60	10.03	-1.68	9.74	-1.38	5.49	-1.52	5.56

Table 2: The mean error (Mean) and the root mean squared error (RMSE) for 11 different observed maturities between 4 Jan 1982 and 30 Sept. 2008. All numbers are measured in basis points, except maturity (Mat.), which is in years.

errors of the 10 year and 20 year yields.[11] However, the arbitrage-free models (1) and (10) fit better over the whole range of maturities than the Svensson model. Even the restricted model that uses a single parameter has a lower RMSE. The specification of the restricted model emphasizes that the new models (1) and (10) may be less hampered by non-identification issues than the Svensson model.[12]

The factors are given in the plots of Figure 2. The real factors Y_{St}, X_{Ft} and Y_{Lt} show

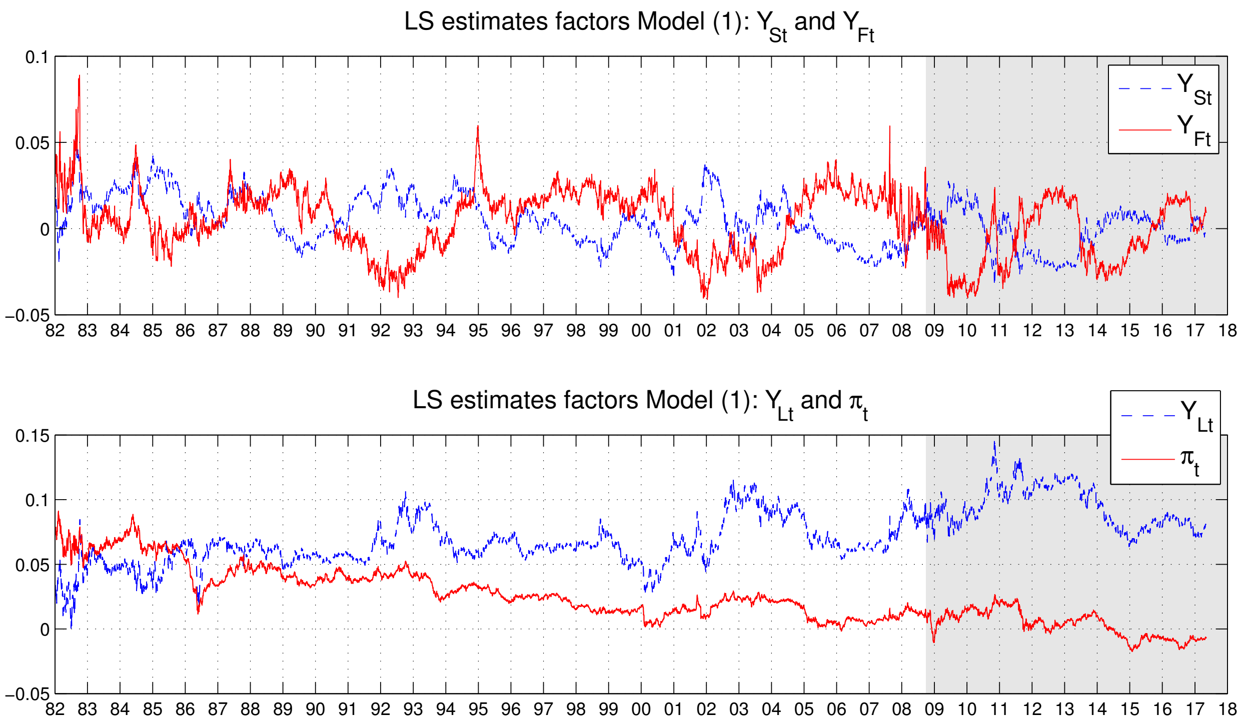

Figure 2: Factors Model (1): Y_{St} and Y_{Ft} in the top panel, Y_{Lt} and $\pi_t = Y_{\pi t} - 0.09$ in the bottom panel. The data in the shaded area have not been used to estimate the parameters.

nice mean reverting movements. Perhaps the relative large values of Y_{Lt} after 30 September 2008, in the grey area, are due to FED interventions.

To focus on the real interest rate, Figure 3 gives two estimates of the 1 month real interest rate. The first estimate is based on Haubrich et al. (2012) (HPR) who used Treasury yields, survey inflation forecasts, and inflation swap rates to estimate, among

[11]Christensen et al. (2009) observe for the dynamic version of the Svensson model, DNSS, there is a clear relationship between the seond curvature factor and the 10-year yield.

[12]Identification and numerical issues when estimating the Svensson model are addressed in (Bolder and Stréliski, 1999; De Pooter, 2007; Gimeno and Nave, 2009, e.g.)

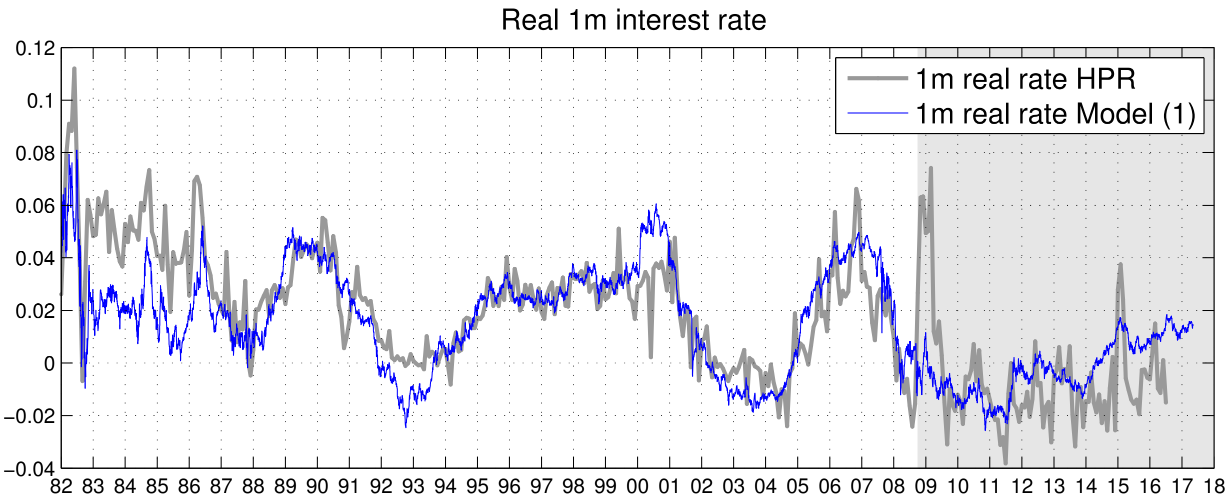

Figure 3: One month interest rate: The 1m real interest rate from Haubrich, Pennacchi and Ritchken (2008) and the 1m real yield based on Model (1). The data in the shaded area have not been used to estimate the parameters.

other quantities, the 1m real rate.[13][14]. The second graph is the real 1m real rate based on Model (1), where only nominal Treasury yields have been used. The level of the Model (1) real rate is not identified, but for $c = 0.09$, the changes can be compared to the HPR rate. The Model (1) rate is less volatile, it shows less trend and looks more stationary, while the HPR rate starts high in the period where inflation was high and ends low, when inflation was low. Although, the inflation component of Model (1) is extremely simple, the implied 1m real rate looks promising as a potential candidate for measuring changes in real interest rates.

Both Y_{St} and Y_{Lt} affect the short rate, and Figure 4 shows a linear transformation of their sum, or $c - r_t$. It covaries quite closely with the slope of the yield curve as expressed by the observed difference between the 10y nominal yield and the 3m nominal yield. The bottom panel of Figure 4 shows the observed slope and a linear transformation of $-Y_{ft}$. Over a substantial time interval the curvature, or futures factor Y_{Ft} seems to be leading. This observation is in agreement with the theoretical result of Section 4.2 that the drift term of Y_{St} is given by $Y_{St}(\mu_{St} - \bar{\mu}_S) - Y_{Ft}$. According to the model, curvature must

[13]Their model is in the completely affine class with four stochastic drivers and seven state variables.

[14]I would like to thank George Pennacchi who made the data available to me.

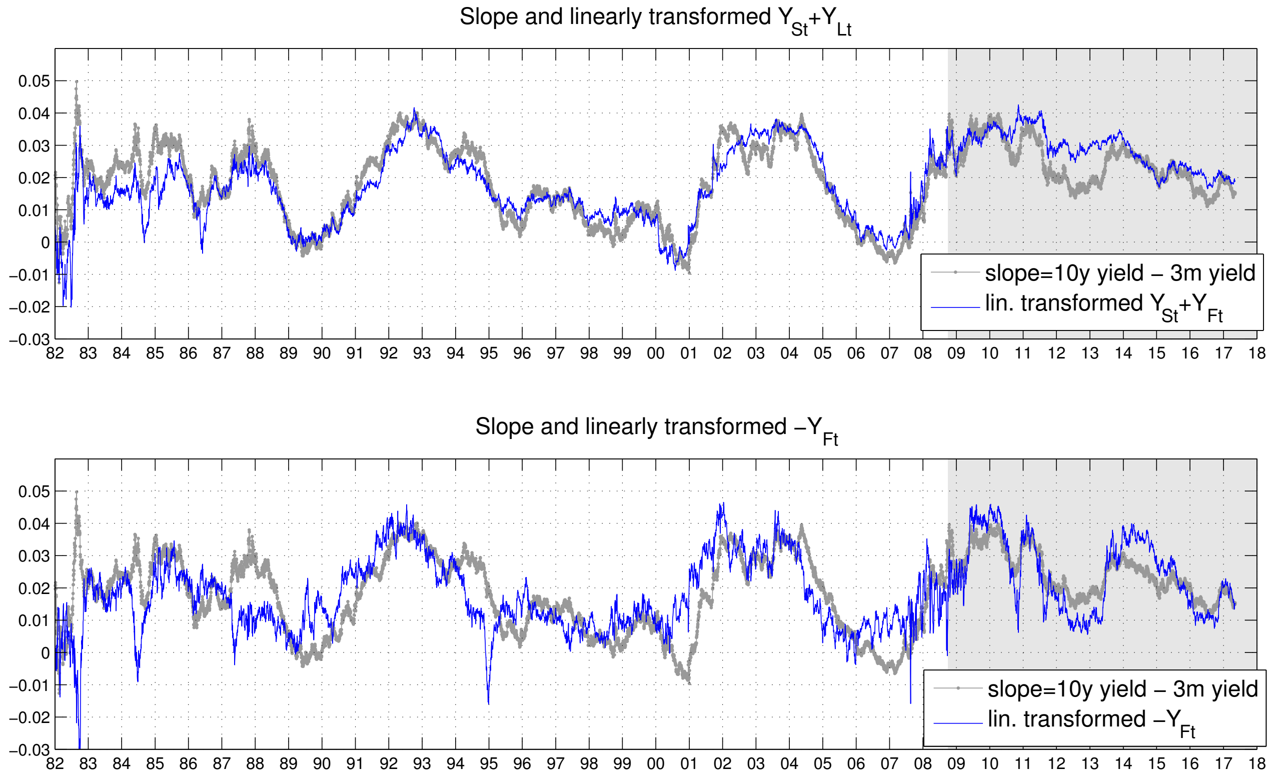

Figure 4: Slope and curvature: The observed slope $y_t(10) - y_t(1/4)$ and linearly transformed $Y_{St} + Y_{Lt}$ with the same sample mean and variance in the top panel, and the observed slope and linearly transformed $-Y_{Ft}$ in the bottom panel. The data in the shaded area have not been used to estimate the parameters.

have predictive potential, where low curvature points in the direction of increasing slopes. Figure 4 provides empirical evidence in line with that hypothesis.

6 Conclusion

The paper provides a parsimonious, arbitrage-free yield model that performs well, when compared to the Nelson-Siegel model, its arbitrage-free extension based on independent factors, and the Svensson model in terms of in-sample fit. The factors can be interpreted in terms of inflation and real yields. Among the real factors we distinguish between the long-bond factor, the short-end slope factor, which determines the short rate together with the long-bond factor, and a curvature or futures factor that we might interpret in terms of real interest rate futures positions. This futures factor is shown theoretically to have predictive potential for the short-end slope factor, which is confirmed empirically by evidence that is in line with this hypothesis.

As a next step, we plan to specify the real factor dynamics. It can be done with considerable flexibility without affecting the functional form of the yield curves as described in this paper. The only requirement is that the dynamics should be specified in an arbitrage-free manner, since the real factors are assumed to be portfolio values. With such a specification attention could center on measuring risk premia and out-of-sample prediction of future yield curves. However, as the Nelson-Siegel family has shown, a parsimonious yield model is a very useful tool by itself. This paper provides a simple, interpretable, accurate and arbitrage-free alternative.

Appendix

A.1 Linearization and Ridge regression

Referring to the ridge regression of Section 2, let $\boldsymbol{y}_t^{obs}$ be an $m \times 1$ vector of observed nominal yields. Let $\hat{\boldsymbol{y}}_t = [\hat{y}_t(\tau_1), \ldots, \hat{y}_t(\tau_m)]'$ be fitted values given a value σ_π, where

$$\hat{y}_t(\tau) = \widehat{Y}_{\pi t} - \frac{\sigma_\pi^2 \tau^2}{6} - \tau^{-1} \log \left\{ 1 + \sum_{i=S,F,L} \widehat{Y}_{it} h_i(\tau) \right\},$$

and $h_S(\tau)$, $h_F(\tau)$ and $h_L(\tau)$ are implicitly defined based on model (1). The fitted factors $\widehat{\boldsymbol{Y}}_t(\tau) = \left[\widehat{Y}_{\pi t}(\tau), \widehat{Y}_{St}(\tau), \widehat{Y}_{Ft}(\tau), \widehat{Y}_{Lt}(\tau)\right]'$ can be computed using $\boldsymbol{H}_t = [\boldsymbol{H}_t(\tau_1)', \ldots, \boldsymbol{H}_t(\tau_m)']'$, where

$$\boldsymbol{H}_t(\tau) = \left[1, \ \frac{-\left[h_S(\tau), \ h_F(\tau), \ h_L(\tau)\right]}{\tau \left\{ 1 + \sum_{i=S,F,L} h_i(\tau) \widehat{Y}_{it} \right\}} \right],$$

such that $\widehat{\boldsymbol{Y}}_t = \widehat{\boldsymbol{Y}}_{t-1} + (\boldsymbol{H}_{t-1}' \boldsymbol{H}_{t-1} + \alpha \boldsymbol{I}_4)^{-1} \boldsymbol{H}_{t-1}' (\boldsymbol{y}_t^{obs} - \hat{\boldsymbol{y}}_{t-1})$, where $\boldsymbol{I}_4$ is the 4×4 identity matrix. I use $\alpha = 0.001$.

A.2 Real bond prices

With reference to the real yield curve of Section 3.2, let the real stochastic discount factor, be driven by three factors,

$$\mathrm{d}\Lambda_t^{\mathrm{R}} = -\Lambda_t^{\mathrm{R}} \left(r_t^{\mathrm{R}} \, \mathrm{d}t + \sum_{j=1}^{3} \lambda_{jt} \, \mathrm{d}W_{jt}^{\mathrm{R}} \right),$$

where r_t^{R} is the real short rate and $\lambda_{1,t}$, $\lambda_{2,t}$ and $\lambda_{3,t}$ are three prices of risk, related to the independent Brownian motions W_1^{R}, W_2^{R} and W_3^{R}. Without need to specify the prices of risk, the expected value of the real stochastic discount factor is given by $\mathrm{E}_t\left(\Lambda_T^{\mathrm{R}}\right) = \Lambda_t^{\mathrm{R}} - \mathrm{E}_t\left(\int_t^T \Lambda_u^{\mathrm{R}} r_u^{\mathrm{R}} \, \mathrm{d}u\right)$. The real bond prices are thus given by

$$P_t^{\mathrm{R}}(T) = \mathrm{E}_t(\Lambda_T^{\mathrm{R}} / \Lambda_t^{\mathrm{R}}) = 1 - \int_t^T \mathrm{E}_t(r_u^{\mathrm{R}} \Lambda_u^{\mathrm{R}} / \Lambda_t^{\mathrm{R}}) \, \mathrm{d}u.$$

As $r^{\mathrm{R}} = c - X_{1t} - X_{2t} - X_{3t}$ and $\mathrm{E}(\Lambda_T^{\mathrm{R}} X_{iT}) = \Lambda_t^{\mathrm{R}} X_{it} e^{-\bar{\mu}_i^{\mathrm{R}}(T-t)}$, the real bond prices satisfy

$$P_t^{\mathrm{R}}(T) = 1 - c\int_t^T P_t^{\mathrm{R}}(u)\,\mathrm{d}u + \sum_{i=1}^{3} X_{it}\int_t^T e^{-\bar{\mu}_i^{\mathrm{R}}(u-t)}\,\mathrm{d}u,$$

which is expressed more compactly as a differential equation by

$$\frac{\mathrm{d}P_t^{\mathrm{R}}(T)}{\mathrm{d}T} = -cP_t^{\mathrm{R}}(T) + \sum_{i=1}^{3} X_{it}e^{-\bar{\mu}_i^{\mathrm{R}}\tau}.$$

As $P_t^{\mathrm{R}}(t) = 1$, the general solution is given by $P_t^{\mathrm{R}}(T) = e^{-c\tau}\left\{1 + \sum_{i=1}^{m} X_{it}k(\bar{\mu}_i^{\mathrm{R}} - c, \tau)\right\}$, where $k(\delta, \tau) = \int_0^\tau e^{-\delta s}\,\mathrm{d}s = \delta^{-1}(1 - e^{-\delta\tau})$. The real forward rates are given by

$$F_t^{\mathrm{R}}(T) = -\frac{\mathrm{dlog}\,P_t^{\mathrm{R}}(T)}{\mathrm{d}T} = c - \frac{\sum_{i=1}^{3} X_{it}e^{-\bar{\mu}_i^{\mathrm{R}}\tau}}{P_t^{\mathrm{R}}(T)}.$$

A.3 US Data

Referring to Section 5, I use daily data on constant maturity Treasury yields from the H.15 release by the Federal Reserve Board, which are publicly available via Federal Reserve Economic Database (FRED) of the Federal Reserve Bank of St. Louis.[15] The dataset contains daily observations from January 4, 1982 to April 19, 2017, covering $n = 8826$ observation days, with 1, 3 and 6 months and 1, 2, 3, 5, 7, 10, 20 and 30 years maturities. Treasury yields represent par yields on Treasury bonds paying a semi-annual coupon. Zero-coupon yields are bootstrapped from Treasury yields by assuming constant forward rates between observed maturities.

The 1 month, 20 and 30 year yields are only partially observed over the sample period. Publication of the 30 year maturity started before January 4, 1982, but it was interrupted from February 19, 2002, through February 8, 2006, as the Treasury was no longer auctioning 30 year bonds. The Treasury publishes extrapolation factors to construct an estimate the 30 year yield and these estimates are used for that period.[16] The 20 year yield is published

[15] See `https://research.stlouisfed.org/fred2`

[16] See for the extrapolation factors and their usage: `http://www.treas.gov/offices/domestic-finance/debt-management/interest-rate/ltcompositeindex.shtml`

starting from October 1, 1993 and the 1 month yield is published starting from 31 July, 2001. Both are treated as missing before these starting dates.

References

C. Almeida and J. Vicente. The role of no-arbitrage in forecasting: lessons from a parametric term structure model. *Journal of Banking and Finance*, 32:2695–2705, 2008.

P. A. Bekker and K. E. Bouwman. The term-structure of interest rates and macro-portfolio returns. *Working Paper*, 2011.

T. Björk and B. J. Christensen. Interest rate dynamics and consistent forward rate curves. *Mathematical Finance*, 9:323–348, 1999.

D. Bolder and D. Stréliski. Yield curve modelling at the Bank of Canada. Technical Report 84, Bank of Canada, 1999.

C. G. Bowsher and R. Meeks. The dynamics of economic functions: Modeling and forecasting the yield curve. *Journal of the American Statistical Association*, 103(484):1419–1437, 2008.

J. H. E. Christensen, F. X. Diebold, and G. D. Rudebusch. An arbitrage-free generalized nelson-siegel term structure model. *Econometrics Journal*, 12:C33–C64, 2009.

J. H. E. Christensen, F. X. Diebold, and G. D. Rudebusch. The affine arbitrage-free class of nelson-siegel term structure models. *Journal of Econometrics*, 164:4–20, 2011.

M. De Pooter. Examining the Nelson-Siegel class of term structure models. Tinbergen Institute Discussion paper TI 2007-043/4, 2007.

F. X. Diebold and C. Li. Forecasting the term structure of government bond yields. *Journal of Econometrics*, 130:337–364, 2006.

F. X. Diebold, G. D. Rudebusch, and B. Aruoba. The macroeconomy and the yield curve: a dynamic latent factor approach. *Journal of Econometrics*, 131:309—338, 2006.

P. H. Dybvig, J. E. Ingersoll, Jr, and S. A. Ross. Long forward and zero-coupon rates can never fall. *The Journal of Business*, 69:1–25, 1996.

D. Filipović. A note on the Nelson-Siegel family. *Mathematical Finance*, 9:349–359, 1999.

Bank for International Settlements. Zero-coupon yield curves Technical Documentation. BIS papers 25, Bank for International Settlements, 2005.

X. Gabaix. Linearity-generating processes: A modelling tool yielding closed forms for asset prices. Working paper, New York University, 2009.

R. Gimeno and J.M. Nave. A genetic algorithm eestimation of the term structure of interest rrates. volume 53, pages 2236–2250. 2009.

J.G. Haubrich, G. Pennacchi, and P. Ritchken. Inflation expectations, real rates, and risk premia: Evidence from inflation swaps. *The Review of Financial Studies*, 25(5): 1588–1629, 2012.

F. Hubalek, I. Klein, and J. Teichmann. A general proof of the dybvig-ingersoll-ross-theorem: Long forward rates can never fall. *Mathematical Finance*, 12(4):447–451, 2002.

C. Nelson and A. Siegel. Parsimonious modeling of yield curves. *Journal of Business*, 60: 473–489, 1987.

L. E. O. Svensson. Estimating forward interest rates with the extended Nelson-Siegel method. *Quaterly Review,* Sveriges Riksbank, 3:13–26, 1995.

O. Vasicek. An equilibrium characterization of the term structure. *Journal of Financial Economics*, 5:177–188, 1977.

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