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Abstract This paper develops a framework that allows the order in which consumers visit firms to depend on a priori available product information and consumer preferences. Consumers first visit the firm which is most likely to offer the product best to their liking. Prices and profits turn out to be higher than under the traditional assumption of a random search order. Under the proposed search rule consumers obtain on average a better match and search less often. These gains in efficiency result in higher total welfare, although consumers are worse off under the alternative search rule due to higher prices.

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1 Introduction

On many markets there are multiple suppliers who offer products which differ in several dimensions and, in order to find the best offer on the market, consumers will have to search. Consumers might search randomly amongst the options, however, one's preferences might influence the search order. For instance, if one wants to decide what to watch on television, one can consult websites which suggest programs based on the preferences entered in an earlier stage. If one wants to buy a new DVD, shopping websites suggest titles based on the order history. When people are using an online search engine or comparison website to find a product, they might enter not only the product name but also specify some characteristics they like. If a person wants to purchase a car with high MPG, she will not start shopping at a car-dealer who advertises SUV's. Similarly, an employee of a bookstore will not suggest to start looking in the comics section when a consumer indicates that she likes classical literature. It is clear that consumers often start searching where they are likely to find a product close to their preferences, and this choice can be made after consulting advertisements, comparison websites, consultancy agencies, or acquaintances.

The existing literature does not allow consumer preferences and available product information to play a role in the consumers search process. This paper takes this assumption away. To model the role of the preferences in the search order, I assume that a product consists of so-called communicable and incommunicable horizontal characteristics. Consider, for instance, a consumer wanting to buy a laptop. In order to find a suitable model she gathers information. For instance by visiting a comparison website and entering her preferences on communicable attributes, such as screen size and color. Based upon this information a ranking of suppliers is obtained by the consumer and she can calculate the utility associated with the communicated characteristics. The consumer then visits the top suggested firm in order to find out its price and inspect the product's incommunicable characteristics, such as whether the laptop is convenient to handle and whether the screen is readable from different angles. She can then buy the laptop or decide to continue her search until she does find an offer to her liking. For simplicity this paper assumes that if the consumer decides to continue her search, she does so in a random fashion and does not use the provided ranking any further. I expect, however, that my findings carry over to the more general framework in which she does use it. Furthermore, note that my model can also be justified if one is willing to assume that consumers forget about the ranking of firms once they visit the first firm or when the source of the consumer's information (comparison website/acquaintances/consultancy agencies) only provides the best suggestion.

I find that prices and profits are higher when consumers use provided product information in their search process. This is because firms exploit the fact that consumers first visit the firm with the highest communicable attributes and are therefore less likely to visit a competitor. In effect, products are ex-ante less homogeneous for consumers when product characteristics are communicated.

My model extends the seminal work of Anderson and Renault (1999). In that article consumers visit firms in a random order, by introducing communicable characteristics I allow consumer preferences to influence the search order. When, in my framework, a product only consists of incommunicable characteristics consumers search at random and the model of Anderson and Renault (1999) results. However, when all product characteristics are available to consumers prior to search the Diamond (1971) paradox emerges. The result in Anderson and Renault (1999) that prices are non-decreasing in search costs carries over to my work.

This paper provides some interesting insights on the welfare effects of search as well. When product information is used to determine the search order consumers obtain on average a better match, as they start searching at the firm where they are most likely to find the product that fits their tastes the best. Moreover, consumers are less likely to continue search in each stage of the process. This is because they know that for certain characteristics the product of the first firm is the best on the market. Hence, consumers spend less on search costs. These two effects ensure that total welfare is higher under this alternative search strategy than in case of random search. However, consumers turn out to be worse off under the alternative search strategy as these two effects are outweighed by the higher prices consumers pay.

My findings contribute to the literature on consumer search with differentiated products, which is built around the classic papers by Wolinsky (1986) and Anderson and Renault (1999). These and subsequent works often assume totally random search by consumers amongst firms. However, there are some exceptions. Arbatskaya (2007) considers a market with homogeneous products where consumers face heterogeneous search costs and search in an exogenously given order. Firms have knowledge about this search order, and they charge higher prices when they are visited in an early stage. In the work of Zhou (2011) consumers also search in a fixed order, however, products are horizontally differentiated. He finds that prices increase in the search order, since firms visited in a later stage exploit the fact that consumers who sample them must have relatively low valuations for the offered product of earlier sampled firms. Other literature in which the search order is non-random includes Wilson (2010), who considers a homogeneous good market in which firm's search costs are endogenized. Firms can obfuscate, which makes it more time-consuming for consumers to inspect a product and learn its price. In equilibrium Wilson finds that consumers are more likely to visit a firm with low search costs. A similar approach is taken by Ellison and Wolitzky (2013), but they assume consumers can not observe the level of obfuscation prior to arrival. In the empirical work of Hortaçsu and Syverson (2004) the sampling probability of a firm is proxied by advertising expenditures. Haan and Moraga-González (2011) and Armstrong et al. (2009) also consider non-random search models. Armstrong et al. (2009) allow one firm to be prominent in the market, and this firm is always sampled first. I, however, do not impose this a-symmetry and, a priori, every firm is equally likely to be sampled first by a consumer. Moreover, and most importantly, in my case the sampling depends on consumer preferences. The work by Haan and Moraga-González (2011) takes a different approach. They consider a consumer search model in which consumers first visit the firm whose advertising is most salient. Finally, Haan et al. (2015) consider a duopoly in which prices are advertised along with some product characteristics. Consumers first sample the firm at which they expect to find the best deal, as in my model. They find that prices are decreasing in search costs because it is more important to attract consumers on their first visit when these costs are high.

The remainder of this paper is organized as follows. Section 2 introduces the model. In section 3 equilibrium prices are derived. Section 4 presents the benchmark model. Comparative statics are discussed in section 5. In section 6 I conduct a welfare-analysis. Section 7 concludes.

2 The model

I consider a market with $n \geq 2$ firms selling horizontally differentiated products. Firms face constant marginal costs, which I normalize to zero. Demand is assumed to be inelastic. Without loss of generality the market size is normalized to 1. A consumer buying from firm $i \in \{1, \ldots, n\}$ receives utility

$$u_i = (1-\lambda)\varepsilon_i + \lambda v_i - p_i.$$

Here p_i is the price charged. $(1-\lambda)\varepsilon_i + \lambda v_i$ is the stochastic match value between a consumer and product i. Match values are independently distributed across products, moreover, I assume ε_i and v_i are independently distributed random variables. ε_i is the realization of a distribution F and is referred to as the Incommunicable Part of the Match Value, *IPMV* henceforth. v_i is the realization of a distribution H and is referred to as the Communicable Part of the Match Value, *CPMV* from now on. The total stochastic match value, denoted by $t_i = (1-\lambda)\varepsilon_i + \lambda v_i$, has a distribution which is the weighted convolution of F and H, which is denoted by M. Let f, h and m be the densities associated with F, H and M, respectively. f and h are taken to be continuous. $[a_F, b_F]$ and $[a_H, b_H]$ respectively denote the supports of F and H on the extended real line. Let the bounds on t_i be denoted by

$$a = (1-\lambda)a_F + \lambda a_H \quad \text{and} \quad b = (1-\lambda)b_F + \lambda b_H.$$

Firms can not discriminate in prices as they are unable to observe match values. In order to find the best combination of match value and price consumers search sequential among firms with perfect recall. The price charged by a firm can only be discovered by visiting the firm, for which a consumer has to incur search cost s. Consumers have some information about the match with firms and use this to determine their search order. This strategy is presented below, and will be referred to as *directed search.*

Stage 1 Consumers are informed about which firm has the highest CPMV on the market, for example by means of a comparison website, advertisements, an acquaintance, or an intermediary. One might imagine a consumer looking to buy a laptop consulting an advisor. The advisor knows some of the characteristics of the laptops on offer on the market. For instance, she knows which suppliers offer laptops with small screens and are convenient to travel with or which firms sale laptops with Arabic or American keyboards. However, before buying a laptop a consumer might also want to consider how it handles and whether the screen is readable from different angles. Hence, the advisor knows the communicable characteristics a consumer would get if she would buy from a particular supplier, but has no information about the incommunicable characteristics. When the consumer visits the advisor, she reveals her preferences. Based on this information, the advisor tells the consumer at which supplier she is expected to find a laptop that fits her preferences the best. The consumer can then calculate the CPMV based upon this information.[2]

Stage 2 The consumer first visits the firm with the highest CPMV. Upon arrival she observes the price the firm is charging and the total match value of the product, including the IPMV. The consumer takes this into account when calculating the expected benefit of continuing search, which she compares to the costs of doing so. If the search costs are higher than the expected benefit of search she will buy at the current supplier, otherwise the consumer continues her search by randomly visiting another firm. In calculating the expected benefit of searching, the consumer incorporates the fact that for any other product the CPMV is smaller than at the first firm.[3]

Notice that the directed search rule is a generalization of the one presented in the standard consumer search models, see for instance Anderson and Renault (1999). In those models, consumers search completely at random without using any product information. The assumption of not using product information is relaxed in my model by allowing consumers to determine the first shop they visit based on the advise of intermediating agencies, advertisements, or comparison websites.

[2]Notice that the model allows for the case that the consumer is not able to make her preferences fully known to the advisor. This friction can be captured in the IPMV. The advise of the intermediating firm might not be optimal due to this lack of knowledge, and consumers might for this reason find a better product on the market if they continue their search beyond the advised firm.

[3]The model allows for an alternative interpretation. In that case consumers obtain all the CPMV's in Stage 1 (possibly through reading advertisements) and visit in Stage 2 the firm with the highest CPMV first. If a consumer decides to search onwards, she does not use the information of the CPMV's of the other firms, possibly because she forgot them. Only the CPMV value of the first firm is then used under directed search. Future research might still look into the possibility of generalizing the search rule further and allowing for the consumer to remember every CPMV. In such a model the consumer faces, in each stage, a different expected benefit of continuing search, depending on the CPMV of the next best firm. This complicates the model severely.

3 The pricing Nash equilibrium

In this section I derive a symmetric Nash equilibrium in prices. As I look for a symmetric Nash equilibrium, I need to consider the best response of firm $i \in \{1, \ldots, n\}$ when all other firms set some price p^*. Setting p^* should be optimal for firm i as well. Below I derive the demand and profit function for firm i.

Consider a consumer before entering the market. She gets informed about which firm has $v_m = \max_{j \in \{1,\ldots,n\}}\{v_j\}$, the highest CPMV, and she visits this firm first. Suppose a consumer has decided to visit firm i first: $v_m = v_i$. If she decides to buy from firm i she will receive an utility of $(1-\lambda)\varepsilon_i + \lambda v_i - p_i$. If she continues search and buys from firm j she receives $(1-\lambda)\varepsilon_j + \lambda v_j - p^*$. Define $\Delta = p_i - p^*$ and $x \equiv (1-\lambda)\varepsilon_i + \lambda v_i - \Delta$. When the consumer arrives at firm i she learns x. The consumer is better off at firm j when $t_j > x$. The expected benefit of continuing searching (net of search costs) from firm i is thus given by:

$$g(x, v_m) = E(t_j - x | v_j \leq v_m) = \int_{-\infty}^{v_m} \int_{\frac{x-\lambda v_j}{1-\lambda}}^{\infty} ((1-\lambda)\varepsilon_j + \lambda v_j - x) f(\varepsilon_j) \mathrm{d}\varepsilon_j \frac{h(v_j)}{H(v^m)} \mathrm{d}v_j.$$

Notice that the consumer does take into account that she has already visited the firm with CPMV v_m and therefore the CPMV's for the remaining firms will be lower. This is why the conditional density $h(v_j)/H(v_m)$ figures in this expression.

$g(x, v_m)$ is strictly decreasing in x and goes from $+\infty$ to zero as x goes from $-\infty$ to $+\infty$. Let $\hat{x}$ be implicitly defined by $g(\hat{x}, v_m) = s$, which exists and is unique for each v_m given the above and $s \in (0, \infty)$. It follows that for a consumer it is beneficial to continue search if the total match value at a firm is lower than $\hat{x}$. Notice that $\hat{x}$ depends on v_m, moreover, the random search model à la Anderson and Renault (1999) is obtained when v_m is replaced by b_H in $g(x, v_m)$ or when $\lambda = 0$. In addition, as $s > 0$, $\hat{x} < (1-\lambda)b_F + \lambda v_m$.

I now derive demand for firm i. As I am considering a symmetric Nash equilibrium, the consumer expects that every shop is charging price p^*, and does not anticipate firm i charging p_i. Suppose $v_i = v_m$, so a consumer arrives at firm i first. This consumer does not continue her search and buys from firm i whenever $(1-\lambda)\varepsilon_i + \lambda v_m > \hat{x}$. This happens, for a given v_m, with probability $1 - F\left(\frac{\hat{x}+\Delta-\lambda v_m}{1-\lambda}\right)$. The probability that the firm i has the highest CPMV on the market is $H(v_m)^{n-1}$. Therefore, the fraction of the population arriving at i first and staying there to buy can be found by multiplying these two terms, weighing this term with density of v_m, and then integrating over v_m. This procedure yields

$$Q_1(p_i, p^*) = \int_{-\infty}^{\infty} H(v_m)^{n-1} \left[1 - F\left(\frac{\hat{x} + \Delta - \lambda v_m}{1-\lambda}\right)\right] h(v_m) \mathrm{d}v_m. \tag{1}$$

Now suppose firm i is visited as l^{th} firm, $l \in \{2, \ldots, n\}$. Let the subscripts of the CPMV and IPMV denote the order in which firms are visited. In this scenario the consumer must have visited first a firm with CPMV v_m and she must have rejected the deal offered there. For a given v_m this happens with probability $F\left(\frac{\hat{x}-\lambda v_m}{1-\lambda}\right)$. Subsequently the consumer must have visited firms 2 up till and including $l-1$ and also rejected their offers, implying that $(1-\lambda)\varepsilon_j + \lambda v_j < \hat{x}$ for $j \in \{2, \ldots, l-1\}$. The probability that the consumer continues search from firm j, for a given v_m, equals

$$R(\hat{x}, v_m) = \int_{-\infty}^{v_m} F\left(\frac{\hat{x} - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j,$$

When the consumer arrives after $l-1$ firms at firm i she should buy there. Hence, we need $(1-\lambda)\varepsilon_i + \lambda v_i - p_i > \hat{x} - p^*$ and similar as above this happens, for a given v_m, with probability

$$\int_{-\infty}^{v_m} \left[1 - F\left(\frac{\hat{x} + \Delta - \lambda v_i}{1-\lambda}\right)\right] h(v_i)\mathrm{d}v_i.$$

These probabilities already impose that $v_j < v_m$ for $j \in \{2, \ldots, l-1\}$.and $v_i < v_m$. However, we also need that $v_j < v_m$ for the firms the consumer does not visit, which is captured by the factor $H(v_m)^{n-l}$. By taking these factors into account, weighing them with the density of v_m, and by integrating over v_m one finds that the probability that a consumer buys from firm i upon arrival, given that i is sampled as l^{th} firm, equals

$$\begin{aligned} Q_l(p_i, p^*) \quad &= \quad \int_{-\infty}^{\infty} \int_{-\infty}^{v_m} F\left(\frac{\hat{x} - \lambda v_m}{1-\lambda}\right) R(\hat{x}, v_m)^{l-2} \\ &\cdot \left[1 - F\left(\frac{\hat{x} + \Delta - \lambda v_i}{1-\lambda}\right)\right] H(v_m)^{n-l} h(v_i)\mathrm{d}v_i h(v_m)\mathrm{d}v_m, \end{aligned} \tag{2}$$

In addition there are consumers who buy from firm i after they initially rejected its offer and visited all firms. The term *comebacks* will be used for these consumers. First consider consumers who visited firm i first ($v_i = \max_{j\in\{1,\ldots,n\}}\{v_j\}$) and return there in the end. The probability that firm i offers a better deal than firm j, for given CPMV's, equals $F\left(\frac{(1-\lambda)\varepsilon_i + \lambda v_m - \Delta - \lambda v_j}{1-\lambda}\right)$. Therefore, we find that demand for firm i from comebacks who visited i first equals

$$Y^1(p_i, p^*) = \int_{-\infty}^{\infty} \int_{-\infty}^{\frac{\hat{x}+\Delta-\lambda v_m}{1-\lambda}} \left(\int_{-\infty}^{v_m} F\left(\frac{(1-\lambda)\varepsilon_i + \lambda v_m - \Delta - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right)^{n-1} f(\varepsilon_i)\mathrm{d}\varepsilon_i h(v_m)\mathrm{d}v_m,$$

where the upperbound on the second integral ensures that initially the consumer continued search from firm i. Now consider consumers who visit firm i as firm $l \in \{2, \ldots, n\}$, so $v_i < v_m$, and buys there after considering all the other offers on the market. Similarly as above, we find that this happens with probability:

$$Y^2(p_i,p^*) = (n-1)\int_{-\infty}^{\infty}\int_{-\infty}^{v_m}\int_{-\infty}^{\frac{\hat{x}+\Delta-\lambda v_i}{1-\lambda}}\left[\int_{-\infty}^{v_m} F\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_i-\Delta-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right]^{n-2}$$
$$\cdot F\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_i-\Delta-\lambda v_m}{1-\lambda}\right)f(\varepsilon_i)\mathrm{d}\varepsilon_i h(v_i)\mathrm{d}v_i h(v_m)\mathrm{d}v_m.$$

It follows that total demand for firm i charging p_i, given all other firms charge p^*, is:

$$D^D(p_i,p^*) \quad = \quad Q_1(p_i,p^*)+\sum_{l=2}^{n} Q_l(p_i,p^*)+Y^1(p_i,p^*)+Y^2(p_i,p^*). \tag{3}$$

Profits of firm i are then given by

$$\Pi(p_i,p^*) = p_i D^D(p_i;p^*). \tag{4}$$

Proposition 1. *Let f and h be continuously differentiable densities on the supports $[a_F,b_F]$ and $[a_H,b_H]$. Furthermore, let $0<s<E(t_i)-a$. When the symmetric Nash equilibrium exists its price under directed search for any $\lambda\in(0,1)$ is given by:*

$$p^* = \frac{-1}{n\frac{\partial D^D}{\partial p_i}(p^*,p^*)}, \tag{5}$$

with equilibrium profits $\Pi^*=\frac{p^*}{n}$. A sufficient condition for existence of the equilibrium is $f'(\varepsilon)\geq 0\ \forall\varepsilon$. In the case of $\lambda=1$ or $s\geq E(t_i)-a$, firms charge infinite prices in equilibrium.

It is necessary that $s>0$, otherwise consumers search freely, and a situation of perfect information occurs in which prices will drop to zero. The condition $s<E(t_i)-a$ is imposed to ensure that there is search on the market. If this requirement would not be met, search cost would be so high that it is unbeneficial for consumers to search onward if all firms charge the same price, even if the match value at the current firm is the worst possible. Anderson and Renault (1999) already pointed to the similarity of this setting to that of Diamond (1971): for any price set by the competitors, a firm can increase its price without affecting its demand. Hence, in equilibrium all firms set infinite prices. I find that this situation is analogous to that of the case of $\lambda=1$ under the directed search regime.

Equation (5) is found by equating the derivative of (4) at p^* to zero. The remainder of the proof that (5) gives a global maximum when $f'(\varepsilon)\geq 0\ \forall\varepsilon$ is presented in Appendix A. The Diamond-type argument that prices and profits go to infinity when $s\geq E(t_i)-a$ is trivial and is omitted.

The condition $f'(\varepsilon)\geq 0\ \forall\varepsilon$ is satisfied by the uniform distribution, and more generally for any distribution $F(\varepsilon) = \varepsilon^\kappa$ with $\kappa \geq 1$. Establishing the existence of equilibrium for the somewhat more general conditions given by Anderson and Renault (1999) is not my central concern. This paper's aim is to quantify the value of information.

Note that the Proposition does not treat the case $\lambda = 0$. In that case consumers do not value the CPMV and basing their search order upon it makes no sense. The model then collapses to that of random search, which is treated in the next section and acts as a benchmark.

4 Benchmark

I now derive equilibrium prices under the assumption of random search. The price in the symmetric Nash equilibrium under this scenario is denoted by p^r. The demand of firm i under the random search rule can be derived similar fashion as above and in the expression below, where $\Delta_r = p_i - p^r$, it is presented. More details can be found in Anderson and Renault (1999).

$$\begin{aligned} D^r(p_i, p^r) &= \frac{1}{n}\int_{-\infty}^{\infty}\left[1 - F\left(\frac{\bar{x} + \Delta_r - \lambda v_i}{1-\lambda}\right)\right] h(v_i)\mathrm{d}v_i \\ &+ \frac{1}{n}\sum_{l=2}^{n}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[1 - F\left(\frac{\bar{x} + \Delta_r - \lambda v_i}{1-\lambda}\right)\right] F\left(\frac{\bar{x} - \lambda v_j}{1-\lambda}\right)\right. \\ &\left.\cdot\, R(\bar{x}, b_H)^{l-2} h(v_i)\mathrm{d}v_i h(v_j)\mathrm{d}v_j\right] + \int_{-\infty}^{\bar{x}+\Delta_r} M(t_i - \Delta_r)^{n-1} m(t_i)\mathrm{d}t_i \\ &= \frac{1 - M(\bar{x} + \Delta_r)}{n}\frac{1 - M(\bar{x})^n}{1 - M(\bar{x})} + \int_{-\infty}^{\bar{x}+\Delta_r} M(t_i - \Delta_r)^{n-1} m(t_i)\mathrm{d}t_i, \end{aligned} \tag{6}$$

as $R(\bar{x}, b_H) = M(\bar{x})$. Here $\bar{x}$ solves $g(x, b_H) = s$. Since $s > 0$, $\bar{x} < b$.

Profits for firm i under the random search regime are:

$$\Pi^r(p_i, p^r) = p_i D^r(p_i; p^r). \tag{7}$$

Proposition 2. *Let f and h be continuously differentiable densities on the supports $[a_F, b_F]$ and $[a_H, b_H]$. Furthermore, let $0 < s < E(t_i) - a$. When the symmetric Nash equilibrium exists its price under random search for any $\lambda \in [0, 1]$ is given by:*

$$p^r = \frac{-1}{n\frac{\partial D^r}{\partial p_i}(p^r; p^r)}, \tag{8}$$

with equilibrium profits $\Pi^{*r} = \frac{p^r}{n}$. A sufficient condition for existence is of the equilibrium is

$f'(\varepsilon_i) \geq 0$. *In the case of $s \geq E(t_i) - a$, firms charge infinite prices in equilibrium.*

A proof of this Proposition can be found in Anderson and Renault (1999).

5 Comparative statics

Due to the complexity of the model I restrict attention to standard uniformly distributed CPMV and IPMV and 2 firms in the remainder of this section. Moreover, I assume $\lambda \in (0, 0.5]$ and that search costs are sufficiently small: $s \leq \frac{1-\lambda}{8}$. This last condition ensures that there will be no consumers who refrain from searching beyond the first sampled firm even before they know the realization of the IPMV. Appendix B includes a proof of this last claim and presents several Lemma's in which $\hat{x}$, $\bar{x}$, $\frac{\partial D^D}{\partial p_i}(p^*, p^*)$ and $\frac{\partial D^r}{\partial p_i}(p^r, p^r)$ are derived explicitly for uniformly distributed match values and 2 firms. Using these Lemma's and numerical methods the following set of Propositions is derived.

Proposition 3. $p^* \geq p^r$.

The Proposition states that firms charge higher prices when consumers use directed instead of random search. Under directed search consumers will first visit the firm that offers the product with the highest expected utility, which reduces demand elasticity.

One interpretation of the result is that disclosing product's horizontal attributes leads to higher prices. This is also found Meurer and Stahl (1994) and Anderson and Renault (2009), although in different settings. These papers consider buyers who observe prices and are not able to search for the best product match. However, some consumers might be informed about product characteristics through advertising, which leads to higher prices.

Proposition 4. *The equilibrium price p^* is non-decreasing in search costs.*

The intuition behind this result is that when search costs are higher it is more costly for consumers to inspect the option at the next firm. Firms know that consumers are thus less likely to continue searching and will charge a higher price. For the equilibrium price under random search, p^r, a similar result can be established, see Anderson and Renault (1999).

Proposition 5.

1. *p^* is decreasing in the CPMV weight λ if $s < \tilde{s}$, where $\tilde{s}$ is a function of λ.*

2. *p^* is increasing in λ if $s > \tilde{s}$.*

This Proposition is the result of two mechanisms which work in opposite directions. To see this, first consider the impact of λ upon the prices under random search, p^r. A change in λ does not effect the search decision of consumers directly, as they search random. However, the change does affect the distribution of the total match value, $t_i = (1 - \lambda)\varepsilon_i + \lambda v_i$. An increase in $\lambda \in (0, 0.5)$ leads to a lower variance of t_i, but leaves its expected value unaffected. A lower variation in the total match value means less product differentation. As Anderson and Renault (1999) shows, less differentiation leads to less monopoly power for firms and thus lower prices. When consumers use directed search the increase of λ has the additional effect that consumers are better informed: for a larger part of the match value they know that the first firm they visit offers the best option. Hence, the expected return from searching decreases in λ under directed search and leads to upward pressure on prices. This effect only dominates the other effect when search costs are sufficiently large. After all, when search costs are small consumers are likely to search onward, even if the weight on the CPMV is increased.

The discussion above shows that λ can not directly be interpreted as the amount of product information made available to consumers, as a change of it alters the distribution of the match value as well. Hence, λ should be fixed and the CPMV might be used to determine the search order or not.

Corollary 1. $\Pi^* \geq \Pi^{*r}$.

Proof of Corollary 1. All firms have an equal market share (1/2) in each equilibrium, as they charge the same price as their competitor, whether search is random or directed. The result now follows, since $p^* \geq p^r$ by Lemma 3. ■

The Propositions in this section are established for uniformly distributed match values and a duopoly. Model complexity prevents me to generalize these results or establish them analytically. However, I do allow for more general distributions and more than 2 firms when analyzing the welfare effects of product information in the next section.

6 Welfare

Throughout this section I assume that $\lambda \neq 0$ and $s > 0$. It is convenient to define

$$t_i^m = (1-\lambda)\varepsilon_i + \lambda v_m, \text{ with } v_m \geq v_j \;\; \forall j.$$

Proposition 6. *Consumers search strictly less under directed search than under random search. In each stage of the search process the consumers is less likely to continue search when using directed search.*

Proof of Proposition 6. First consider the decision whether to continue search when a consumer has already arrived at at least two firms. Recall $t_i = (1-\lambda)\varepsilon_i + \lambda v_i$. Notice that the maximum of $n-1$ independent random variables with distribution H has $(n-1)H(v_m)^{n-2}h(v_m)$ as its probability density function. Under directed search a consumer continues search when $t_i \le \hat{x}$, which happens with probability

$$\int_{-\infty}^{\infty} \int_{-\infty}^{v_m} F\left(\frac{\hat{x} - \lambda v_i}{1-\lambda}\right) h(v_i)\mathrm{d}v_i (n-1) H(v_m)^{n-2} h(v_m)\mathrm{d}v_m. \tag{9}$$

In Appendix A it is shown that $\hat{x} \le \bar{x}$ for all v_m, therefore it follows that this probability equals at most

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F\left(\frac{\bar{x} - \lambda v_i}{1-\lambda}\right) h(v_i)\mathrm{d}v_i (n-1) H(v_m)^{n-2} h(v_m)\mathrm{d}v_m = \int_{-\infty}^{\infty} F\left(\frac{\bar{x} - \lambda v_i}{1-\lambda}\right) h(v_i)\mathrm{d}v_i, \tag{10}$$

which is exactly the probability of continuing search under random search. Hence, under the directed regime the consumer who has already left the first firm continues searching at most as often as in the random search regime.

Now consider the probability of searching under the different regimes when a consumer has arrived at the first firm. In the random search case one has a match value of t_i while under the directed regime one has a match value t_i^m. Straightforwardly, one obtains $\Pr[t_i \le \bar{x}] \ge \Pr[t_i^m \le \bar{x}] \ge \Pr[t_i^m \le \hat{x}]$. The first inequality holds strictly if $v_i \ne v_m$. So, a consumer under the directed regime is less likely to continue search from the first firm than under the random search rule. Therefore, expected search expenditures are strictly lower under the directed regime. ■

Proposition 7. *The expected total match value a consumer obtains under directed search is higher than under random search.*

Proof of Proposition 7. The possibility of consumers returning to a firm after visiting all firms is irrelevant for the analysis. When such an event happens, the expected match value a consumer obtains is the same under random and directed search because she can choose from the same set of firms. Let v_m, the highest CPMV on the market, be fixed. Define

$$Z = \lambda v_m + E[(1-\lambda)\varepsilon_j] \quad \text{and} \quad W = E[\lambda v_i + (1-\lambda)\varepsilon_i | v_i < v_m].$$

Z is the expected match value of the firm with the highest CPMV, say firm j. W is the expected match value at the other firms. Notice that $Z \ge W$. The likelihood of a consumer arriving at firm j equals 1 under directed search, as this firm will be sampled first. Under random search it is strictly lower as a consumer might sample another firm first and buy there. To show that the expected match value under directed search is higher it thus suffices to show that consumers are less likely to continue search from firm j. Under random search this happens with probability $\Pr[t_j^m \le \bar{x}]$ as consumers do not realize this is the firm with the highest CPMV. Under directed search it is lower and equals $\Pr[t_j^m \le \hat{x}]$. ■

For an individual consumer it is optimal to minimize the search costs and maximize the expected match value. In both cases directed search outperforms random search, implying that this will be the strategy followed by consumers. Moreover, as prices are merely transfers in a covered market, total welfare is higher under the directed regime than under the random search.[4]

Corollary 2. *Total welfare under directed search is higher than under random search.*

Having considered total welfare and profits I now turn to consumer welfare. I derive an expression for the expected match value and search expenditures under directed and random search. I start with directed search. Notice that $nH(v_m)^{n-1}$ is the density of the maximum of n CPMV's and $(n-1)H(v_m)^{n-2}$ that of the maximum of $n-1$ CPMV's. Hence, the expected match from buying at the first firm equals

$$E(t_i^m|t_i^m > \hat{x}) = \int_{-\infty}^{\infty} \int_{\frac{\hat{x}-\lambda v_m}{1-\lambda}}^{\infty} \frac{[(1-\lambda)\varepsilon + \lambda v_m]f(\varepsilon)}{1 - F\left(\frac{\hat{x}-\lambda v_m}{1-\lambda}\right)} nH(v_m)^{n-1}\mathrm{d}\varepsilon h(v_m)\mathrm{d}v_m,$$

while the expected match from buying directly from the l^{th} visited firm, $l \in \{2, \ldots, n\}$, equals

$$E(t_i|t_i > \hat{x}) = \int_{-\infty}^{\infty} \int_{-\infty}^{v_m} \int_{\frac{\hat{x}-\lambda v}{1-\lambda}}^{\infty} \frac{[(1-\lambda)\varepsilon + \lambda v]f(\varepsilon)}{1 - F(\frac{\hat{x}-\lambda v}{1-\lambda})} h(v)(n-1)H(v_m)^{n-2}h(v_m)\mathrm{d}\varepsilon\mathrm{d}v\mathrm{d}v_m.$$

When a consumer returns to an earlier visited firm her expected match is the expectation of the maximum of n match values. By considering first the case that $\max_{i=1,\ldots n}\{t_i\} \neq t^m$ and then the case that $\max_{i=1,\ldots n}\{t_i\} = t^m$ we find that this expected match value equals

$$E\left(\max_{i=1,\ldots n}\{t_i\}\middle| t_i < \hat{x}\right) = \int_{-\infty}^{\infty} \int_{-\infty}^{v_m} \int_{-\infty}^{\frac{\hat{x}-\lambda v}{1-\lambda}} \frac{[(1-\lambda)\varepsilon_i + \lambda v_i]f(\varepsilon_i)}{F\left(\frac{\hat{x}-\lambda v}{1-\lambda}\right)}$$
$$\cdot \left[\int_{-\infty}^{v_m} F\left(\frac{(1-\lambda)\varepsilon_i + \lambda v_i - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right]^{n-2} F\left(\frac{(1-\lambda)\varepsilon_i + \lambda v_i - \lambda v_m}{1-\lambda}\right) \mathrm{d}\varepsilon_i h(v_i)\mathrm{d}v_i h(v_m)\mathrm{d}v_m$$
$$+ \int_{-\infty}^{\infty} \int_{-\infty}^{\frac{\hat{x}-\lambda v_m}{1-\lambda}} \frac{[(1-\lambda)\varepsilon_i + \lambda v_m]f(\varepsilon_i)}{F\left(\frac{\hat{x}-\lambda v_m}{1-\lambda}\right)} \left[\int_{-\infty}^{v_m} F\left(\frac{(1-\lambda)\varepsilon_i + \lambda v_m - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right]^{n-1} \mathrm{d}\varepsilon_i h(v_m)\mathrm{d}v_m.$$

Here $f(\varepsilon)/F\left(\frac{\hat{x}-\lambda v}{1-\lambda}\right)$ is the distribution of ε_i conditional on $t_i < \hat{x}$ and terms as $F\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_i-\lambda v_j}{1-\lambda}\right)$ ensure $t_i \geq t_j$.

[4] If the model would allow for nonpurchase than the higher equilibrium prices under directed search might possibly offset the positive effects of lower search cost and higher expected match values and lead to a welfare loss.

There are $nQ_1(p^*, p^*)$ consumers who buy at the first firm they arrive at. Here $Q_1(p^*, p^*)$ is given in equation (1), this term is multiplied with n as there are n firms active who can have the highest CPMV for a consumer. Similarly, the number of consumers buying from firm $l \in \{2, \ldots, n\}$ without having visited it before is given by $(n-l+1)Q_l(p^*, p^*)$, where $Q_l(p^*, p^*)$ is given in (2). Finally, the number of comebacks on the market is given by

$$\Pr\left(t_i < \hat{x}\ \forall i \in \{1, \ldots n\}\right) = \int_{-\infty}^{\infty} F\left(\frac{\hat{x} - \lambda v_m}{1-\lambda}\right) R(\hat{x}, v_m)^{n-1} h(v_m) \mathrm{d}v_m.$$

Using these expressions I find that total welfare under directed search is given by

$$\begin{aligned} SW_D = nQ_1(p^*, p^*)[-s + E(t_i^m | t_i^m > \hat{x})] + \sum_{l=2}^{n} (n-l+1)Q_l(p^*, p^*)[-ls + E(t_i | t_i > \hat{x})] \\ + \Pr\left(t_i < \hat{x}\ \forall i \in \{1, \ldots n\}\right)\left[-ns + E\left(\max_{i=1,\ldots n}\{t_i\} | t_i < \hat{x}\right)\right]. \end{aligned}$$

Social welfare resulting from random search can be derived in a similar fashion and equals

$$SW_R = [1 - M(\bar{x})] \sum_{l=1}^{n} M(\bar{x})^{l-1}[-ls + E(t_i | t_i \geq \bar{x})] + M(\bar{x})^n \left[-ns + E\left(\max_{i=1,\ldots n}\{t_i\} \middle| t_i < \bar{x}\right)\right].$$

Corollary 2 established that $SW_D > SW_R$. However, Proposition 3 shows that $p^* \geq p^r$. So, on the one hand, consumers pay higher prices under the directed search regime, but on the other hand, they obtain a higher match values and spend less on search cost when searching directly. I evaluate $SW_D - p^* - (SW_R - p^r)$ to determine whether consumers are worse off under directed search. I apply numerical methods due to the complexity of the expression. Again I will restrict attention to sufficiently small search costs, uniformly distributed CPMV's and IPMV's, $\lambda \in (0, 0.5]$ and $n = 2$. Appendix B presents for this setting $\frac{\partial D^D}{\partial p_i}(p^*, p^*)$ and $\frac{\partial D^r}{\partial p_i}(p^r, p^r)$ from which p^* and p^r directly follow. I find the following result.

Proposition 8. *Consumer surplus under directed search is lower than under random search.*

7 Conclusion

In this paper I incorporated the idea that consumers base the order in which they visit firms on their own preferences and available product information. Consumers might derive this information from for instance advertisements, comparison websites, acquaintances or consultancy agencies. Based on the gained information consumers first sample the firm where they expect to find the product that gives them the highest utility. The model fits into the literature on consumer search with differentiated products.

Under the alternative search rule which I propose I find that prices and profits are higher as compared to the case of random search, which is generally assumed by the literature. The driving force behind this result is that firms realize that consumers will first sample the firm at which they expect to find the product that fits their tastes the best. This gives firms some monopoly power over these consumers, which is exploited in equilibrium. Disclosing product information effectively allows firms to differentiate their products and charge higher prices. In line with existing literature I find that prices are increasing in search costs, as higher search costs reduce demand elasticity. In the extreme case when all product characteristics are communicable to consumers prior to search the Diamond paradox arises again.

Total welfare is higher when consumers use directed rather than random search. This is the result of two effects. First, consumers obtain on average a higher match value under directed search. This is because they start their search at a firm which offers a product that, in certain dimensions, fits their tastes the best. Second, consumers are less likely to search onward in each stage of the process when they use directed search. This is because they know they will never find a product that fits their preferences better for certain aspects.

Consumer welfare turns out to be lower under directed search, as the positive effects of better matches and lower search expenditures are outweighed by the higher prices.

Future research might extend the presented model and allow the entire search order to depend upon available product information and consumer preferences. Such an extension is far from trivial since it takes away the stationary character of the search process as the expected benefit of continuing search will then depend upon the available product information of the next best firm.

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A General distributions

First I give a useful Lemma and its proof.

Lemma 1. $g(x, v^m) = \int\limits_{-\infty}^{v^m} \int\limits_{\frac{x-\lambda v_j}{1-\lambda}}^{\infty} ((1-\lambda)\varepsilon_j + \lambda v_j - x) f(\varepsilon_j) d\varepsilon_j \frac{h(v_j)}{H(v^m)} dv_j$ *is decreasing in x and increasing in v^m. In addition, $\hat{x} \leq \bar{x}$ where $\hat{x}$ solves $g(x, v^m) = s$ and $\bar{x}$ solves $g(x, b) = s$.*

Proof of Lemma 1. By inspecting the expression of $g(x, v^m)$ it immediately follows that it is decreasing in x. Since

$$\frac{\partial g(x, v^m)}{\partial v^m} = \int\limits_{\frac{x-\lambda v^m}{1-\lambda}}^{\infty} ((1-\lambda)\varepsilon_j + \lambda v^m - x) f(\varepsilon_j) \mathrm{d}\varepsilon_j \frac{h(v^m)}{H(v^m)}$$

$$- \int\limits_{-\infty}^{v^m} \int\limits_{\frac{x-\lambda v_j}{1-\lambda}}^{\infty} ((1-\lambda)\varepsilon_j + \lambda v_j - x) f(\varepsilon_j) \mathrm{d}\varepsilon_j \frac{h(v_j) h(v^m)}{H(v^m)^2} \mathrm{d}v_j$$

$$> \int\limits_{\frac{x-\lambda v^m}{1-\lambda}}^{\infty} ((1-\lambda)\varepsilon_j + \lambda v^m - x) f(\varepsilon_j) \mathrm{d}\varepsilon_j \frac{h(v^m)}{H(v^m)}$$

$$-\int_{-\infty}^{v^m}\int_{\frac{x-\lambda v^m}{1-\lambda}}^{\infty}((1-\lambda)\varepsilon_j+\lambda v^m-x)f(\varepsilon_j)\mathrm{d}\varepsilon_j\frac{h(v_j)h(v^m)}{H(v^m)^2}\mathrm{d}v_j=0$$

$g(x, v^m)$ is increasing in v^m. Combining these results gives $\hat{x} \leq \bar{x}$. ■

Proof of Proposition 1. I show that profit function is concave for $\lambda \in (0,1)$, the case $\lambda = 1$ will be treated later. Define

$$u_i=\varepsilon_i-\frac{\Delta}{1-\lambda}\text{ and }\Theta=\left\{u_i:u_i\in\left[a_F-\frac{\Delta}{1-\lambda},\frac{\hat{x}-\lambda v_m}{1-\lambda}\right]\right\}. \tag{11}$$

Using this and using integration by parts one finds[5]:

$$\begin{aligned}
&\int_{-\infty}^{\infty}\int_{-\infty}^{\frac{\hat{x}+\Delta-\lambda v_m}{1-\lambda}}\left(\int_{-\infty}^{v_m}F\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_m-\Delta-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)^{n-1}f(\varepsilon_i)\mathrm{d}\varepsilon_i h(v_m)\mathrm{d}v_m\\
=&\int_{-\infty}^{\infty}\int_{\Theta}\left(1-F\left(u_i+\frac{\Delta}{1-\lambda}\right)\right)\mathrm{d}\left(\int_{-\infty}^{v_m}F\left(u_i+\frac{\lambda v_m-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)^{n-1}h(v_m)\mathrm{d}v_m\\
&+\int_{-\infty}^{\infty}\left(\int_{-\infty}^{v_m}F\left(a_F+\frac{\lambda v_m-\Delta-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)^{n-1}h(v_m)\mathrm{d}v_m\\
&-\int_{-\infty}^{\infty}\left[1-F\left(\frac{\hat{x}-\lambda v_m+\Delta}{1-\lambda}\right)\right]\left(\int_{-\infty}^{v_m}F\left(\frac{\hat{x}-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)^{n-1}h(v_m)\mathrm{d}v_m.
\end{aligned}\tag{12}$$

Define

$$T(u,v_i,v_m)=\left[\int_{-\infty}^{v_m}F\left(u_i+\frac{\lambda v_i-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right]^{n-2}F\left(u_i+\frac{\lambda v_i-\lambda v_m}{1-\lambda}\right).$$

Applying this definition and using integration by parts one finds[6]:

$$\sum_{l=2}^{n}\int_{-\infty}^{\infty}\int_{-\infty}^{v_m}\int_{-\infty}^{\frac{\hat{x}+\Delta-\lambda v_i}{1-\lambda}}\left[\int_{-\infty}^{v_m}F\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_i-\Delta-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right]^{n-2}$$

[5] Start the computation with the first term after the equality sign. Apply integration by parts on the inner integral of this expression. That is, calculate $\int_\Theta \alpha(u_i)\beta'(u_i)dx = \alpha(\frac{\hat{x}-\lambda v_m}{1-\lambda})\beta(\frac{\hat{x}-\lambda v_m}{1-\lambda}) - \alpha(a_F - \frac{\Delta}{1-\lambda})\beta(a_F-\frac{\Delta}{1-\lambda}) - \int_\Theta \beta(u_i)\alpha'(u_i)\mathrm{d}u_i$ with $\beta'(u_i) = \mathrm{d}\left(\int_{-\infty}^{v_m} F\left(u_i+\frac{\lambda v_m-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)^{n-1}$ and $\alpha(u_i) = \left(1-F\left(u_i+\frac{\Delta}{1-\lambda}\right)\right)$. Bringing the term $\int_\Theta \beta(u_i)\alpha'(u_i)\mathrm{d}u_i$ to the other side of the equality sign then gives the result. In Anderson and Renault (1999) a similar approach is used to calculate $D_A(p_i,p^*)$ and $D_B(p_i,p^*)$ at page 733.

[6] Use the same approach as in footnote 5.

$$
\begin{aligned}
&\cdot F\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_i-\Delta-\lambda v_m}{1-\lambda}\right)f(\varepsilon_i)\mathrm{d}\varepsilon_i h(v_i)\mathrm{d}v_i h(v_m)\mathrm{d}v_m \\
= \;& \sum_{l=2}^{n}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_{\Theta}\left(1-F\left(u_i+\frac{\Delta}{1-\lambda}\right)\right)\mathrm{d}T(u_i,v_i,v_m)-\left(1-F\left(\frac{\hat{x}-\lambda v_i+\Delta}{1-\lambda}\right)\right)\right. \\
&\left.\cdot\, T\left(\frac{\hat{x}-\lambda v_i}{1-\lambda},v_i,v_m\right)+T\left(a_F-\frac{\Delta}{1-\lambda},v_i,v_m\right)\right]h(v_i)\mathrm{d}v_i h(v_j)\mathrm{d}v_m. \qquad (13)
\end{aligned}
$$

Using (12) and (13) to rewrite demand from comebacks in (3) and applying some algebra gives

$$
D^D(p_i,p^*)=\int_{-\infty}^{\infty}\left[D_1(p_i,v_m)+D_2(p_i,v_m)+D_3(p_i,v_m)+D_4(p_i,v_m)\right]h(v_m)\mathrm{d}v_m, \qquad (14)
$$

where

$$
\begin{aligned}
D_1(p_i,v_m) &= \left[1-F\left(\frac{\hat{x}+\Delta-\lambda v_m}{1-\lambda}\right)\right]\left[H(v_m)^{n-1}-\left(\int_{-\infty}^{v_m}F\left(\frac{\hat{x}-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)^{n-1}\right] \\
D_2(p_i,v_m) &= \int_{-\infty}^{v_m}\left[1-F\left(\frac{\hat{x}+\Delta-\lambda v_i}{1-\lambda}\right)\right]\sum_{l=2}^{n}\left[H(v_m)^{n-l}\left(\int_{-\infty}^{v_m}F\left(\frac{\hat{x}-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)^{l-2}\right. \\
&\quad\left.-\left(\int_{-\infty}^{v_m}F\left(\frac{\hat{x}-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)^{n-2}\right]F\left(\frac{\hat{x}-\lambda v_m}{1-\lambda}\right)h(v_i)\mathrm{d}v_i \\
D_3(p_i,v_m) &= \int_{\Theta}\left(1-F\left(u_i+\frac{\Delta}{1-\lambda}\right)\right)\mathrm{d}\left(\int_{-\infty}^{v_m}F\left(u_i+\frac{\lambda v_m-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)^{n-1} \\
&\quad+\left(\int_{-\infty}^{v^m}F\left(a_F+\frac{\lambda v^m-\Delta-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)^{n-1} \\
D_4(p_i,v_m) &= \sum_{l=2}^{n}\int_{-\infty}^{v_m}\left[\int_{a_F-\frac{\Delta}{1-\lambda}}^{\frac{\hat{x}-\lambda v_i}{1-\lambda}}\left(1-F\left(u_i+\frac{\Delta}{1-\lambda}\right)\right)\mathrm{d}T(u_i,v_i,v_m)\right. \\
&\quad\left.+T\left(a_F-\frac{\Delta}{1-\lambda},v_i,v_m\right)\right]h(v_i)\mathrm{d}v_i.
\end{aligned}
$$

Notice that:

$$
H(v_m)^{n-1}-\left(\int_{-\infty}^{v_m}F\left(\frac{\hat{x}-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)^{n-1}\geq H(v_m)^{n-1}-\left(\int_{-\infty}^{v_m}h(v_j)\mathrm{d}v_j\right)^{n-1}\geq 0, \qquad (15)
$$

and similarly for all $l \in \{2,3,\dots,n\}$:

$$H(v_m)^{n-l}\left(\int_{-\infty}^{v_m} F\left(\frac{\hat{x}-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)^{l-2} - \left(\int_{-\infty}^{v_m} F\left(\frac{\hat{x}-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)^{n-2} \geq 0. \tag{16}$$

It follows that every term in (14), and therefore demand, is non-increasing in p_i.

Let $\pi_k(p_i,p^*) = p_i D_k(p_i, v_m)$, for $k \in \{1,2,3,4\}$. Then firm i her profits are $\Pi(p_i,p^*) = \int_{-\infty}^{\infty} \sum_{k=1}^{4} \pi_k(p_i,p^*)h(v_m)\mathrm{d}v_m$. I show $\pi_k(p_i,p^*)$ is concave for all $k \in \{1,2,3,4\}$.

By (15) π_1 is concave whenever

$$-\frac{2}{1-\lambda}f\left(\frac{\hat{x}+\Delta-\lambda v_m}{1-\lambda}\right) - \frac{p_i}{(1-\lambda)^2}f'\left(\frac{\hat{x}+\Delta-\lambda v_m}{1-\lambda}\right) \leq 0.$$

As $f'(\varepsilon) \geq 0 \ \forall \varepsilon$ this inequality holds. π_2 can be rewritten as a positive constant times $\int_{-\infty}^{v_m}\left[1-F\left(\frac{\hat{x}+\Delta-\lambda v_i}{1-\lambda}\right)\right]h(v_i)\mathrm{d}v_i$. Therefore $\frac{\partial^2 \pi_2}{\partial p_i^2}$ is proportional to:

$$\int_{-\infty}^{v_m}\left[-\frac{2}{1-\lambda}f\left(\frac{\hat{x}+\Delta-\lambda v_i}{1-\lambda}\right) - \frac{p_i}{(1-\lambda)^2}f'\left(\frac{\hat{x}+\Delta-\lambda v_i}{1-\lambda}\right)\right]h(v_i)\mathrm{d}v_i.$$

Since $f'(\varepsilon) \geq 0 \ \forall \varepsilon$ this expression is non-positive. Therefore π_2 is concave. $\frac{\partial^2 \pi_3}{\partial p_i^2}$ equals

$$\int_{\Theta}\left[-\frac{2}{1-\lambda}f\left(u_i+\frac{\Delta}{1-\lambda}\right) - \frac{p_i}{(1-\lambda)^2}f'\left(u_i+\frac{\Delta}{1-\lambda}\right)\right]$$
$$\cdot \mathrm{d}\left(\int_{-\infty}^{v_m} F\left(u_i+\frac{\lambda v_m-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)^{n-1} - p_i\frac{(n-1)f(a_F)}{(1-\lambda)^2}$$
$$\cdot\left(\int_{-\infty}^{v_m} F\left(a_F+\frac{\lambda v_m-\Delta-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)^{n-2}\left(\int_{-\infty}^{v_m} f\left(a_F+\frac{\lambda v_m-\Delta-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right).$$

By again using $f'(\varepsilon) \geq 0 \ \forall \varepsilon$ it straightforwardly follows that $\frac{\partial^2 \pi_3}{\partial p_i^2} \leq 0$. $\frac{\partial^2 \pi_4}{\partial p_i^2}$ is equal to

$$\sum_{l=2}^{n}\int_{-\infty}^{v_m}\int_{\Theta}\left[-\frac{2}{1-\lambda}f\left(u_i+\frac{\Delta}{1-\lambda}\right) - \frac{p_i}{(1-\lambda)^2}f'\left(u_i+\frac{\Delta}{1-\lambda}\right)\right]\mathrm{d}T(u_i,v_i,v_m)h(v_i)\mathrm{d}v_i.$$
$$-\sum_{l=2}^{n}\int_{-\infty}^{v_m}\frac{p_i}{(1-\lambda)^2}f(a_F)(n-2)\left[\int_{-\infty}^{v_m} F\left(a_F+\frac{\lambda v_i-\Delta-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right]^{n-3}$$

$$\cdot \left[\int_{-\infty}^{v_m} f\left(a_F + \frac{\lambda v_i - \Delta - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j \right] F\left(a_F + \frac{\lambda v_i - \Delta - \lambda v_m}{1-\lambda}\right) h(v_i)\mathrm{d}v_i$$

$$-\sum_{l=2}^{n} \int_{-\infty}^{v_m} \frac{p_i f(a_F)}{(1-\lambda)^2} \left[\int_{-\infty}^{v_m} F\left(a_F + \frac{\lambda v_i - \Delta - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j \right]^{n-2}$$

$$\cdot f\left(a_F + \frac{\lambda v_i - \Delta - \lambda v_m}{1-\lambda}\right) h(v_i)\mathrm{d}v_i.$$

All terms in the expression are non-positive since $f'(\varepsilon) \geq 0 \;\; \forall \varepsilon$, which leads to the conclusion that $\pi_4(p_i, p^*)$ is concave.

Because the sum of concave functions is concave it follows by these arguments that a sufficient condition for the symmetric Nash equilibrium to exists is $f'(\varepsilon) \geq 0 \;\; \forall \varepsilon$ when $\lambda \in (0,1)$.

Now suppose $\lambda = 1$. Then consumers perfectly observe the entire match value in the first stage of the game, and the IPMV is zero. However, they still have to search for the best price. An argument that is analogous to that of Diamond shows that the best response to any market price is an increase of the price. Prices will explode under the covered market assumption. ■

B Uniformly distributed match values

To derive an explicit expressions for $\frac{\partial D^D}{\partial p_i}(p^*, p^*)$ and $\frac{\partial D^r}{\partial p_i}(p^r, p^r)$ it is convenient to write out the density of a convex combination of two uniformly distributed variables.

The convolution of two uniform distributions. Let X and Y be uniformly distributed on $[0, b_F]$ and $[0, b_H]$, respectively. The density of $Z = \lambda Y + (1-\lambda)X$, denoted by m, is given by:

$$m(z) = \begin{cases} 0 & \text{if } z \leq 0 \\ \frac{z}{\lambda b_H (1-\lambda) b_F} & \text{if } z \in (0, \min\{\lambda b_H, (1-\lambda) b_F\}] \\ \frac{\min\{\lambda b_H, (1-\lambda) b_F\}}{\lambda b_H (1-\lambda) b_F} & \text{if } z \in [\min\{\lambda b_H, (1-\lambda) b_F, \max\{\lambda b_H, (1-\lambda) b_F\}] \\ \frac{\lambda b_H + (1-\lambda) b_F - z}{\lambda b_H (1-\lambda) b_F} & \text{if } z \in [\max\{\lambda b_H, (1-\lambda) b_F\}, \lambda b_H + (1-\lambda) b_F) \\ 0 & \text{if } z \geq \lambda b_H + (1-\lambda) b_F. \end{cases}$$

Moreover, the following Lemma will be useful.

Lemma 2. *For uniformly distributed match values and $\lambda \in (0, 0.5]$ the following statements hold.*

(a) When $s \in [\frac{\lambda^2 v_m^2}{6(1-\lambda)}, \frac{1-\lambda}{8}]$ then $\hat{x} = 1 - \lambda + \frac{1}{2}\lambda v_m - \sqrt{2(1-\lambda)s - \frac{\lambda^2 v_m^2}{12}}$.

(b) When $s \in [\frac{\lambda^2}{6(1-\lambda)}, \frac{1-\lambda}{8}]$ *then* $\bar{x} = 1 - \frac{1}{2}\lambda - \sqrt{2(1-\lambda)s - \frac{\lambda^2}{12}}$.

(c) When $s < \frac{\lambda^2 v_m^2}{6(1-\lambda)}$ *then* $\hat{x} = 1 - \lambda + \lambda v_m - (6 v_m \lambda(1-\lambda)s)^{1/3}$.

(d) When $s < \frac{\lambda^2}{6(1-\lambda)}$ *then* $\bar{x} = 1 - (6\lambda(1-\lambda)s)^{1/3}$.

When $s < \frac{1-\lambda}{8}$ *then* $\hat{x} \geq \lambda v_m$ *for all* v_m *and the decision to continue search will depend upon the realization of a consumer's IPMV.*

Proof of Lemma 2. Suppose $\hat{x} \in [\lambda v_m, 1-\lambda]$, then $g(x, v_m)$ reduces to

$$
\begin{aligned}
g(x,v_m) &= \int_0^{v_m} \int_{\frac{x-\lambda v_j}{1-\lambda}}^{1} ((1-\lambda)\varepsilon_j + \lambda v_j - x)\mathrm{d}\varepsilon_j \frac{1}{v_m}\mathrm{d}v_j = \int_0^{v_m} \frac{(1-\lambda+\lambda v_j - x)^2}{2(1-\lambda)v_m}\mathrm{d}v_j \\
&= \frac{1}{6(1-\lambda)}\left(\lambda^2 v_m^2 - 3\lambda v_m x - 3\lambda^2 v_m + 3\lambda v_m + 3x^2 + 6\lambda x - 6x + 3\lambda^2 - 6\lambda + 3\right).
\end{aligned}
$$

Equating this to s and solving for x gives $\hat{x} = 1 - \lambda + \frac{1}{2}\lambda v_m - \sqrt{2(1-\lambda)s - \frac{\lambda^2 v_m^2}{12}}$. As $\hat{x} \leq 1-\lambda$ this puts the condition $s \geq \frac{\lambda^2 v_m^2}{6(1-\lambda)}$ on s. However, when s is sufficiently large it might happen that $1 - \lambda + \frac{1}{2}\lambda v_m - \sqrt{2(1-\lambda)s - \frac{\lambda^2 v_m^2}{12}} < \lambda v_m$ for sufficiently small v_m, contradicting $\hat{x} \geq \lambda v_m$. In that case there are some consumers who will decide not to continue search based upon their CPMV, independent of their IPMV. I assume $s \leq \frac{1-\lambda}{8}$ to keep the model tractable. The expression for $\bar{x}$ and the conditions imposed on s for case (b) are found by taking $v_m = 1$ in part (a).

Now suppose $\hat{x} > 1-\lambda$, then one finds

$$
g(x,v_m) \quad = \quad \int_{\frac{x-1+\lambda}{\lambda}}^{v_m} \frac{(1-\lambda+\lambda v_j - x)^2}{2(1-\lambda)v_m}\mathrm{d}v_j = \frac{(1-\lambda+\lambda v_m - x)^3}{6 v_m \lambda(1-\lambda)}.
$$

Solving $g(\hat{x}, v_m) = s$ gives $\hat{x} = 1 - \lambda + \lambda v_m - (6 v_m \lambda(1-\lambda)s)^{1/3}$. Setting $v_m = 1$ in this expression gives $\bar{x}$. The conditions on s for this case now straightforwardly follow. ■

The following Lemma gives explicit versions of $\frac{\partial D^D}{\partial p_i}(p^*, p^*)$ and $\frac{\partial D^r}{\partial p_i}(p^r, p^r)$.

Lemma 3. *Let* $\check{v} = \frac{1}{\lambda}\sqrt{6(1-\lambda)s}$. *For uniformly distributed match values,* $s \leq \frac{1-\lambda}{8}$ *and* $\lambda \in (0, 0.5]$ *the following statements hold.*

(a) When $s \leq \min\{\frac{1-\lambda}{8}, \frac{\lambda^2}{6(1-\lambda)}\}$ *then*

$$
\frac{\partial D^D}{\partial p_i}(p^*, p^*) = -\frac{1}{2(1-\lambda)} - \int_{\check{v}}^{1} \frac{-2\lambda v_m (6 v_m \lambda(1-\lambda)s)^{1/3} + (1-\lambda)^2}{2\lambda(1-\lambda)^2}\mathrm{d}v_m
$$

$$-\int_0^{\check{v}} \frac{4v_m - 2 - 3\lambda v_m + 2\lambda - 3\lambda v_m^2 - 2(2v_m - 1)\sqrt{2(1-\lambda)s - \frac{\lambda^2 v_m^2}{12}}}{2(1-\lambda)^2} \mathrm{d}v_m. \tag{17}$$

(b) When $s \in \left(\frac{\lambda^2}{6(1-\lambda)}, \frac{1-\lambda}{8}\right)$*, a situation which can not occur when* λ *sufficiently large, then*

$$\frac{\partial D^D}{\partial p_i}(p^*, p^*) = -\frac{1}{2(1-\lambda)}$$

$$-\int_0^{1} \frac{4v_m - 2 - 3\lambda v_m + 2\lambda - 3\lambda v_m^2 - 2(2v_m - 1)\sqrt{2(1-\lambda)s - \frac{\lambda^2 v_m^2}{12}}}{2(1-\lambda)^2} \mathrm{d}v_m. \tag{18}$$

(c) When $s \leq \min\{\frac{1-\lambda}{8}, \frac{\lambda^2}{6(1-\lambda)}\}$*, then* $\bar{x} = 1 - (6\lambda(1-\lambda)s)^{1/3}$ *and*

$$\frac{\partial D^r}{\partial p_i}(p^r, p^r) = -\frac{\bar{x}^3 - 3\bar{x}^2 + 3\bar{x} - 16\lambda^3 + 12\lambda^2 - 1}{12\lambda^2(1-\lambda)^2}. \tag{19}$$

(d) When $s \in \left(\frac{\lambda^2}{6(1-\lambda)}, \frac{1-\lambda}{8}\right)$*, then* $\bar{x} = 1 - \frac{1}{2}\lambda - \sqrt{2(1-\lambda)s - \frac{\lambda^2}{12}}$ *and*

$$\frac{\partial D^r}{\partial p_i}(p^r, p^r) = -\frac{6(1+\bar{x}) - 11\lambda}{12(1-\lambda)^2}. \tag{20}$$

Proof of Lemma 3. By Lemma 2 $\hat{x} = 1 - \lambda + \lambda v_m - (6v_m\lambda(1-\lambda)s)^{1/3}$ for $v_m \geq \check{v}$ and $\hat{x} = 1 - \lambda + \frac{1}{2}\lambda v_m - \sqrt{2(1-\lambda)s - \frac{\lambda^2 v_m^2}{12}}$ for $v_m < \check{v}$. First assume $\check{v} \leq 1$, that is, $s \leq \frac{\lambda^2}{6(1-\lambda)}$.

The derivative of the first three lines of (3) with respect to p_i, at $\Delta = 0$, is

$$-\int_{-\infty}^{\infty} H(v_m)^{n-1} f\left(\frac{\hat{x} - \lambda v_m}{1-\lambda}\right) \frac{1}{1-\lambda} h(v_m) \mathrm{d}v_m - \sum_{l=2}^{n} \left[\int_{-\infty}^{\infty}\int_{-\infty}^{v_m} f\left(\frac{\hat{x} - \lambda v_i}{1-\lambda}\right)\right.$$
$$\left.\cdot\frac{1}{1-\lambda} H(v_m)^{n-l} F\left(\frac{\hat{x} - \lambda v_m}{1-\lambda}\right) R(\hat{x}, v_m)^{l-2} h(v_i)\mathrm{d}v_i h(v_m)\mathrm{d}v_m\right]. \tag{21}$$

Note $v_m > \check{v}$ if and only if $\hat{x} > 1 - \lambda$. Therefore (21) becomes for $n = 2$

$$-\frac{1}{2(1-\lambda)} - \left[\int_{\check{v}}^{1}\int_{\frac{\hat{x}-1+\lambda}{\lambda}}^{v_m} \mathrm{d}v_i \frac{1}{1-\lambda}\frac{\hat{x} - \lambda v_m}{1-\lambda}\mathrm{d}v_m + \int_0^{\check{v}}\int_0^{v_m} \mathrm{d}v_i \frac{1}{1-\lambda}\frac{\hat{x} - \lambda v_m}{1-\lambda}\mathrm{d}v_m\right]. \tag{22}$$

For $n = 2$ and uniformly distributed match values the derivative of the last four lines of

(3) with respect to p_i at $\Delta = 0$ becomes:

$$\frac{1}{1-\lambda}\int_{-\infty}^{\infty}\left(\int_{-\infty}^{v_m} F\left(\frac{\hat{x}-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right) f\left(\frac{\hat{x}-\lambda v^m}{1-\lambda}\right)h(v_m)\mathrm{d}v_m$$
$$-\frac{1}{1-\lambda}\int_{-\infty}^{\infty}\int_{-\infty}^{\frac{\hat{x}-\lambda v_m}{1-\lambda}}\left(\int_{-\infty}^{v_m} f\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_m-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right) f(\varepsilon_i)\mathrm{d}\varepsilon_i h(v_m)\mathrm{d}v_m$$
$$+\frac{1}{1-\lambda}\int_{-\infty}^{\infty}\int_{-\infty}^{v_m} F\left(\frac{\hat{x}-\lambda v_m}{1-\lambda}\right) f\left(\frac{\hat{x}-\lambda v_i}{1-\lambda}\right)h(v_i)\mathrm{d}v_i h(v_m)\mathrm{d}v_m$$
$$-\frac{1}{1-\lambda}\int_{-\infty}^{\infty}\int_{-\infty}^{v_m}\int_{-\infty}^{\frac{\hat{x}-\lambda v_i}{1-\lambda}} f\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_i-\lambda v_m}{1-\lambda}\right) f(\varepsilon_i)\mathrm{d}\varepsilon_i h(v_i)\mathrm{d}v_i h(v_m)\mathrm{d}v_m.$$

This equals

$$\frac{1}{1-\lambda}\int_0^{\check{v}}\int_0^{v_m}\frac{\hat{x}-\lambda v_j}{1-\lambda}\mathrm{d}v_j\mathrm{d}v_m+\frac{1}{1-\lambda}\int_{\check{v}}^{1}\left(\frac{\hat{x}-1+\lambda}{\lambda}+\int_{\frac{\hat{x}-1+\lambda}{\lambda}}^{v_m}\frac{\hat{x}-\lambda v_j}{1-\lambda}\mathrm{d}v_j\right)\mathrm{d}v_m$$
$$-\frac{1}{1-\lambda}\int_0^{\check{v}}\int_0^{\frac{\hat{x}-\lambda v_m}{1-\lambda}}\int_0^{v_m}\mathrm{d}v_j\mathrm{d}\varepsilon_i\mathrm{d}v_m-\frac{1}{1-\lambda}\int_{\check{v}}^{1}\int_0^{\frac{\hat{x}-\lambda v_m}{1-\lambda}}\int_{\frac{(1-\lambda)\varepsilon-(1-\lambda)+\lambda v_m}{\lambda}}^{v_m}\mathrm{d}v_j\mathrm{d}\varepsilon_i\mathrm{d}v_m$$
$$+\frac{1}{1-\lambda}\int_{\check{v}}^{1}\frac{\hat{x}-\lambda v_m}{1-\lambda}\left(v_m-\frac{\hat{x}-1+\lambda}{\lambda}\right)dv_m+\frac{1}{1-\lambda}\int_0^{\check{v}}\frac{\hat{x}-\lambda v_m}{1-\lambda}\mathrm{d}v_m$$
$$-\frac{1}{1-\lambda}\int_0^{1}\int_0^{v_m}\int_{\frac{\lambda v_m-\lambda v_i}{1-\lambda}}^{\frac{\hat{x}-\lambda v_i}{1-\lambda}}\mathrm{d}\varepsilon_i\mathrm{d}v_i\mathrm{d}v_m. \tag{23}$$

Adding (23) to (22) and some algebra leads to the expression given in part (a) of the Lemma. Part (b) can be derived in a similar fashion and by noticing that $\check{v} > 1$ when $s > \frac{\lambda^2}{6(1-\lambda)}$.

Now consider random search. The derivative of (6) with respect to p_i at $\Delta = 0$ is for $n = 2$:

$$-\frac{1}{2(1-\lambda)}\left[1+\int_{-\infty}^{\infty} F\left(\frac{\bar{x}-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right]\int_{-\infty}^{\infty} f\left(\frac{\bar{x}-\lambda v_i}{1-\lambda}\right)h(v_i)\mathrm{d}v_i$$
$$+M(\bar{x})m(\bar{x})-\int_{-\infty}^{\bar{x}} m(t_i)^2\mathrm{d}t_i. \tag{24}$$

First suppose $s > \frac{\lambda^2}{6(1-\lambda)}$, then $\bar{x} \in [\lambda, 1-\lambda)$. Hence, $f\left(\frac{\bar{x}-\lambda v_i}{1-\lambda}\right) = 1$ and $F\left(\frac{\bar{x}-\lambda v_j}{1-\lambda}\right) = \frac{\bar{x}-\lambda v_j}{1-\lambda}$ for all $v_i, v_j \in [0,1]$. When one uses $m(t_i)$ for the uniform distributions given at the start of this Appendix one finds after some calculations:

$$M(\bar{x})m(\bar{x}) - \int\limits_{-\infty}^{\bar{x}} m(t_i)^2 \mathrm{d}t_i = \frac{2\bar{x}-\lambda}{2(1-\lambda)^2} - \frac{3\bar{x}-2\lambda}{3(1-\lambda)^2} = \frac{\lambda}{6(1-\lambda)^2}.$$

Combining these findings with (24) and Lemma 2 results in part (d) of the Lemma.

Next consider $s \leq \frac{\lambda^2}{6(1-\lambda)}$, then $\bar{x} \geq 1-\lambda$. Hence, $\int\limits_{-\infty}^{\infty} f\left(\frac{\bar{x}-\lambda v_i}{1-\lambda}\right) h(v_i)\mathrm{d}v_i = \frac{1-\bar{x}}{\lambda}$ and $M(\bar{x}) = \int\limits_{-\infty}^{\infty} F\left(\frac{\bar{x}-\lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j = \frac{2\bar{x}-1+2\lambda-2\lambda^2-\bar{x}^2}{2\lambda(1-\lambda)}$. Furthermore, using $m(t_i)$ as given for the uniform case at the start of this Appendix gives $\int\limits_{-\infty}^{\bar{x}} m(t_i)^2 \mathrm{d}t_i = \frac{3\lambda^2-4\lambda^3+3\bar{x}-3\bar{x}^2+\bar{x}^3-1}{3\lambda^2(1-\lambda)^2}$. Combining these findings with (24) and Lemma 2 results in part (c) of the Lemma. ■

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