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# Price and Match-Value Advertising with Directed Consumer Search* 

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#### Abstract

We present a consumer search model in which firms sell products with two product attributes that are horizontally differentiated. One attribute is observable without visiting the firm, while the other can only be discovered upon visiting the store. Search is directed as consumers will be more inclined to visit a firm where they like the observable characteristics. Moreover, firms can influence the order of search by adjusting prices and/or by providing match-value information. We show that price advertising leads to lower prices and profits. With price advertising, a lower price not only retains more consumers, but is also more likely to attract them. Second, we how that with price advertising equilibrium prices and profits decrease in search costs. With higher search costs consumers are less likely to walk away, hence firms are more eager to attract them in the first place. Unless price advertising is prohibitively costly, price advertising will occur in equilibrium. Firms do not want to reveal match-value information, provided search costs are realistically low.


JEL Classification: D83, L13.
Keywords: directed consumer search, price advertising, match-value advertising

[^0]
## 1 Introduction

Most of the consumer search literature assumes that consumers search randomly. In a standard model with symmetric firms, a consumer picks one firm at random, pays it a costly visit in order to learn price and/or product characteristics, and on the basis of those decides whether to pay a costly visit to the next randomly selected non-visited firm. And so and so forth. Assuming that search is random is reasonable when firm offerings are truly identical a priori. This is the case, for example, when products are differentiated but consumers are completely uninformed about the differences at the time they engage in search. Also, it requires that prices can only be learned after costly search.

In the real world, however, things are often different. First, it has become very easy for consumers to compare prices. There are many search engines that list all prices without requiring any search effort on behalf of the consumer. Second, it is also very easy to find at least some characteristics of a product or a firm without paying it a costly visit; yet, some product characteristics are much harder (or even impossible) to discern from home and can only be discovered upon search. For example, a consumer that wants to buy a car can easily find all technical specifications in specialized magazines or on the Internet. However, she still has to make a costly visit to the dealer to be able to kick the tyres and take it for a test drive. Similarly, a consumer that wants to buy a new pair of jeans may find many details and pictures of this product online, which allows her to check whether she likes the design. But she still has to make a costly visit to the stor $\rrbracket^{1}$ in order to try the jeans on and decide whether she likes their fit.

In these situations, consumers search will be directed. The typical consumer will start searching the products that look ex-ante more promising to her, either because some of the easily observable characteristics are more appealing or because the product is cheaper. This paper develops a tractable duopoly model for such directed search, To do so, we build on Anderson and Renault's (1999) model of consumer search with differentiated products. In our model firms sell products with two product attributes that are horizontally differentiated. One attribute is observable without visiting the firm,

[^1]while the other can only be discovered upon visiting the store and physically interacting with the product. Because of the observable characteristics, search is directed.

Amongst others, our framework allows us to study situations in which prices are readily observable, so consumers only have to search to learn product characteristics. In Anderson and Renault's (1999) framework allowing for such price advertising does not yield a tractable solution (see also Armstrong and Zhou, 2011, p. F381). We examine how search costs affect equilibrium prices, both wen prices are advertised and when they are not. We also study whether firms have an incentive to make their prices public.

Firms can also influence the order of search by providing additional match-value information (cf. Lewis and Sappington, 1994; Johnson and Myatt, 2006; Anderson and Renault, 2006). We also study the incentives of the firms to reveal information that allows consumers to learn their match value, and how these incentives depend on whether prices are observable.

Our main results are as follows. First, and perhaps most surprisingly, we find that with price advertising equilibrium prices and profits decrease in search costs. With higher search costs consumers are less likely to walk away from a firm they visit. This gives firms more of an incentive to try to attract those consumers, which they can do by charging a lower price. Second, we show that in equilibrium prices are lower when they are advertised. As a result profits are also lower. With price advertising, a lower price not only affects whether a consumer who pays a visit to the firm buys its products, but also whether the consumer chooses to visit in the first place. This gives firms more of an incentive to lower prices. Third, despite this being the case, we show that if firms have the choice whether or not to advertise their price, the unique symmetric equilibrium has them both doing so. Hence, price advertising is a prisoner's dilemma. In deriving this result, we assume that consumers have what we coin consistent wary beliefs. If they see that a firm defects by hiding its price, their belief concerning the price that this firm charges should be correct.

We also consider firm incentives to disclose match-value information. In that extension, we assume that one product characteristic is always readily observable, whereas the firm can choose whether to also make the other observable. We show that the firms' equilibrium choice crucially depends on whether prices are observable or not, and on the magnitude of
search costs. With unobservable prices the unique equilibrium has both firms concealing match-value information to consumers. The intuition is as follows. If consumers have all product information, the market would break down because of the Diamond (1971) paradox, which is also the central principle in the related Anderson and Renault's (2006) study of advertising content. By concealing some product information, firms can avoid the temptation to hold up consumers in the standard Diamond fashion. With incomplete information, the demand from consumers that visit this firm is no longer completely inelastic, and therefore any price increase leads to a decrease in demand.

With observable prices such Diamond (1971) issues do not arise. Still, in that case, the unique symmetric pure strategy equilibrium has both firms concealing match-value information provided search cost are reasonably low. When prices are observable, it is never an equilibrium for both firms to reveal match-value information.

Unless advertising costs are prohibitive, our paper thus suggests that models where prices are advertised to consumers make the most sense. Whether match values also are, depends on the amount of search costs. In the context of the Wolinsky (1986) model, firms would want to advertise prices. In the context of the Perloff/Salop (1985) model, firm would want to conceal some product characteristics.

The remainder of this paper is structured as follows. In the next section, we discuss the related literature. We set up the model in Section 3. We solve for the equilibrium in Section 4, and show that prices are decreasing in search costs. Section 5 considers the case in which prices are hidden. In that case, prices are increasing in search costs. Section 6 examines the incentives of a firm to advertise its price while Section 7 focuses on the question whether a firm wants to disclose match-value information to consumers. Section 8 concludes. Most proofs are relegated to an Appendix.

## 2 Related literature

Our model builds on the literature on search with differentiated products, pioneered by Wolinsky (1986) and Anderson and Renault (1999). Yet, different from those papers, we assume that firms are not visited at random. Other papers also drop that assumption.

Arbatskaya (2007) studies a model with otherwise homogeneous products and heterogeneous search costs, where search order is exogenously given. She finds that prices fall in the order of search: a consumer that walks away from a firm reveals that she has low search costs, giving the next firm an incentive to charge a lower price. Zhou (2011) finds the opposite effect in a model with differentiated products. A consumer that walks away now reveals that she did not like that product much, which gives more market power to the next firm in line. A similar result is found in Armstrong et al. (2009), who study a search market with differentiated products where one firm is always visited first, while the other firms are sampled randomly if at all.

In Haan and Moraga-Gonzalez (2011), firms can also influence the order of search. In that paper, they do so by advertising. A firm that advertises more attracts a higher share of consumer visits. In equilibrium prices increase in search costs, but advertising also does, hence profits may decrease.

In both Armstrong and Zhou (2011) and Shen (2015) prices also serve to direct search. As mentioned above, in Wolinsky's (1986) framework an equilibrium in pure-strategies fails to exist when prices are observable and it is extremely hard to characterise mixedstrategy equilibria. Armstrong and Zhou (2011) present an alternative Hotelling-type model where match utilities are negatively correlated. Consumers only observe match values after costly search and firms can advertise prices on a price comparison website. Consumers first visit the firm with the lowest price. By construction, upon learning that firm's match value they can immediately infer the match value the other firm offers. The equilibrium involves price mixing. Similar to our model, average prices decrease as search costs rise. Shen (2015) embeds the Wolinsky framework into a Hotelling model and finds that equilibrium prices may either increase or decrease in search costs. In spirit, this is similar to what we do but we do not restrict attention to perfectly negatively correlated match utilities. Moreover, we also allow firms to influence the order of search by providing match-value advertising ${ }^{2}$

[^2]We believe that our result that equilibrium prices decrease in search costs when prices can direct search, is much more general than the framework we consider. Indeed, Ursu (2014) and Garcia and Shelegia (2015) also find that equilibrium prices decrease in search costs. In these papers, even though prices are not ex-ante observable by consumers, the dynamics are such that the current price of a firm does have a direct influence on the number of consumers that search the firm in the future.

Our paper also relates to the literature on the disclosure of horizontal match-value information. Most of this literature discusses the question how much information firms will provide. In one of the earliest papers, Lewis and Sappington (1994), a monopolist finds it optimal to either provide all information, or not at all, but never some intermediate amount $3^{3}$ Anderson and Renault (2009) study a two-stage oligopoly where firms first provide match-value information and then compete. Firms then choose to reveal all product information. Product information relaxes price competition and this effect dominates the possibly opposing effects found in a monopoly setting.

Anderson and Renault (2006) analyze a monopolist's choice of advertising content and the information disclosed to consumers in a search environment. They show that a monopolist always wants to disclose its price as doing so avoids a hold-up problem. We also find that firms want to advertise their price, but in our case that is due to a businessstealing argument. Both papers find that with unobservable prices firms prefer to conceal match-value information. This is to avoid a hold-up problem whereby consumers would be charged more than what they expect before visiting.

## 3 The Model

Setup Consider a market where 2 single-product firms compete in prices to sell horizontally differentiated products to a unit mass of consumers. Production costs are normalized
prices are observable, which is opposite to what Bakos suggested.
${ }^{3}$ Johnson and Myatt (2006) argue that such match-value advertising is a special case of their general theory of demand rotations, and show that profits are U-shaped in the dispersion of consumers valuations. Hence the result of Lewis and Sappington (1994): firms typically prefer either very high (all match-value information) or very low (no match-value information) levels of dispersion of consumer valuations. BarIsaac et al. (2010) and Wang (2013) present applications where firms prefer partial information disclosure over all or nothing strategies. The latter involves adding costly search to the model.
to zero. A consumer incurs a cost $s$ if she visits a firm. This search cost has to be incurred even if the consumer is fully informed and merely has to visit a firm to obtain the product. Search is sequential and recall is costless. Consumer $j$ that buys product $i$ at price $p_{i}$ obtains utility

$$
\begin{equation*}
u_{i j}\left(p_{i j}\right)=\varepsilon_{i j}+\eta_{i j}-p_{i} . \tag{1}
\end{equation*}
$$

The term $\varepsilon_{i j}+\eta_{i j}$ is the valuation of product $i$ by consumer $j$, and can be interpreted as the match value between $j$ and $i$. It consists of two components: the observable component $\eta_{i j}$ reflects characteristics that can be readily observed, while the opaque component $\varepsilon_{i j}$ reflects characteristics that can only be observed upon visiting the firm. For example, a consumer that wants to buy a car can readily observe its design and exact specifications, so these would be part of the observable component $\eta_{i j} \stackrel{4}{4}^{4}$ Yet, before buying, she would first like to kick the tires and take it for a test drive to be able to evaluate the feel of the car; this would be part of the opaque component $\varepsilon_{i j}$.

The observable component $\eta_{i j}$ differentiates our model from the canonical model of search with differentiated products in e.g. Wolinsky (1986) or Anderson and Renault (1999). It is this component that allows us to have directed search. When prices and distributions of opaque characteristics are equal across firms, a consumer will first visit the firm with the observable characteristic she likes most (see Weitzman, 1979). Moreover, the observable component $\eta_{i j}$ also allows us to analyze cases in which prices are readily observable, so firms can additionally direct search by adjusting their prices. In a model with only an opaque component, a pure strategy equilibrium would then fail to exist ${ }_{5}$

As in Anderson and Renault (1999), we assume that the utility of not buying is sufficiently negative such that all consumers always buy in equilibrium. This allows us to compute the equilibrium price in closed form. Other than in the welfare analysis, this assumption has no important bearing on our results. We will focus on symmetric pure-

[^3]strategy Nash equilibria (SNE). For ease of exposition, we will omit the consumer-specific index $j$ when doing so does not cause confusion.

Distribution functions Values of $\varepsilon_{i j}$ and $\eta_{i j}$ are private information of consumer $j$. It is common knowledge that $\varepsilon_{i j}$ and $\eta_{i j}$ are independently and identically distributed across consumers and firms with distribution functions $F(\varepsilon)$ and $G(\eta)$, respectively. Both $F$ and $G$ are continuously differentiable and the corresponding density functions $f(\varepsilon)$ and $g(\eta)$ are log-concave and non-negative on the entire real line. The assumption of infinite support has the big advantage that demand functions do not have kinks, which greatly simplifies the analysis. Nevertheless, our main results do not depend on this assumption, as we show later when we consider the case where both match values are distributed uniformly on closed intervals. The demand of a firm then has 7 kinks, which makes the analysis of existence of equilibrium quite cumbersome (details in the Appendix).

For our analysis, it will prove useful to define the difference between the observable components for both firms as $\Delta_{\eta} \equiv \eta_{2}-\eta_{1}$. We denote the distribution function of $\Delta_{\eta}$ by $\Gamma\left(\Delta_{\eta}\right)$, its density function by $\gamma\left(\Delta_{\eta}\right)$. Note that

$$
\Gamma\left(\Delta_{\eta}\right)=\operatorname{Pr}\left(\eta_{2}<\eta_{1}+\Delta_{\eta}\right)=\int_{-\infty}^{\infty} \int_{-\infty}^{\eta_{1}+\Delta_{\eta}} d G\left(\eta_{2}\right) d G\left(\eta_{1}\right)=\int_{-\infty}^{\infty} G\left(\eta+\Delta_{\eta}\right) d G(\eta)
$$

hence

$$
\gamma\left(\Delta_{\eta}\right)=\int_{-\infty}^{\infty} g\left(\eta+\Delta_{\eta}\right) d G(\eta)
$$

The density of $\Delta_{\eta}$ can be shown to have the following properties:

Lemma 1. $\gamma\left(\Delta_{\eta}\right)$ is log-concave and symmetric around zero; moreover, $E\left(\Delta_{\eta}\right)=0$.

Proof. In appendix.

The consumer search rule Suppose we are in an equilibrium where all firms charge the same price $p^{*}$. Suppose a consumer has visited a firm, say $i$, has observed utility $\varepsilon_{i}+\eta_{i}-p^{*}$ and is contemplating whether to visit the other firm, $k$. Buying at firm $k$ gives higher utility whenever $\varepsilon_{k}>x \equiv \varepsilon_{i}+\eta_{i}-\eta_{k}$. The expected gains of paying a costly visit
to $k$ are thus given by

$$
\begin{equation*}
h(x, s) \equiv \int_{x}^{\infty}\left(\varepsilon_{k}-x\right) d F\left(\varepsilon_{k}\right)-s \tag{2}
\end{equation*}
$$

Define the reservation value $\hat{x}$ as the solution to $h(\hat{x}, s)=0$. As the right-hand side of (2) is strictly decreasing in $x$, we have that the consumer buys product $i$ without visiting firm $k$ whenever $x>\hat{x}$, hence $\varepsilon_{i}>\hat{x}-\eta_{i}+\eta_{k}$. Otherwise, she does visit firm $k$.

In a symmetric equilibrium, the reservation utilities of this consumer at firms $i$ and $k$ are thus given by $\hat{x}+\eta_{i}$ and $\hat{x}+\eta_{k}$, respectively. Following Weitzman (1979), a consumer that searches to maximize expected utility should first visit the firm where her reservation utility is the highest. This implies that search is directed here and consumers for whom $\eta_{i}>\eta_{j}$ start their search at firm $i$. Upon learning the match value (and possibly the price) at that firm, they use the stopping rule described above.

## 4 Advertised prices

In this section, we consider the case that prices are readily observable. Hence, consumers do not have to visit a firm before learning its price. There can be many reasons for that. For example, there is a price comparison site that lists all prices. Alternatively, firms simply advertise their price. In the remainder of this paper, we will stick to the last interpretation. In section 6 we will endogenize a firm's decision to advertise its price. Here, we derive a symmetric equilibrium price $p_{A}^{*}$ for the case that prices are advertised. In this analysis, search is directed not only by the differences in observable characteristics but also by price differences. That is, by its choice of price an individual firm can affect the share of consumers that choose to initiate their search at its premises.

Suppose a firm, say 1 , deviates from the tentative equilibrium price $p_{A}^{*}$ by charging $p_{1} \neq p_{A}^{*}$. Define $\Delta_{p} \equiv p_{A}^{*}-p_{1}$. With observable prices, reservation utilities at firms 1 and 2 are $\hat{x}+\eta_{1}-p_{1}$ and $\hat{x}+\eta_{2}-p_{A}^{*}$, respectively. Hence, consumers for whom $\eta_{1}-p_{1} \geq \eta_{2}-p_{A}^{*}$ (or $\Delta_{\eta} \leq \Delta_{p}$ ), will first visit firm 1, while the others will first visit firm 2. Let $D_{1}^{A}\left(p_{1}, p_{A}^{*} ; \hat{x}\right)$ denote total demand for firm 1 . The superscript $A$ reflects that prices are advertised; the argument $\hat{x}$ indicates that this demand also depends on the magnitude of search costs.

Demand for firm 1 consists of two components. First, some consumers first visit firm

1 and also end up buying product 1 . We denote demand from this source as $q_{11}^{A}\left(p_{1}, p_{A}^{*} ; \hat{x}\right)$, where the first subscript denotes where the consumer starts searching, and the second denotes where she ends up buying. Second, there are consumers who visit firm 2 first, but choose to walk away from it to inspect product 1 and end up buying it. Demand from these consumers is denoted $q_{21}^{A}\left(p_{1}, p_{A}^{*} ; \hat{x}\right)$. Naturally,

$$
\begin{equation*}
D_{1}\left(p_{1}, p_{A}^{*} ; \hat{x}\right)=q_{11}^{A}\left(p_{1}, p_{A}^{*} ; \hat{x}\right)+q_{21}^{A}\left(p_{1}, p_{A}^{*} ; \hat{x}\right) . \tag{3}
\end{equation*}
$$

For the first group, we have

$$
\begin{align*}
q_{11}^{A}\left(p_{1}, p_{A}^{*} ; \hat{x}\right) & =\int_{-\infty}^{\Delta_{p}}\left(1-F\left(\hat{x}+\Delta_{\eta}-\Delta_{p}\right)\right) d \Gamma\left(\Delta_{\eta}\right) \\
& +\int_{-\infty}^{\Delta_{p}} \int_{-\infty}^{\hat{x}+\Delta_{\eta}-\Delta_{p}} F\left(\varepsilon-\Delta_{\eta}+\Delta_{p}\right) d F(\varepsilon) d \Gamma\left(\Delta_{\eta}\right) \tag{4}
\end{align*}
$$

This can be seen as follows. First, some consumers first visit firm 1, and decide to buy there without visiting firm 2. Such consumers necessarily have $\Delta_{\eta} \leq \Delta_{p}$ (they first visit firm 1) and $\varepsilon_{1} \geq \hat{x}+\Delta_{\eta}-\Delta_{p}$ (they find an $\varepsilon_{1}$ that does not make it worthwhile to visit firm 2). The first term of (4) reflects the joint probability of these events. Second, there are consumers who first visit firm 1, then decide to also visit firm 2, but do end up buying product 1. Such consumers have $\Delta_{\eta} \leq \Delta_{p}$ (they first visit firm 1), $\varepsilon_{1}<\hat{x}+\Delta_{\eta}-\Delta_{p}$ (they find it worthwhile to visit firm 2), and $\varepsilon_{2}<\varepsilon_{1}-\Delta_{\eta}+\Delta_{p}$ (they learn that firm 2 offers a worse deal). The second term of (4) reflects the joint probability of these events.

For the second group in (3), we have

$$
\begin{align*}
q_{21}^{A}\left(p_{1}, p_{A}^{*} ; \hat{x}\right) & =\int_{\Delta_{p}}^{\infty} \int_{-\infty}^{\hat{x}} F\left(\varepsilon-\Delta_{\eta}+\Delta_{p}\right) d F(\varepsilon) d \Gamma\left(\Delta_{\eta}\right) \\
& +\int_{\Delta_{p}}^{\infty} F\left(\hat{x}-\Delta_{\eta}+\Delta_{p}\right)(1-F(\hat{x})) d \Gamma\left(\Delta_{\eta}\right) \tag{5}
\end{align*}
$$

These consumers have $\Delta_{\eta}>\Delta_{p}$ (they first visit firm 2), $\varepsilon_{2}<\hat{x}-\Delta_{\eta}+\Delta_{p}$ (they decide to also visit firm 1), and $\varepsilon_{2}<\varepsilon_{1}-\Delta_{\eta}+\Delta_{p}$ (they learn that firm 2 offers the worse deal). The joint probability of these events is reflected in (5). ${ }^{6}$

[^4]The payoff to firm 1 is

$$
\begin{equation*}
\pi_{1}^{A}\left(p_{1}, p_{A}^{*} ; \hat{x}\right)=p_{1} D_{1}^{A}\left(p_{1}, p_{A}^{*} ; \hat{x}\right) . \tag{6}
\end{equation*}
$$

Taking the first-order condition (henceforth FOC) and imposing symmetry, we obtain:
Proposition 2. If a SNE with advertised prices exists, we have

$$
\begin{equation*}
p_{A}^{*}=\frac{1}{4 \int_{-\infty}^{0}\left[f\left(\hat{x}+\Delta_{\eta}\right)(1-F(\hat{x}))+\int_{-\infty}^{\hat{x}+\Delta_{\eta}} f\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right] d \Gamma\left(\Delta_{\eta}\right)+2 \gamma(0)(1-F(\hat{x}))^{2}} . \tag{7}
\end{equation*}
$$

Per-firm profits are $\pi_{A}^{*}=\frac{1}{2} p_{A}^{*}$. Equilibrium prices and profits decrease with search costs.
Proof. In appendix.
Hence, in this set-up, we are able to derive an explicit expression for the equilibrium price - provided that it exists. Most interestingly, we find that higher search costs imply lower prices. This runs counter to most of the consumer search literature. The intuition is as follows. Different from e.g. Anderson and Renault (1999), we assume that prices are readily observable, and consumers only have to search for product characteristics. Hence, prices serve to direct search. With higher search costs, a consumer that visits is less likely to continue search. Hence, with higher search costs, firms are more eager to attract consumers in the first place. The only way they can do so, is by setting a lower price. This argument suggests that this result is driven by the assumption that prices are readily observable. In the next section, we show that that is indeed the case.

Regarding existence of the SNE, we show in the Appendix that the payoff function (6) is strictly concave in a neighborhood of $p_{A}^{*}$. Yet, it may not be globally quasi-concave even with log-concavity of the densities $\gamma$ and $f$, as the sum of log-concave functions (in this case, $q_{11}^{A}$ and $q_{21}^{A}$ ) need not be log-concave. 7 We have numerically checked $\log$-concavity of (6) for several distributions. In the Appendix we look at the cases when $\eta$ and $\varepsilon$ are either both normally distributed (Figure 1a); both follow a Gumbel distribution (Figure 1 b ); or are both uniformly distributed (Figure 2). In all these cases, (6) is quasi-concave

[^5]provided that the dispersion of $\eta$ 's is sufficiently large 8 At least in those cases, we thus have that the SNE in Proposition 1 indeed exists.

Example: the uniform distribution As an example, we consider the case where match values $\eta$ and $\varepsilon$ are uniformly distributed. Hence, in this example, we do not have infinite support. Assume that $\eta \in[\beta-\bar{\eta}, \beta+\bar{\eta}]$, with $\bar{\eta}$ sufficiently large (cf. footnote 8), and $\varepsilon \in[\alpha-\bar{\varepsilon}, \alpha+\bar{\varepsilon}]$. An increase in $\beta$ and $\alpha$ thus raise the mean of $\eta$ and $\varepsilon$, respectively, while an increase in $\bar{\eta}$ and $\bar{\varepsilon}$ raise the variance of $\eta$ and $\varepsilon$. Note that for our analysis we only need the distribution of $\Delta_{\eta}$, not that of the individual $\eta$ 's. Hence, the parameter $\beta$ will not affect the analysis. For ease of exposition, we set $\beta=\bar{\eta}$, so $\eta$ is distributed on $[0,2 \bar{\eta}]$. When then have

$$
\Gamma(z) \equiv \operatorname{Pr}\left(\Delta_{\eta}<z\right)=\int_{0}^{\min \left\{\max \left\{z+\eta_{1}, 2 \bar{\eta}\right\}, 0\right\}} \int_{0}^{2 \bar{\eta}} d G\left(\eta_{1}\right) d G\left(\eta_{2}\right) .
$$

With a uniform distribution of $G$ on $[0,2 \bar{\eta}]$ this implies

$$
\Gamma(z)=\left\{\begin{array}{cl}
\frac{1}{8 \bar{\eta}^{2}}(2 \bar{\eta}+z)^{2} & \text { if } z \leq 0 ; \\
1-\frac{1}{8 \bar{\eta}^{2}}(2 \bar{\eta}-z)^{2} & \text { if } z \geq 0 .
\end{array}\right.
$$

which in turn implies

$$
\gamma(z)= \begin{cases}\frac{1}{4 \bar{\eta}^{2}}(2 \bar{\eta}+z) & \text { if } z \leq 0  \tag{8}\\ \frac{1}{4 \bar{\eta}^{2}}(2 \bar{\eta}-z) & \text { if } z \geq 0 .\end{cases}
$$

Moreover

$$
\begin{array}{ll}
G(\eta)=\frac{\eta-(\beta-\bar{\eta})}{2 \bar{n}} ; & g(\eta)=\frac{1}{2 \bar{n}} ; \\
F(\varepsilon)=\frac{\varepsilon-(\alpha \bar{\varepsilon})}{2 \bar{\varepsilon}} ; & f(\varepsilon)=\frac{1}{2 \bar{\varepsilon}} . \tag{9}
\end{array}
$$

For consistency with the analysis above, we assume that regardless of the value of $\Delta_{\eta}$ a consumer may walk away from the firm that she chooses to visit first. In other words, even with, say, the highest possible $\Delta_{\eta}$, she will still continue search if she finds the worst possible $\varepsilon_{1}$ at firm 1. Hence, we need $\alpha-\bar{\varepsilon}<\hat{x}-2 \bar{\eta}$. Without this assumption, our expressions for $q_{11}^{A}$ and $q_{21}^{A}$ would be different, but our qualitative results would still hold. ${ }_{-}$

[^6]From (2), we now have

$$
h(x, s)=\int_{x}^{\alpha+\bar{\varepsilon}} \frac{\left(\varepsilon_{k}-x\right)}{2 \bar{\varepsilon}} d \varepsilon
$$

Equating this to $s$ yields $\sqrt{10}$

$$
\begin{equation*}
\hat{x}=\alpha+\bar{\varepsilon}-2 \sqrt{\bar{\varepsilon} S} \tag{10}
\end{equation*}
$$

The condition $\alpha-\bar{\varepsilon}<\hat{x}-2 \bar{\eta}$ then becomes

$$
\begin{equation*}
\bar{\eta}<\bar{\varepsilon}-\sqrt{\bar{\varepsilon} s} . \tag{11}
\end{equation*}
$$

Hence, we need that the observable component is less noisy than the opaque one. If (11) is not satisfied, we could still derive an equilibrium price but the analysis would be much more cumbersome. Note that for (11) to be satisfied, it is necessary to have $s<\bar{\varepsilon}$.

For uniform distributions, the equilibrium price in (7) can re rewritten as:

$$
\begin{equation*}
\frac{1}{p_{A}^{*}}=4 \int_{-2 \bar{\eta}}^{0}\left[\frac{1}{2 \bar{\varepsilon}}\left(\frac{\bar{\varepsilon}+\alpha-\hat{x}}{2 \bar{\varepsilon}}\right)+\int_{\alpha-\bar{\varepsilon}}^{\hat{x}+\Delta_{\eta}}\left(\frac{1}{2 \bar{\varepsilon}}\right)^{2} d \varepsilon\right] \gamma\left(\Delta_{\eta}\right) d \Delta_{\eta}+\frac{1}{\bar{\eta}}\left(\frac{\bar{\varepsilon}+\alpha-\hat{x}}{2 \bar{\varepsilon}}\right)^{2} \tag{12}
\end{equation*}
$$

Plugging (10) into (12) yields

$$
\begin{aligned}
\frac{1}{p_{A}^{*}} & =4 \int_{-2 \bar{\eta}}^{0}\left[\frac{1}{2 \bar{\varepsilon}}\left(\sqrt{\frac{s}{\bar{\varepsilon}}}\right)+\int_{\alpha-\bar{\varepsilon}}^{\hat{x}+\Delta_{\eta}}\left(\frac{1}{2 \bar{\varepsilon}}\right)^{2} d \varepsilon\right]\left(\frac{2 \bar{\eta}+\Delta_{\eta}}{4 \bar{\eta}^{2}}\right) d \Delta_{\eta}+\frac{s}{\bar{\eta} \bar{\varepsilon}} \\
& =4 \int_{-2 \bar{\eta}}^{0}\left[\frac{1}{2 \bar{\varepsilon}}\left(\sqrt{\frac{s}{\bar{\varepsilon}}}\right)+\frac{2 \bar{\varepsilon}-2 \sqrt{\bar{\varepsilon} s}+\Delta_{\eta}}{4 \bar{\varepsilon}^{2}}\right]\left(\frac{2 \bar{\eta}+\Delta_{\eta}}{4 \bar{\eta}^{2}}\right) d \Delta_{\eta}+\frac{s}{\bar{\eta} \bar{\varepsilon}} \\
& =4 \int_{-2 \bar{\eta}}^{0}\left(\frac{2 \bar{\varepsilon}+\Delta_{\eta}}{4 \bar{\varepsilon}^{2}}\right)\left(\frac{2 \bar{\eta}+\Delta_{\eta}}{4 \bar{\eta}^{2}}\right) d \Delta_{\eta}+\frac{s}{\bar{\eta} \bar{\varepsilon}}=\frac{3 \bar{\varepsilon}-\bar{\eta}}{3 \bar{\varepsilon}^{2}}+\frac{s}{\bar{\eta} \bar{\varepsilon}} .
\end{aligned}
$$

Hence, the equilibrium price simplifies to:

$$
p_{A}^{*}=\frac{3 \bar{\varepsilon}^{2} \bar{\eta}}{3 \bar{\varepsilon} \bar{\eta}+3 s \bar{\varepsilon}-\bar{\eta}^{2}}
$$

As $\bar{\eta}<\bar{\varepsilon}$, the price is indeed positive. Moreover, it is immediate that the equilibrium price decreases in search costs. Also note that $p_{A}^{*}$ does not depend on $\alpha$ or $\beta$; as the market is fully covered, an increase in either $\alpha$ or $\beta$ simply means that both products become more attractive to the same extent; hence this does not affect pricing. Finally

[^7]note that
$$
\frac{\partial p_{A}^{*}}{\partial \bar{\varepsilon}}=\frac{3 \bar{\varepsilon} \bar{\eta}\left(3 \bar{\varepsilon} \bar{\eta}+3 s \bar{\varepsilon}-2 \bar{\eta}^{2}\right)}{\left(3 \bar{\varepsilon} \bar{\eta}+3 s \bar{\varepsilon}-\bar{\eta}^{2}\right)^{2}}>0
$$
where the inequality follows directly from $\bar{\eta}<\bar{\varepsilon}$, and moreover
$$
\frac{\partial p_{A}^{*}}{\partial \bar{\eta}}=\frac{3 \bar{\varepsilon}^{2}\left(\bar{\eta}^{2}+3 s \bar{\varepsilon}\right)}{\left(3 \bar{\varepsilon} \bar{\eta}+3 s \bar{\varepsilon}-\bar{\eta}^{2}\right)^{2}}>0
$$

Hence an increase in the dispersion of consumer valuations, which implies a higher degree of product differentiation, results in higher prices.

## 5 Non-advertised prices

In this section, we derive a symmetric equilibrium price $p_{N}^{*}$ for the case in which consumers can only observe a firm's price after visiting it. This is the usual assumption in consumer search models. Search will still be directed, but only because of the observable characteristic. As prices are not observed before searching, an individual firm is unable to influence the share of consumers that choose to start searching at its premises.

The analysis is very similar to that in the previous section. Suppose firm 1 deviates from the tentative equilibrium price $p_{N}^{*}$ by charging $p_{1} \neq p_{N}^{*}$. As before, let $\Delta_{p} \equiv p_{N}^{*}-p_{1}$. Consumers expect both firms to charge $p_{N}^{*}$; hence their reservation utilities at firms 1 and 2 are $\hat{x}+\eta_{1}-p_{N}^{*}$ and $\hat{x}+\eta_{2}-p_{N}^{*}$, respectively. Given this, a consumer will first visit firm 1 if and only if $\Delta_{\eta} \equiv \eta_{2}-\eta_{1}<0$. Total demand $D_{1}^{N}\left(p_{1}, p_{N}^{*} ; \hat{x}\right)$ of firm 1 again consists of two components: $q_{11}^{N}\left(p_{1}, p_{N}^{*} ; \hat{x}\right)$ from consumers that first visit firm 1 , and $q_{21}^{N}\left(p_{1}, p_{N}^{*} ; \hat{x}\right)$ from those that first visit firm 2, where superscripts $N$ denote that prices are not advertised. The expressions for $q_{11}^{N}$ and $q_{21}^{N}$ now differ from $q_{11}^{A}$ and $q_{21}^{A}$.

Consumers that first visit firm 1 and decide to buy without visiting firm 2 now have $\Delta_{\eta}<0$ and $\varepsilon_{1} \geq \hat{x}+\Delta_{\eta}-\Delta_{p}$. Those that first visit firm 1, then visit firm 2, but end up buying product 1 have $\Delta_{\eta}<0 ; \varepsilon_{1}<\hat{x}+\Delta_{\eta}-\Delta_{p}$ and $\varepsilon_{1} \geq \varepsilon_{2}-\Delta_{\eta}-\Delta_{p}$. Hence

$$
q_{11}^{N}=\int_{-\infty}^{0}\left(1-F\left(\hat{x}+\Delta_{\eta}-\Delta_{p}\right)+\int_{-\infty}^{\hat{x}+\Delta_{\eta}-\Delta_{p}} F\left(\varepsilon-\Delta_{\eta}+\Delta_{p}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right) .
$$

A consumer that first visits firm 2 has $\Delta_{\eta} \geq 0$. She decides to also visit firm 1 if $\varepsilon_{2}<\hat{x}-\Delta_{\eta}$,
as she expects firm 1 to charge the equilibrium price $p_{N}^{*}$. Upon observing the match value and price of firm 1 , she buys product 1 if $\varepsilon_{2}<\varepsilon_{1}-\Delta_{\eta}+\Delta_{p}$. Hence

$$
q_{21}^{N}=\int_{0}^{\infty}\left(\int_{-\infty}^{\hat{x}-\Delta_{p}} F\left(\varepsilon-\Delta_{\eta}+\Delta_{p}\right) d F(\varepsilon)+F\left(\hat{x}-\Delta_{\eta}\right)\left(1-F\left(\hat{x}-\Delta_{p}\right)\right)\right) d \Gamma\left(\Delta_{\eta}\right),
$$

where the first term reflects the case that $\varepsilon_{1}<\hat{x}-\Delta_{p}$, and the second term the case that $\varepsilon_{1} \geq \hat{x}-\Delta_{p}$. Total profits of firm 1 are

$$
\begin{equation*}
\pi_{1}^{N}\left(p_{1}, p_{N}^{*}\right)=p_{1} D_{1}^{N}\left(p_{1}, p_{N}^{*} ; \hat{x}\right) \tag{13}
\end{equation*}
$$

Taking the FOC and imposing symmetry, we obtain:
Proposition 3. If a SNE with concealed prices exists, we have

$$
\begin{equation*}
p_{N}^{*}=\frac{1}{2 \int_{-\infty}^{0}\left(f\left(\hat{x}+\Delta_{\eta}\right)(1-F(\hat{x}))+2 \int_{-\infty}^{\hat{x}+\Delta_{\eta}} f\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right)} \tag{14}
\end{equation*}
$$

Per-firm profits are $\pi_{N}^{*}=\frac{1}{2} p_{N}^{*}$. Equilibrium prices and profits increase with search cost. Proof. In appendix.

Hence equilibrium prices and profits now indeed increase in search costs, as they do in the standard consumer search model with differentiated products. Again, higher search costs imply that a consumer that visits a firm is less likely to continue search. That gives firms more market power which leads to higher prices. With advertised prices this effect is also present, but is dominated by the effect that lower prices then also attract more consumers in the first place, a channel that is ruled out when prices are not advertised.

Again, existence of equilibrium is an issue. In the Appendix we show that also for this case, the payoff (13) is typically quasi-concave (see Figure 3a for normally distributed, Figure 3b for Gumbel distributed and Figure 4 for uniformly distributed match values).

Comparing the equilibrium prices in Propositions 2 and 3, it is easy to see that $p_{N}^{*} \geq$ $p_{A}^{*}$. Hence, price observability leads to lower prices. Also note that this price difference increases as search costs increase; with $s=0$, we have $p_{N}^{*}=p_{A}^{*}$ by construction, as prices are effectively observable in both cases. As $s$ increases, $p_{A}^{*}$ decreases while $p_{N}^{*}$ increases, increasing the gap between the two.

Example: the uniform distribution. We again consider the case where match values $\eta$ and $\varepsilon$ are uniformly distributed with supports $[\beta-\bar{\eta}, \beta+\bar{\eta}]$ and $[\alpha-\bar{\varepsilon}, \alpha+\bar{\varepsilon}]$, respectively. Note that $\hat{x}$ is still given by (10). Plugging (10) into (14), using (8) and (9) yields

$$
\begin{aligned}
\frac{1}{p_{N}^{*}} & =2 \int_{-2 \bar{\eta}}^{0}\left[\frac{1}{2 \bar{\varepsilon}}\left(\sqrt{\frac{s}{\bar{\varepsilon}}}\right)+\frac{2 \bar{\varepsilon}-2 \sqrt{\bar{\varepsilon} s}+\Delta_{\eta}}{2 \bar{\varepsilon}^{2}}\right]\left(\frac{2 \bar{\eta}+\Delta_{\eta}}{4 \bar{\eta}^{2}}\right) d \Delta_{\eta} \\
& =2 \int_{-2 \bar{\eta}}^{0}\left(\frac{2 \bar{\varepsilon}+\Delta_{\eta}-\sqrt{s \bar{\varepsilon}}}{2 \bar{\varepsilon}^{2}}\right)\left(\frac{2 \bar{\eta}+\Delta_{\eta}}{4 \bar{\eta}^{2}}\right) d \Delta_{\eta}=\frac{6 \bar{\varepsilon}-2 \bar{\eta}-3 \sqrt{s \bar{\varepsilon}}}{6 \bar{\varepsilon}^{2}} .
\end{aligned}
$$

Hence

$$
p_{N}^{*}=\frac{6 \bar{\varepsilon}^{2}}{6 \bar{\varepsilon}-2 \bar{\eta}-3 \sqrt{s \bar{\varepsilon}}} .
$$

With $\eta<\bar{\varepsilon}$ and $s<\bar{\varepsilon}$, the equilibrium price is again positive. From the expression, it is immediate that the price increases in search costs. The derivative with respect to $\bar{\eta}$ is also clearly positive. Moreover,

$$
\frac{\partial p_{N}^{*}}{\partial \bar{\varepsilon}}=\frac{3 \bar{\varepsilon}(12 \bar{\varepsilon}-8 \bar{\eta}-9 \sqrt{s \bar{\varepsilon}})}{(2 \bar{\eta}-6 \bar{\varepsilon}+3 \sqrt{s \bar{\varepsilon}})^{2}}>0
$$

because with $\bar{\eta}<\bar{\varepsilon}-\sqrt{\bar{\varepsilon} s}$ we have $12 \bar{\varepsilon}-8 \bar{\eta}-9 \sqrt{s \bar{\varepsilon}}>4 \bar{\varepsilon}-\sqrt{s \bar{\varepsilon}}>0$ as $s<\bar{\varepsilon}$. Hence, also in this case, more dispersion in consumer valuations, and hence more product differentiation, yields higher prices.

## 6 The decision to advertise the price

Above, we considered the cases that either all prices are advertised and readily observable before a consumer starts searching, or that all prices are not advertised and only observed after engaging in costly search. In this section we endogenize the advertising decision. First, we study whether an equilibrium exists in which firms do not advertise their price. Second, we study whether it can be an equilibrium for firms to advertise their price. For simplicity, we assume that firms can advertise their price for free.

Concealing the price We first study whether an equilibrium exists in which firms do not advertise their price. To do so, we take the equilibrium with non-advertised prices of

Proposition 3 and check whether an individual firm can gain by changing its price and advertising it. That turns out to be the case:

Proposition 4. A SNE where firms choose not to advertise their prices does not exist.
Proof. A sketch of the proof is as follows. Suppose firm 1 deviates by charging $p_{1} \neq p_{N}^{*}$ and advertising it. All consumers can then readily observe the price of firm 1, while they still assume that firm 2 charges $p_{N}^{*}$. Hence, profits of firm 1 then equal $\pi_{1}^{A}\left(p_{1}, p_{N}^{*} ; \hat{x}\right)$ which by construction are its profits when prices for firms 1 and 2 are known to be $p_{1}$ and $p_{N}^{*}$, respectively. As $\partial \pi_{1}^{A}\left(p_{N}^{*}, p_{N}^{*} ; \hat{x}\right) / \partial p_{1}<0$ and $\pi_{A}\left(p_{N}^{*}, p_{N}^{*}\right)=\pi_{N}\left(p_{N}^{*}, p_{N}^{*}\right)$, we have that in the equilibrium with no advertising firm 1 indeed has an incentive to deviate to a lower price and advertising that price. Details in Appendix.

One implication of this result is that the standard model of search with differentiated products, as proposed by Wolinsky (1986) can only be justified in situations where advertising frictions exist, and it is prohibitively costly for firms to advertise their price.

Advertising the price We now study whether an equilibrium exists in which both firms do advertise their price. To verify this, we need to check that no firm has an incentive to deviate from the equilibrium with advertised prices given in Proposition 1, by concealing its price.

Suppose that firm 1 deviates by concealing its price, while firm 2 sticks to the tentative equilibrium by charging price $p_{A}^{*}$ and advertising it. Let $p_{1} \neq p_{A}^{*}$ be the price charged by firm 1. Since $p_{1}$ is not advertised, it now becomes crucial what consumers believe concerning the price that firm 1 charges. Borrowing from the literature on vertical restraints, consumers could have passive beliefs (Hart and Tirole, 1990), and expect firm 1 to still charge $p_{A}^{*}$, despite the fact that it now conceals its price. Similar to the reasoning behind the Diamond (1971) paradox, firm 1 would then have an incentive to hold up consumers: given that consumers would visit the firm as usual, the deviant firm would gain by raising its price. Of course, rational consumers should anticipate that, which implies that passive beliefs are not the most natural assumption to make.

One alternative is to have wary beliefs (see McAfee and Schwartz, 1994). In the context of vertical contracting a downstream firm has wary beliefs if, after being offered
an out-of-equilibrium contract, it assumes that the other downstream firms receive offers that are the upstream monopolist's optimal choices given the offer it made to this firm. Yet, that refinement is not readily applicable in our context. We cannot simply assume that consumers believe that a firm that conceals its price sets the profit-maximizing price given its concealment - as that optimal price will in turn depend on consumer's beliefs.

We therefore introduce consistent wary beliefs; consumers expect the concealing firm to set a price $p_{1}$ such that it is indeed optimal for the concealing firm to set $p_{1}$, given that consumers believe it will do so. Specifically, if we denote the belief of consumers conditional on the firm having deviated from equilibrium by not adverting its price by $p_{1}^{e}$, we require this belief to satisfy the following:

$$
\begin{equation*}
p_{1}^{e}=\arg \max _{p_{1}} \pi_{N}\left(p_{1}, p_{A}^{*} ; \hat{x}, p_{1}^{e}\right) \tag{15}
\end{equation*}
$$

But that immediately implies:

Proposition 5. With consistent wary beliefs, an equilibrium where prices are advertised always exists.

Proof. Consider the equilibrium where both firms charge $p_{A}^{*}$ and advertise that price. Firm 1's profits then are $\pi_{A}\left(p_{A}^{*}, p_{A}^{*} ; \hat{x}\right)$. From (15), with consistent wary beliefs, any deviation $p_{1}$ necessarily has $p_{1}=\pi_{N}\left(p_{1}, p_{A}^{*} ; \hat{x}\right)$. As consumer beliefs are correct, any such deviation is equivalent to firm 1 advertising its deviation price. That is, it must be the case that $\pi_{N}\left(p_{1}, p_{A}^{*} ; \hat{x}\right)=\pi_{A}\left(p_{1}, p_{A}^{*} ; \hat{x}\right)$. Clearly, any deviation to $p_{1} \neq p_{A}^{*}$ is then unprofitable since by construction $p_{A}^{*}=\arg \max _{p_{1}} \pi_{A}\left(p_{1}, p_{A}^{*} ; \hat{x}\right){ }^{11}$

## 7 Match-value advertising

In the previous section, we studied the incentives of firms to advertise their price. We showed that, under reasonable consumer beliefs, the unique symmetric equilibrium has both firms advertising their price.

[^8]In this section, we study the incentive for firms to advertise product characteristics. In doing so, we slightly deviate from the interpretation of the model that we used so far. Above, we assumed that $\eta$ can be observed at no costs, while $\varepsilon$ can only observed after visiting the firm, as it concerns e.g. the fit of a pair of jeans or the feel of a car.

We now relax that assumption, and interpret $\varepsilon$ as a characteristic that firms can choose either to hide or to advertise. Of course, it is hard to imagine that any ad can convey the fit of a pair of jeans. But arguably, firms through ads can give information about many characteristics, by means of detailed specifications, pictures, etc., that a consumer would otherwise only be able to obtain by visiting the firm. Hence, in this section, we assume that $\eta$ is a match value that is readily observable, and that $\varepsilon$ is a match value that consumers can learn either from visiting the firm, or from seeing its ads - provided the firm chooses to advertise product information that allows the consumer to learn its $\varepsilon$.

We do this for the two cases analysed above; first for the case when prices are not observable, then for the observable prices case.

It is important to note that firms cannot directly advertise match values; each consumer has a different $\varepsilon_{i j}$ and firms are never able to observe these. When we discuss "match-value advertising", we thus refer to a situation where a firm advertises product characteristics in such detail that each consumer is able to perfectly learn its $\varepsilon_{i j}$.

### 7.1 Match-value only advertising

We start by studying whether firms want to advertise information on product characteristics when prices cannot be advertised. As argued in the introduction, there are circumstances in which price advertising is not feasible. For example, a firm may sell many different products. Alternatively, due to cost volatility sellers may not be willing to commit to advertises prices.

Proposition 6. Assume that prices are not advertised and consumers have consistent wary beliefs. The unique symmetric equilibrium then has neither firm reveal information concerning the match value $\varepsilon$. In this equilibrium, price is equal to $p_{N}^{*}$ given by Proposition 3.

Proof. Essentially, this is a variation on the Diamond paradox. If firm 1 reveals $\varepsilon_{1}$ and consumers expect it to set a price $p_{1}^{e}$, firm 1 is always better off charging $p_{1}=p_{1}^{e}+s$ instead, as by doing so it will not lose any customers. With consistent wary beliefs, consumers anticipate this and revealing $\varepsilon$ would cause the market for product 1 to break down. Hence, such a deviation is not be profitable. Moreover, if both firms would reveal $\varepsilon$, we would find ourselves in the standard Diamond paradox. Hence, the equilibrium described in the Proposition is unique. Details in appendix.

By revealing information on the match value $\varepsilon_{1}$ firm 1 creates a hold-up problem which is unfavourable for itself. In fact, for every expectation held by the consumers $p_{1}^{e}$, firm 1 gains by raising its price to $p_{1}^{e}+s$. Such a price increase does not affect its sales, neither to the consumers who choose to visit firm 1 and buy there directly, nor to the consumers who first visit firm 2 and eventually choose to walk away from it and buy from firm 1.

### 7.2 Match-value and price advertising

We now examine the case when prices can be advertised. From Proposition 5 we know that if firms are given the opportunity to advertise prices then they will do so and the equilibrium price will be given by $p_{A}^{*}$ in Proposition 2. Starting from such an equilibrium with advertised prices, we now ask whether an individual firm wants to deviate by providing information on the match value $\varepsilon_{1}$ and possibly changing its price. After having done so, we study whether it can be an equilibrium for both firms to advertise information about $\varepsilon$.

Concealing information about $\varepsilon$ We first study whether it can be an equilibrium for both firms to advertise their price, but to conceal their $\varepsilon$. If that is the case, neither firm should have an incentive to reveal its $\varepsilon$, while possibly also changing its price to $p_{1} \neq p_{A}^{*}$. As a result of this deviation, consumers become fully informed concerning the offering of firm 1. We first derive how that affects demand of firm 1. The reservation utility for visiting firm 2 is again given by $\hat{x}+\eta_{2}-p_{A}^{*}$. Consumers have to pay $s$ to buy product 1 .

A consumer will thus visit firm 1 directly and buy there if

$$
\begin{equation*}
\varepsilon_{1}>\hat{x}+s+\Delta_{\eta}-\Delta_{p} . \tag{16}
\end{equation*}
$$

where again $\Delta_{p} \equiv p_{A}^{*}-p_{1}$ and $\Delta_{\eta}=\eta_{2}-\eta_{1}$. If (16) does not hold she first visits firm 2 and will still buy from firm 1 if $\varepsilon_{1}>\varepsilon_{2}-\Delta_{\eta}+\Delta_{p}-s$. Demand for firm 1 then is
$D_{1}^{A A}=\int_{-\infty}^{\infty}\left(1-F\left(\hat{x}+s+\Delta_{\eta}-\Delta_{p}\right)+\int_{-\infty}^{\hat{x}+s+\Delta_{\eta}-\Delta_{p}} F\left(\varepsilon-\Delta_{\eta}+\Delta_{p}-s\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right)$

Comparing demand in (17) and that in (4) and (5) reveals that providing information on $\varepsilon_{1}$ results in a significant change in the composition of demand. Some consumers that buy immediately from firm 2 absent the defection are actually better off buying from firm 1 , but never find out that firm 1 is offering them a better deal in terms of a high $\varepsilon_{1}$. They do now, as they can readily observe $\varepsilon_{1} \cdot \frac{12}{2}$ This represents a source of demand increase. On the other hand, consumers who used to first visit firm 1 decide no longer to do so as they learn in advance that their $\varepsilon_{1}$ is low ${ }^{13}$ Also, there are fewer consumers that first visit 1 , then go to 2 , but return to 1 to buy there ${ }^{[14]}$ and fewer consumers that still come

[^9]$$
g=\int_{\Delta_{p}}^{\infty}\left(1-F\left(\hat{x}-\Delta_{\eta}+\Delta_{p}\right)\right)\left(1-F\left(\hat{x}+\Delta_{\eta}-\Delta_{p}+s\right)\right) d \Gamma\left(\Delta_{\eta}\right)
$$
${ }^{13}$ This decrease amounts to
$$
\int_{-\infty}^{\Delta_{p}}\left(F\left(\hat{x}+s+\Delta_{\eta}-\Delta_{p}\right)-F\left(\hat{x}+\Delta_{\eta}-\Delta_{p}\right)\right) d \Gamma\left(\Delta_{\eta}\right)>0
$$

Not all these consumers are lost as some of them now go to firm 2 and return to firm 1 . These are

$$
\int_{-\infty}^{\Delta_{p}} \int_{\hat{x}+\Delta_{\eta}-\Delta_{p}}^{\hat{x}+\Delta_{\eta}-\Delta_{p}+s} F\left(\varepsilon-\Delta_{\eta}+\Delta_{p}-s\right) d F(\varepsilon) d \Gamma\left(\Delta_{\eta}\right) .
$$

${ }^{14}$ Demand decrease from this source amounts to

$$
\int_{-\infty}^{\Delta_{p}} \int_{-\infty}^{\hat{x}+\Delta_{\eta}-\Delta_{p}}\left(F\left(\varepsilon-\Delta_{\eta}+\Delta_{p}\right)-F\left(\varepsilon-\Delta_{\eta}+\Delta_{p}-s\right)\right) d \Gamma\left(\Delta_{\eta}\right)>0
$$



Figure 1: Demand and payoff comparison disclose/conceal match-value information
to buy from firm 1 after visiting firm $2{ }^{15}$ All this amounts to a demand decrease.
It is very difficult to evaluate the net effect on demand analytically. We therefore resort to a numerical analysis. Figure 1a gives the two demand functions when matchvalues follow a standard normal distribution and search costs equal $s=2.5$. Providing match-value information then yields a rightwards rotation of the demand curve, increasing demand for high, but decreasing it for low prices. The Figure also gives the equilibrium price $p_{A}^{*}$ and the best deviation price when revealing $\varepsilon$, which is $p_{1}^{A A}$. The demand rotation clearly gives the deviant an incentive to raise its price. Whether this deviation is profitable depends on search costs. Figure 1b shows that the deviation is only profitable for high search costs.

The intuition is as follows. First, consider the case of small search costs. Suppose that firm 1 reveals information about $\varepsilon$, while firm 2 conceals it. Ceteris paribus, more consumers will then visit firm 2 first. When both conceal, they will share the first-visits equally. But when firm 1 reveals, even consumers that have a $\varepsilon$ slightly above the median are willing to first check out firm 2 ; if it turns out that their $\varepsilon_{2}$ is very low, they still have the option to buy from firm 1. Hence, with low $s$ they are willing to take the gamble to

[^10]see what firm 2 has on offer. This implies that by revealing information, demand for firm 1 decreases. However, the pool of consumers that do end up at firm 1 will on average have a higher valuation for its product, so it can charge a higher price. Still, the lower-demand effect dominates. That is no longer true for high search costs. The number of consumers that go to both firms will then be very low, which implies that the lower-demand effect is only moderate; there are fewer consumers that are willing to check out firm 2, even if they have a favorable $\varepsilon_{1}$. At the same time, the higher-valuation effect is now stronger; consumers that do end up buying from firm 1 are those with a high valuation for its product. Hence, the latter effect now dominates.

Note however that the search costs for which concealing is no longer an equilibrium, are unreasonably high. From the Figure, this is only true when search costs are at least the same order of magnitude as the equilibrium price. Hence, only in markets where checking out one firm is about as costly as buying the product, it is no longer an equilibrium for both firms to conceal information about $\varepsilon$.

Revealing information about $\varepsilon$ Now start from the situation in which both firms reveal their $\varepsilon$. This implies that consumers have full information concerning prices and match values of both firms. Essentially, this brings us in the Perloff and Salop (1985) model. The only difference is that consumers have to incur costs $s$ when buying their preferred product. If one firm now decides to conceal (i.e. not advertise) its $\varepsilon$, this has effects that are essentially the mirror image of those described above. We do not discuss these in detail for the sake of brevity.

Figure 2 gives an analysis that is very similar to that in Figure 1, using the same parameters. The left-hand panel shows that hiding one's $\varepsilon$ now implies a leftward rotation of the demand curve, decreasing demand for high, but increasing it for low prices ( $D_{1}^{P S}$ is demand for firm 1 when it reveals $\varepsilon$ (PS for Perloff-Salop), $D_{1}^{H}$ is its demand when it conceals). As a result, the best deviation price when concealing $\left(p_{1}^{H}\right)$ is now lower than the equilibrium price with advertising $\left(p^{P S}\right)$. From the right-hand panel, it is always profitable to defect by hiding $\varepsilon$. Hence, it is never an equilibrium for both firms to advertise their $\varepsilon$.

The intuition for this result is very similar to that given above. Like there, firms have


Figure 2: Demand and payoff comparison disclose/conceal match-value information
an advantage if they conceal information while the other firms reveals it. Hence, starting out from a situation where they both reveal information, they now have an incentive to defect by concealing it.

Summing up We thus found the following:

Result 1. Assume that prices are advertised. Then, if match values are normally distributed, it is an equilibrium for both firms to conceal information concerning the match value $\varepsilon$, provided that search costs are sufficiently low. It is never an equilibrium for both firms to reveal information concerning $\varepsilon$.

Note that beliefs are not an issue in this analysis, as prices are always observable.

## 8 Conclusion

In this paper we presented a consumer search model where firms sell products with various characteristics, some observable, others unobservable before search. As consumers are more inclined to visit a firm where they like the observable characteristics, search is directed. In our model firms can also influence the direction of search. One way to do so is by adjusting prices since consumers prefer to visit firms whose prices are lower. Another way is by providing match-value information.

We first showed that price advertising leads to lower prices and profits. With price advertising, a lower price not only retains more consumers, but is also more likely to
attract them. Also, with price advertising equilibrium prices and profits decrease in search costs. With higher search costs consumers are less likely to walk away, hence firms are more eager to attract them in the first place. Unless price advertising is prohibitively costly, price advertising will occur in equilibrium.

Secondly, we showed that firm incentives to disclose match-value information depends critically on whether prices are observable to consumers or not. With unobservable prices no firm has an incentive to reveal match values to consumers because of the typical holdup problem that arises when consumers have to incur visiting costs to buy products. With observable prices firms also have no incentive to reveal match value information, provided that search costs are not unreasonably high.

Altogether our results suggest a clear picture: when frictions are sizable, models where prices are announced to consumers while match values are not, seem the most sensible.

## Appendix

Proof of Lemma 1. For $\log$-concavity, define $\phi\left(\eta, \Delta_{\eta}\right) \equiv g\left(\eta+\Delta_{\eta}\right)$. As $g(\eta)$ is $\log$ concave in $\eta$, we immediately have that $\phi\left(\eta, \Delta_{\eta}\right)$ is log-concave in $\eta$ and $\Delta_{\eta}$, hence $\phi\left(\eta, \Delta_{\eta}\right) g(\eta)$ is $\log$-concave in $\eta$ and $\Delta_{\eta}$. With $\gamma\left(\Delta_{\eta}\right)=\int \phi\left(\eta, \Delta_{\eta}\right) g(\eta) d \eta$, the PrékopaLeindler inequality immediately implies that $\gamma\left(\Delta_{\eta}\right)$ is logconcave. For symmetry, note that

$$
\begin{aligned}
\gamma\left(-\Delta_{\eta}\right)=\int_{-\infty}^{\infty} g\left(\eta-\Delta_{\eta}\right) d G(\eta) & =\int_{-\infty}^{\infty} g(\eta) d G\left(\eta-\Delta_{\eta}\right) \\
& =\int_{-\infty}^{\infty} g\left(\eta+\Delta_{\eta}\right) d G(\eta)=\gamma\left(\Delta_{\eta}\right)
\end{aligned}
$$

As $\Gamma(0)=1 / 2$, symmetry implies $E\left(\Delta_{\eta}\right)=0$.

Proof of Proposition 2. We first show how to obtain the equilibrium price in (7). After taking the derivative of the payoff (6) with respect to $p_{1}$ and setting $p_{1}=p_{A}^{*}$, we
obtain the following equation

$$
\begin{aligned}
\int_{-\infty}^{0}\left(1-F\left(\hat{x}+\Delta_{\eta}\right)-p_{A}^{*} f\left(\hat{x}+\Delta_{\eta}\right)(1-F(\hat{x}))\right) d \Gamma\left(\Delta_{\eta}\right) & + \\
\int_{-\infty}^{0} \int_{-\infty}^{\hat{x}+\Delta_{\eta}}\left(F\left(\varepsilon-\Delta_{\eta}\right)-p_{A}^{*} f\left(\varepsilon-\Delta_{\eta}\right)\right) d F(\varepsilon) d \Gamma\left(\Delta_{\eta}\right) & + \\
\int_{0}^{\infty}(1-F(\hat{x}))\left(F\left(\hat{x}-\Delta_{\eta}\right)-p_{A}^{*} f\left(\hat{x}-\Delta_{\eta}\right)\right) d \Gamma\left(\Delta_{\eta}\right) & + \\
\int_{0}^{\infty} \int_{-\infty}^{\hat{x}}\left(F\left(\varepsilon-\Delta_{\eta}\right)-p_{A}^{*} f\left(\varepsilon-\Delta_{\eta}\right)\right) d F(\varepsilon) d \Gamma\left(\Delta_{\eta}\right)-\gamma(0) p_{A}^{*}(1-F(\hat{x}))^{2}= & 0(19)
\end{aligned}
$$

Integration by parts yields

$$
\begin{aligned}
\int_{-\infty}^{\hat{x}} F\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon) & =F\left(\hat{x}-\Delta_{\eta}\right) F(\hat{x})-\int_{-\infty}^{\hat{x}} F(\varepsilon) d F\left(\varepsilon-\Delta_{\eta}\right) \\
& =F\left(\hat{x}-\Delta_{\eta}\right) F(\hat{x})-\int_{-\infty}^{\hat{x}-\Delta_{\eta}} F\left(\varepsilon+\Delta_{\eta}\right) d F(\varepsilon) .
\end{aligned}
$$

Moreover

$$
\int_{-\infty}^{\hat{x}} p_{A}^{*} f\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)=\int_{-\infty}^{\hat{x}} p_{A}^{*} f(\varepsilon) d F\left(\varepsilon-\Delta_{\eta}\right)=\int_{-\infty}^{\hat{x}-\Delta_{\eta}} p_{A}^{*} f\left(\varepsilon+\Delta_{\eta}\right) d F(\varepsilon)
$$

Secondly, because of the symmetry of $\gamma\left(\Delta_{\eta}\right)$,

$$
\int_{-\infty .}^{0} f\left(\hat{x}+\Delta_{\eta}\right) d \Gamma\left(\Delta_{\eta}\right)=\int_{0}^{\infty} f\left(\hat{x}-\Delta_{\eta}\right) d \Gamma\left(\Delta_{\eta}\right)
$$

and

$$
\begin{aligned}
& \int_{-\infty}^{0} \int_{-\infty}^{\hat{x}+\Delta_{\eta}}\left(F\left(\varepsilon-\Delta_{\eta}\right)-p_{A}^{*} f\left(\varepsilon-\Delta_{\eta}\right)\right) d F(\varepsilon) d \Gamma\left(\Delta_{\eta}\right)= \\
& \int_{0}^{\infty} \int_{-\infty}^{\hat{x}-\Delta_{\eta}}\left(F\left(\varepsilon+\Delta_{\eta}\right)-p_{A}^{*} f\left(\varepsilon+\Delta_{\eta}\right)\right) d F(\varepsilon) d \Gamma\left(\Delta_{\eta}\right)
\end{aligned}
$$

As a result, equation (19) can be simplified to

$$
\begin{array}{r}
\int_{-\infty}^{0}\left(1-2 p_{A}^{*} f\left(\hat{x}+\Delta_{\eta}\right)(1-F(\hat{x}))-\int_{-\infty}^{\hat{x}+\Delta_{\eta}}\left(2 p_{A}^{*} f\left(\varepsilon-\Delta_{\eta}\right)\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right)- \\
\gamma(0) p_{A}^{*}(1-F(\hat{x}))^{2}=0 \tag{20}
\end{array}
$$

Solving for $p_{A}^{*}$ gives the expression in (7).

We now show that the payoff function (6) is locally concave around the equilibrium price $p_{A}^{*}$. In fact, note that the demand of firm 1 is

$$
\begin{aligned}
D_{1}\left(p_{1}, p_{A}^{*}\right) & =\int_{-\infty}^{p_{A}^{*}-p_{1}}\left(1-F\left(\hat{x}+\Delta_{\eta}-p_{A}^{*}+p_{1}\right)+\int_{-\infty}^{\hat{x}+\Delta_{\eta}-p_{A}^{*}+p_{1}} F\left(\varepsilon-\Delta_{\eta}-p_{1}+p_{A}^{*}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right) \\
& +\int_{p_{A}^{*}-p_{1}}^{\infty}\left(F\left(\hat{x}-\Delta_{\eta}-p_{1}+p_{A}^{*}\right)(1-F(\hat{x}))+\int_{-\infty}^{\hat{x}} F\left(\varepsilon-\Delta_{\eta}-p_{1}+p_{A}^{*}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right)
\end{aligned}
$$

It is readily seen that the second derivative of $D_{1}\left(p_{1}, p_{A}^{*}\right)$ with respect to $p_{1}$ is

$$
\begin{aligned}
\frac{\partial^{2} D_{1}}{\partial p_{1}^{2}} & =\int_{-\infty}^{p_{A}^{*}-p_{1}}\left(-f^{\prime}\left(\hat{x}+\Delta_{\eta}-p_{A}^{*}+p_{1}\right)(1-F(\hat{x}))+\int_{-\infty}^{\hat{x}+\Delta_{\eta}-p_{A}^{*}+p_{1}} f^{\prime}\left(\varepsilon-\Delta_{\eta}-p_{1}+p_{A}^{*}\right) d F(\varepsilon)\right. \\
& \left.-f(\hat{x}) f\left(\hat{x}+\Delta_{\eta}-p_{A}^{*}+p_{1}\right)\right) d \Gamma\left(\Delta_{\eta}\right) \\
& +\int_{0}^{\infty}\left(f^{\prime}\left(\hat{x}-\Delta_{\eta}-p_{1}+p_{A}^{*}\right)(1-F(\hat{x}))+\int_{-\infty}^{\hat{x}} f^{\prime}\left(\varepsilon-\Delta_{\eta}-p_{1}+p_{A}^{*}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right) \\
& +\gamma^{\prime}\left(p_{A}^{*}-p_{1}\right)(1-F(\hat{x}))^{2}
\end{aligned}
$$

Setting $p_{1}=p_{A}^{*}$ and simplifying gives

$$
\begin{align*}
\left.\frac{\partial^{2} D_{1}}{\partial p_{1}^{2}}\right|_{p_{1}=p_{A}^{*}} & =\int_{-\infty}^{0}\left(-f^{\prime}\left(\hat{x}+\Delta_{\eta}\right)(1-F(\hat{x}))+\int_{-\infty}^{\hat{x}+\Delta_{\eta}} f^{\prime}\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right. \\
& \left.-f(\hat{x}) f\left(\hat{x}+\Delta_{\eta}\right)\right) d \Gamma\left(\Delta_{\eta}\right) \\
& +\int_{0}^{\infty}\left(f^{\prime}\left(\hat{x}-\Delta_{\eta}\right)(1-F(\hat{x}))+\int_{-\infty}^{\hat{x}} f^{\prime}\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right) \tag{21}
\end{align*}
$$

Integrating by parts, we establish that

$$
\begin{aligned}
\int_{-\infty}^{\hat{x}} f^{\prime}\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon) & =\int_{-\infty}^{\hat{x}} f(\varepsilon) d f\left(\varepsilon-\Delta_{\eta}\right) \\
& =f(\hat{x}) f\left(\hat{x}-\Delta_{\eta}\right)-f^{2}(-\infty)-\int_{-\infty}^{\hat{x}} f\left(\varepsilon-\Delta_{\eta}\right) d f(\varepsilon) \\
& =f(\hat{x}) f\left(\hat{x}-\Delta_{\eta}\right)-f^{2}(-\infty)-\int_{-\infty}^{\hat{x}-\Delta_{\eta}} f^{\prime}\left(\varepsilon+\Delta_{\eta}\right) d F(\varepsilon)
\end{aligned}
$$

Then, because of the symmetry of $\gamma\left(\Delta_{\eta}\right)$, we simplify (21) to

$$
\int_{-\infty}^{0}\left(-f^{2}(-\infty)\right) d \Gamma\left(\Delta_{\eta}\right)<0
$$

Since the second derivative of demand is negative in a neighborhood of $p_{A}^{*}$, we conclude


Figure 3: Quasi-concavity of payoff function for Gumbel and Normal distributions (advertised prices)
that the demand function is concave in $p_{1}$ at the equilibrium point which implies that the payoff is locally concave.

We will not attempt here to provide general conditions for existence of equilibrium but instead we will check numerically that the equilibrium exists for some common distributions of match values provided there is sufficient variation in the observable characteristic $\eta$. It is well-known that proving existence of equilibrium in oligopoly models with consumer search like ours is difficult because a firm's demand is made of the sum of various probabilities and even if each of these demands are well-behaved it is not guaranteed that their sum will be. Anderson and Renault (1999) provide a useful discussion in their Appendix. This problem is more severe in settings where prices are observable, like ours. As argued, a pure-strategy SNE then fails to exist in Anderson and Renault (1999). Our fix for this problem is to introduce additional heterogeneity in the model, namely the observable characteristic $\eta$. Obviously, we need sufficient heterogeneity for otherwise we would have the same problem of non-existence of equilibrium.

In Figure 3 we have plotted the payoff function (6) when the distributions of match values are normal (Figure 3a) and Gumbel (Figure 3b). In this Figure we have chosen the variance of $\eta$ 's sufficiently high and clearly the payoff is quasi-concave and therefore the equilibrium price (7), indicated by the dashed vertical line, is indeed an equilibrium.

To illustrate the non-existence of an SNE in pure strategies, Figure 4 plots cases in

(a) $\varepsilon$ : standard normal distrib.; $\eta$ normal distrib. with $\mu_{\eta}=0$ and $\sigma_{\eta}=0.05 ; \hat{x}=5$.

(b) $\varepsilon$ : Gumbel with location 0 and scale $1 ; \eta$ : Gumbel with location 0 and scale $0.05 ; \hat{x}=5$.

Figure 4: Non-quasi-concavity of payoff function for Gumbel and Normal distributions (advertised prices)
which there is little ex-ante heterogeneity across products. As we can see, the tentative equilibrium price indicated by the dashed vertical line is lower and then an individual firm has an incentive to deviate to a higher price, despite selling to fewer consumers.

We finally prove that the equilibrium price $p_{A}^{*}$ decreases as search costs go up. Taking the derivative of the denominator of (7) with respect to $\hat{x}$ gives:

$$
\begin{aligned}
& \frac{\partial}{\partial \hat{x}}\left(2 \int_{-\infty}^{0}\left[f\left(\hat{x}+\Delta_{\eta}\right)(1-F(\hat{x}))+\int_{-\infty}^{\hat{x}+\Delta_{\eta}} f\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right] d \Gamma\left(\Delta_{\eta}\right)+\gamma(0)(1-F(\hat{x}))^{2}\right)= \\
& 2(1-F(\hat{x}))\left[\int_{-\infty}^{0} f^{\prime}\left(\hat{x}+\Delta_{\eta}\right) d \Gamma\left(\Delta_{\eta}\right)-\gamma(0) f(\hat{x})\right]= \\
& 2(1-F(\hat{x}))\left[-f(-\infty) \gamma(-\infty)-\int_{-\infty}^{0} f\left(\hat{x}+\Delta_{\eta}\right) \gamma^{\prime}\left(\Delta_{\eta}\right) d \Delta_{\eta}\right]<0 .
\end{aligned}
$$

The last inequality is true if $\partial \gamma\left(\Delta_{\eta}\right) / \partial \Delta_{\eta}>0$ for $\Delta_{\eta} \in[-\bar{\eta}, 0]$. Because $\gamma\left(\Delta_{\eta}\right)$ is $\log$-concave and it is symmetric with respect to 0 , it has to increase with $\Delta_{\eta}$ if $\Delta_{\eta}<0$ and decrease with $\Delta_{\eta}$ if $\Delta_{\eta}>0$. Since the denominator of (7) decreases in $\hat{x}$, the price then increases in $\hat{x}$ and decreases in search cost $s$. This completes the proof.

The example with uniform distributions We finally observe that for uniform distributions, the equilibrium payoff is also nicely quasi-concave. Because the supports of $\varepsilon$ and $\eta$ are closed intervals, the payoff function of firm $i$ depends on the magnitude of its deviation price. For instance, if $p>\bar{\varepsilon}+\hat{x}-\Delta_{\eta}+p^{*}$ and $\Delta_{\eta} \leq \hat{x}+p^{*}$, then the


Figure 5: Quasi-concavity of payoff function with a uniform distribution (advertised prices). Parameters are $\bar{\varepsilon}=4, \bar{\eta}=3.9, \hat{x}=3.7, \alpha=0$.
probability that a consumer starts searching from firm 2 and arrives at firm 1 equals zero. As a result, we have identified eight intervals in which the deviation price $p$ could be, and we have obtained a differently looking payoff function for every interval $\left[{ }^{16}\right.$ The bounds of the intervals are depicted by dashed lines in Figure 5 .

Proof of Proposition 3 After taking the first derivative of the payoff (13) and applying symmetry, the FOC for firm 1 simplifies to

$$
\begin{array}{r}
\int_{-\infty}^{0}\left(1-F\left(\hat{x}+\Delta_{\eta}\right)+\int_{-\infty}^{\hat{x}+\Delta_{\eta}} F\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right) \\
p_{N}^{*} \int_{-\infty}^{0}\left(f\left(\hat{x}+\Delta_{\eta}\right)(1-F(\hat{x}))+\int_{-\infty}^{\hat{x}+\Delta_{\eta}} f\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right) \\
\int_{0}^{\infty}\left(F\left(\hat{x}-\Delta_{\eta}\right)(1-F(\hat{x}))+\int_{-\infty}^{\hat{x}} F\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right) \\
p_{N}^{*} \int_{0}^{\infty}\left(\int_{-\infty}^{\hat{x}} f\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right)=0 \tag{22}
\end{array}
$$

Because of the symmetry of $\gamma\left(\Delta_{\eta}\right)$ we can state that

$$
\begin{aligned}
p_{N}^{*} \int_{0}^{\infty}\left(\int_{-\infty}^{\hat{x}} f\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right) & =p_{N}^{*} \int_{0}^{\infty}\left(\int_{-\infty}^{\hat{x}-\Delta_{\eta}} f\left(\varepsilon+\Delta_{\eta}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right) \\
& =p_{N}^{*} \int_{-\infty}^{0}\left(\int_{-\infty}^{\hat{x}+\Delta_{\eta}} f\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right) .
\end{aligned}
$$

[^11]Note also that integration by parts gives:

$$
\begin{aligned}
F\left(\hat{x}-\Delta_{\eta}\right)(1-F(\hat{x}))+\int_{-\infty}^{\hat{x}} F\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon) & =F\left(\hat{x}-\Delta_{\eta}\right)-\int_{-\infty}^{\hat{x}} F(\varepsilon) d F\left(\varepsilon-\Delta_{\eta}\right) \\
& =F\left(\hat{x}-\Delta_{\eta}\right)-\int_{-\infty}^{\hat{x}-\Delta_{\eta}} F\left(\varepsilon+\Delta_{\eta}\right) d F(\varepsilon) .
\end{aligned}
$$

Then, because of the symmetry of $\gamma\left(\Delta_{\eta}\right)$

$$
\begin{aligned}
& \int_{0}^{\infty}\left(F\left(\hat{x}-\Delta_{\eta}\right)-\int_{-\infty}^{\hat{x}-\Delta_{\eta}} F\left(\varepsilon+\Delta_{\eta}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right)= \\
& \quad \int_{-\infty}^{0}\left(F\left(\hat{x}+\Delta_{\eta}\right)-\int_{-\infty}^{\hat{x}+\Delta_{\eta}} F\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right)
\end{aligned}
$$

Using these remarks, the FOC (22) simplifies to

$$
\frac{1}{2}-p_{N}^{*} \int_{-\infty}^{0}\left(f\left(\hat{x}+\Delta_{\eta}\right)(1-F(\hat{x}))+2 \int_{-\infty}^{\hat{x}+\Delta_{\eta}} f\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right)=0 .
$$

Solving for $p_{N}^{*}$ gives the expression (14) in the proposition.

We now show that the payoff function is locally concave in a neighborhood of the equilibrium price. The demand of firm 1 is

$$
\begin{aligned}
D_{1}\left(p_{1}, p_{N}^{*}\right) & =\int_{-\infty}^{0}\left(1-F\left(\hat{x}+\Delta_{\eta}-p_{N}^{*}+p_{1}\right)+\int_{-\infty}^{\hat{x}+\Delta_{\eta}-p_{N}^{*}+p_{1}} F\left(\varepsilon-\Delta_{\eta}-p_{1}+p_{N}^{*}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right) \\
& +\int_{0}^{\infty}\left(F\left(\hat{x}-\Delta_{\eta}\right)\left(1-F\left(\hat{x}-p_{N}^{*}+p_{1}\right)\right)+\int_{-\infty}^{\hat{x}-p_{N}^{*}+p_{1}} F\left(\varepsilon-\Delta_{\eta}-p_{1}+p_{N}^{*}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right)
\end{aligned}
$$

The second derivative of $D_{1}\left(p_{1}, p_{N}^{*}\right)$ with respect to $p_{1}$ is as follows

$$
\begin{aligned}
\frac{\partial^{2} D_{1}}{\partial p_{1}^{2}} & =\int_{-\infty}^{0}\left(-f^{\prime}\left(\hat{x}+\Delta_{\eta}-p_{N}^{*}+p_{1}\right)(1-F(\hat{x}))+\int_{-\infty}^{\hat{x}+\Delta_{\eta}-p_{N}^{*}+p_{1}} f^{\prime}\left(\varepsilon-\Delta_{\eta}-p_{1}+p_{N}^{*}\right) d F(\varepsilon)\right. \\
& \left.-f(\hat{x}) f\left(\hat{x}+\Delta_{\eta}-p_{N}^{*}+p_{1}\right)\right) d \Gamma\left(\Delta_{\eta}\right) \\
& -\int_{0}^{\infty}\left(f\left(\hat{x}-\Delta_{\eta}\right) f\left(\hat{x}-p_{N}^{*}+p_{1}\right)-\int_{-\infty}^{\hat{x}-p_{N}^{*}+p_{1}} f^{\prime}\left(\varepsilon-\Delta_{\eta}-p_{1}+p_{N}^{*}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right)
\end{aligned}
$$

Setting $p_{1}=p_{N}^{*}$ we obtain

$$
\begin{align*}
\left.\frac{\partial^{2} D_{1}}{\partial p_{1}^{2}}\right|_{p_{1}=p_{N}^{*}} & =\int_{-\infty}^{0}\left(-f^{\prime}\left(\hat{x}+\Delta_{\eta}\right)(1-F(\hat{x}))+\int_{-\infty}^{\hat{x}+\Delta_{\eta}} f^{\prime}\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right. \\
& \left.-f(\hat{x}) f\left(\hat{x}+\Delta_{\eta}\right)\right) d \Gamma\left(\Delta_{\eta}\right) \\
& -\int_{0}^{\infty}\left(f\left(\hat{x}-\Delta_{\eta}\right) f(\hat{x})-\int_{-\infty}^{\hat{x}} f^{\prime}\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right) \tag{23}
\end{align*}
$$

By applying integration by parts we obtain

$$
\begin{gathered}
f\left(\hat{x}-\Delta_{\eta}\right) f(\hat{x})-\int_{-\infty}^{\hat{x}} f^{\prime}\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)=f\left(\hat{x}-\Delta_{\eta}\right) f(\hat{x})-\int_{-\infty}^{\hat{x}} f(\varepsilon) d f\left(\varepsilon-\Delta_{\eta}\right)= \\
f^{2}(\infty)+\int_{-\infty}^{\hat{x}} f\left(\varepsilon-\Delta_{\eta}\right) d f(\varepsilon)=f^{2}(\infty)+\int_{-\infty}^{\hat{x}} f^{\prime}(\varepsilon) d F\left(\varepsilon-\Delta_{\eta}\right)= \\
f^{2}(\infty)+\int_{-\infty}^{\hat{x}-\Delta_{\eta}} f^{\prime}\left(\varepsilon+\Delta_{\eta}\right) d F(\varepsilon)
\end{gathered}
$$

Therefore, because of the symmetry of $\gamma\left(\Delta_{\eta}\right)$, (23) simplifies to

$$
\begin{aligned}
\left.\frac{\partial^{2} D_{1}}{\partial p_{1}^{2}}\right|_{p_{1}=p_{N}^{*}} & =-\int_{-\infty}^{0}\left(f^{\prime}\left(\hat{x}+\Delta_{\eta}\right)(1-F(\hat{x}))+f(\hat{x}) f\left(\hat{x}+\Delta_{\eta}\right)\right) d \Gamma\left(\Delta_{\eta}\right) \\
& -\frac{1}{2} f^{2}(\infty)
\end{aligned}
$$

The expression under the integral is positive because

$$
\begin{aligned}
& f^{\prime}\left(\hat{x}+\Delta_{\eta}\right)(1-F(\hat{x}))+f(\hat{x}) f\left(\hat{x}+\Delta_{\eta}\right)= \\
&(1-F(\hat{x})) f\left(\hat{x}+\Delta_{\eta}\right)\left(\frac{f^{\prime}\left(\hat{x}+\Delta_{\eta}\right)}{f\left(\hat{x}+\Delta_{\eta}\right)}+\frac{f(\hat{x})}{1-F(\hat{x})}\right)> \\
&(1-F(\hat{x})) f\left(\hat{x}+\Delta_{\eta}\right)\left(\frac{f^{\prime}\left(\hat{x}+\Delta_{\eta}\right)}{f\left(\hat{x}+\Delta_{\eta}\right)}+\frac{f\left(\hat{x}+\Delta_{\eta}\right)}{1-F\left(\hat{x}+\Delta_{\eta}\right)}\right)= \\
&(1-F(\hat{x}))\left(\frac{f^{\prime}\left(\hat{x}+\Delta_{\eta}\right)\left(1-F\left(\hat{x}+\Delta_{\eta}\right)\right)+f^{2}\left(\hat{x}+\Delta_{\eta}\right)}{1-F\left(\hat{x}+\Delta_{\eta}\right)}\right)>0
\end{aligned}
$$

Therefore, we conclude that the demand at the point $p_{1}=p_{2}=p=p_{N}^{*}$ is concave, which implies that the second order derivative at this point is negative and $\pi_{1}$ attains a maximum.

For the existence of equilibrium we proceed as before. In Figure 2 we plot the payoff (13) for the cases where the distributions of match values are Normal and Gumbel. Notice


Figure 6: Quasi-concavity of payoff function for different distributions (non-advertised prices).
that in this case of unobservable prices we do not need that the variation in the observable match values $\eta$ 's is large.

We finally prove that the equilibrium price is increasing in $s$. For this, again, we take the derivative of the denominator of $p^{*}$ with respect to $\hat{x}$. This gives:

$$
\begin{aligned}
\frac{\partial}{\partial \hat{x}}\left(\int_{-\infty}^{0}\left(f\left(\hat{x}+\Delta_{\eta}\right)(1-F(\hat{x}))+2 \int_{-\infty}^{\hat{x}+\Delta_{\eta}} f\left(\varepsilon-\Delta_{\eta}\right) d F(\varepsilon)\right) d \Gamma\left(\Delta_{\eta}\right)\right) & = \\
\int_{-\infty}^{0}\left(f^{\prime}\left(\hat{x}+\Delta_{\eta}\right)(1-F(\hat{x}))+f(\hat{x}) f\left(\hat{x}+\Delta_{\eta}\right)\right) d \Gamma\left(\Delta_{\eta}\right) & = \\
\int_{-\infty}^{0} f\left(\hat{x}+\Delta_{\eta}\right)(1-F(\hat{x}))\left(\frac{f^{\prime}\left(\hat{x}+\Delta_{\eta}\right)}{f\left(\hat{x}+\Delta_{\eta}\right)}+\frac{f(\hat{x})}{1-F(\hat{x})}\right) d \Gamma\left(\Delta_{\eta}\right) & =
\end{aligned}
$$

Because $f(\varepsilon)$ is log-concave and $\Delta_{\eta} \leq 0$,

$$
\frac{f(\hat{x})}{1-F(\hat{x})}>\frac{f\left(\hat{x}+\Delta_{\eta}\right)}{1-F\left(\hat{x}+\Delta_{\eta}\right)}
$$

Then

$$
\frac{f^{\prime}\left(\hat{x}+\Delta_{\eta}\right)}{f\left(\hat{x}+\Delta_{\eta}\right)}+\frac{f(\hat{x})}{1-F(\hat{x})}>\frac{\left(1-F\left(\hat{x}+\Delta_{\eta}\right)\right) f^{\prime}\left(\hat{x}+\Delta_{\eta}\right)+f^{2}\left(\hat{x}+\Delta_{\eta}\right)}{f\left(\hat{x}+\Delta_{\eta}\right)\left(1-F\left(\hat{x}+\Delta_{\eta}\right)\right)}>0 .
$$

We then conclude that the denominator of the price increases in $\hat{x}$ so the price decreases in $\hat{x}$ and increases in $s$.


Figure 7: Uniform distribution (non-advertised prices). Parameters are $\bar{\varepsilon}=4, \bar{\eta}=3.9$, $\hat{x}=3.7, \alpha=0$.

The example with uniform distributions We finally show that the payoff function (13) is quasi-concave when the match values are uniformly distributed. Again, because the match values are distributed in closed intervals, the magnitude of the deviation price affects the expression of a payoff function. We have identified seven intervals in which $p$ could be. In these intervals, the expresions of the payoff are different. The bounds of the intervals are depicted by dashed lines in Figure 7 The exact expressions of the payoff are available upon request.

Proof of Proposition 4 The only thing left to show is that indeed $\partial \pi_{1}^{A}\left(p_{N}^{*}, p_{N}^{*} ; \hat{x}\right) / \partial p_{1}<$ 0 . Taking the derivative of (6) with respect to $p_{1}$ gives:

$$
\begin{aligned}
& \int_{-\infty}^{\Delta_{p}}\left[1-F\left(\hat{x}+\Delta_{\eta}-\Delta_{p}\right)+\int_{-\infty}^{\hat{x}+\Delta_{\eta}-\Delta_{p}} F\left(\varepsilon-\Delta_{\eta}+\Delta_{p}\right) d F(\varepsilon)-\right. \\
& \left.p_{N}^{*} f\left(\hat{x}+\Delta_{\eta}-\Delta_{p}\right)(1-F(\hat{x}))-p_{N}^{*} \int_{-\infty}^{\hat{x}+\Delta_{\eta}-\Delta_{p}} f\left(\varepsilon-\Delta_{\eta}-\Delta_{p}\right) d F(\varepsilon)\right] d \Gamma\left(\Delta_{\eta}\right)+ \\
& \int_{\Delta_{p}}^{\infty}\left[F\left(\hat{x}-\Delta_{\eta}+\Delta_{p}\right)(1-F(\hat{x}))+\int_{-\infty}^{\hat{x}} F\left(\varepsilon-\Delta_{\eta}+\Delta_{p}\right) d F(\varepsilon)-\right. \\
& p_{N}^{*} f\left(\hat{x}-\Delta_{\eta}+\Delta_{p}\right)(1-F(\hat{x}))- \\
& \left.p_{N}^{*} \int_{-\infty}^{\hat{x}} f\left(\varepsilon-\Delta_{\eta}+\Delta_{p}\right) d F(\varepsilon)\right] d \Gamma\left(\Delta_{\eta}\right)-p_{N}^{*}[1-F(\hat{x})]^{2} \gamma \Delta_{p}
\end{aligned}
$$

Because $p_{N}^{*}$ satisfies the equilibrium condition (22), if we evaluate this derivative at $p_{1}=$ $p_{N}^{*}$, we obtain

$$
-p_{N}^{*}(1-F(\hat{x}))\left[(1-F(\hat{x})) \gamma(0)+\int_{0}^{\infty} f\left(\hat{x}-\Delta_{\eta}\right) d \Gamma\left(\Delta_{\eta}\right)\right]<0
$$

Proof of Proposition 6 What is left to show is that in the proposed equilibrium firms indeed have no incentive to reveal their $\varepsilon$ as doing so would cause the market for its product to break down. Suppose firm 1 deviates by revealing information about $\varepsilon_{1}$ and setting a price $p_{1}$. Denote the price consumers expect to see at firm 1 by $p_{1}^{e}$. If the deviation is profitable, we need to have $p_{1}=p_{1}^{e}$.

Consider a consumer who learns $\varepsilon_{1}$ and expects price $p_{1}^{e}$ at firm 1 . Visiting 1 gives her net utility $\varepsilon_{1}+\eta_{1}-p_{1}^{e}-s$. Her reservation value at firm 2 is $\hat{x}+\eta_{2}-p_{N}^{*}$. Hence she will visit firm 1 first if

$$
\begin{equation*}
\varepsilon_{1} \geq \hat{x}+\Delta_{\eta}-p_{N}^{*}+p_{1}^{e}+s \tag{24}
\end{equation*}
$$

and once there, buy if

$$
\begin{equation*}
p_{1} \leq \varepsilon_{1}-\hat{x}-\Delta_{\eta}+p_{N}^{*} . \tag{25}
\end{equation*}
$$

as in that case the cost of visiting firm 1 are already sunk. If (24) does not hold, she first visits firm 2. In that case, she will decide to visit firm 1 anyhow if

$$
\begin{equation*}
\varepsilon_{1}-p_{1}^{e}-s \geq \Delta_{\eta}+\varepsilon_{2}-p_{N}^{*} \tag{26}
\end{equation*}
$$

and actually buy there if it turns out that

$$
\begin{equation*}
p_{1} \leq \varepsilon_{1}-\Delta_{\eta}-\varepsilon_{2}+p_{N}^{*} . \tag{27}
\end{equation*}
$$

as in that case the cost of visiting firm 1 are already sunk.
Given consumer beliefs $p_{1}^{e}$, consider a deviation by firm 1 to $p_{1}=p_{1}^{e}+s$. Such a deviation will not affect the number of consumers that first visit firm that is given by (24). But with $p_{1}=p_{1}^{e}+s$, for those consumers (25) is also satisfied, so demand from them is unaffected. The deviation will also not affect the number of consumers that visits firm 1 after having visited firm 2, given by (26). But with $p_{1}=p_{1}^{e}+s$, for those consumers (25) is still satisfied, so demand from these consumers is unaffected as well. Hence, if consumers would expect a price $p_{1}^{e}$, firm 1 would be tempted to deviate to $p_{1}^{e}+s$, as doing so does not lower sales. Hence, with consistent wary beliefs, no profitable deviation from the equilibrium in the Proposition exists.

## References

Anderson, S. P., and R. Renault (1999): "Pricing, Product Diversity, and Search Costs: a Bertrand-Chamberlin-Diamond model," RAND Journal of Economics, 30, 719-735.
__ (2006): "Advertising Content," American Economic Review, 96(1), 93-113.

- (2009): "Comparative Advertising: Disclosing Horizontal Match Information," RAND Journal Economics, 40(3), 558-581.

Arbatskaya, M. (2007): "Ordered Search," RAND Journal of Economics, 38, 119-127.
Armstrong, M., J. Vickers, and J. Zhou (2009): "Prominence and Consumer Search," RAND Journal of Economics, 40(2), 209-233.

Armstrong, M., and J. Zhou (2011): "Paying for Prominence," Economic Journal, 121(556), F368-F395.

Bakos, J. Y. (1997): "Reducing Buyer Search Costs: Implications for Electronic Marketplaces," Management Science, 43(12), 1676-1692.

Bar-Isaac, H., G. Caruana, and V. Cuñat (2010): "Information Gathering and Marketing," Journal of Economics and Management Strategy, 19, 375-401.

Diamond, P. A. (1971): "A Model of Price Adjustment," Journal of Economic Theory, 3, 156-168.

Harrington, J. E. (2001): "Comment on "Reducing Buyer Search Costs: Implications for Electronic Marketplaces"," Management Science, 47(12), 1727-1732.

Hart, O. D., and J. Tirole (1990): "Vertical integration and market foreclosure," Brookings Papers on Economic Activity, Microeconomics, pp. 205-286.

Johnson, J. P., and D. P. Myatt (2006): "On the Simple Economics of Advertising, Marketing, and Product Design," American Economic Review, 96(3), 756-784.

Lewis, T. R., and D. E. M. Sappington (1994): "Supplying Information to Facilitate Price Discrimination," International Economic Review, 35(2), 309-327.

McAfee, R. P., and M. Schwartz (1994): "Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity," American Economic Review, 84, 210-230.

Perloff, J., and S. Salop (1985): "Equilibrium with Product Differentiation," Review of Economic Studies, 52, 107-120.

Shelegia, S., and D. Garcia (2015): "Consumer Search with Observational Learning," Working Paper 1502, University of Vienna.

Shen, J. (2015): "Ex-Ante Preference in a Consumer Search Market," mimeo, Ohio State University.

Ursu, R. (2014): "The Consumer Never Rings Twice: Firms Compete for Consumer Search Share before Competing for Market Share," mimeo, University of Chicago.

Wang, C. (2013):"Advertising as a Search Deterrent," mimeo, University of Mannheim.

Weitzman, M. L. (1979): "Optimal Search for the Best Alternative," Econometrica, 47(3), 641-654.

Wolinsky, A. (1986): "True Monopolistic Competition as a Result of Imperfect Information," Quarterly Journal of Economics, 101(3), 493-511.

Zhou, J. (2011): "Ordered Search in Differentiated Markets," International Journal of Industrial Organization, 29(2), 253-262.

## List of research reports

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[^0]:    *A previous and preliminary version of this paper was circulated under the title "Consumer Search with Observable and Hidden Characteristics", which we presented at the EARIE 2011 (Stockholm), the IIOC 2012 (George Mason University) and at the VI Conference on the Economics of Advertising and Marketing (Recanati School of Business, Tel Aviv, 2013). The detailed comments of our discussants Justin Johnson, Maarten Janssen and Matthijs Wildenbeest are gratefully acknowledged, as are discussions with Régis Renault. Moraga and Petrikaite gratefully acknowledge financial support from Marie Curie Excellence Grant MEXT-CT-2006-042471.
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[^1]:    ${ }^{1}$ Or go through the hassle of ordering them online and returning them in case they don't fit.

[^2]:    ${ }^{2}$ Our paper is also related to Bakos (1997), who also studies a case in which product and price information can be obtained separately. He claims to show that when product information is readily available but price information is costly to obtain, equilibrium prices decrease in search costs. However, Harrington (2001) shows that the analysis is flawed, and this result is not valid. Our paper analyzes a similar set-up in which prices do decrease in search costs. Interestingly, however, for this we need that

[^3]:    ${ }^{4}$ Alternatively, she can learn these characteristics for all cars at some fixed cost - so the marginal search cost for finding the characteristics of a particular car are zero.
    ${ }^{5}$ This can be seen as follows. Suppose that in that case we had a symmetric equilibrium with $p^{*}>0$. Both firms would then be visited first with equal probability. If firm $i$ would slightly undercut $p^{*}$, however, all consumers would visit firm $i$ first, and it would see a discontinuous increase in its demand. Hence, such a deviation would be profitable. It cannot be an equilibrium to have $p^{*}=0$ either. Both firms would then make zero profits. If firm $i$ deviated to a higher price, all consumers would visit the other firm first, but some would still prefer to buy from $i$, rendering the deviation profitable.

[^4]:    ${ }^{6}$ Note that the first term reflects the case when $\varepsilon_{1}<\hat{x}$ and the second term the case when $\varepsilon_{1} \geq \hat{x}$.

[^5]:    ${ }^{7}$ Anderson and Renault (1999), in a model with only opaque characteristics, show that a sufficient condition for the existence and uniqueness of equilibrium is that the derivative of the density $f$ is always non-decreasing. However, such a condition is incompatible with our full support assumption.

[^6]:    ${ }^{8}$ In the limiting case that the variation in $\eta$ 's goes to zero, we're back in the case where an equilibrium in pure strategies does not exist, see the discussion in footnote 5. That argument still goes through when there is little variation in $\eta$ 's.
    ${ }^{9}$ Note that with infinite support the condition is always satisfied, as she could always observe $\varepsilon_{1}=-\infty$.

[^7]:    ${ }^{10}$ Note that for this analysis to apply, we need that $\hat{x}<\alpha+\bar{\varepsilon}$, which implies $s<\bar{\varepsilon}$.

[^8]:    ${ }^{11}$ Note that the profit-maximizing price of the deviant is clearly different from $p_{A}^{*}$ even with consistent wary beliefs. This is because when taking the FOC of the deviation payoff for a given consumer belief we treat such a belief as a parameter.

[^9]:    ${ }^{12}$ This is the case if $\varepsilon_{2} \geq \hat{x}-\Delta_{\eta}+\Delta_{p}$ (they stop searching at firm 2) and $\varepsilon_{1} \geq \varepsilon_{2}+\Delta_{\eta}-\Delta_{p}$ (they're better off buying at firm 1). As these consumers now also pay $s$, firm 1's demand increase from this source equals:

[^10]:    ${ }^{15}$ Demand decrease from this source amounts to

    $$
    \begin{array}{r}
    \int_{\Delta_{p}}^{\infty} \int_{-\infty}^{\hat{x}}\left(F\left(\varepsilon-\Delta_{\eta}+\Delta_{p}\right)-F\left(\varepsilon-\Delta_{\eta}+\Delta_{p}-s\right)\right) d \Gamma\left(\Delta_{\eta}\right) \\
    +\int_{\Delta_{p}}^{\infty} \int_{\hat{x}}^{\hat{x}+\Delta_{\eta}-\Delta_{p}+s}\left(F\left(\hat{x}-\Delta_{\eta}+\Delta_{p}\right)-F\left(\varepsilon-\Delta_{\eta}+\Delta_{p}-s\right)\right) d \Gamma\left(\Delta_{\eta}\right)>0 . \tag{18}
    \end{array}
    $$

[^11]:    ${ }^{16}$ The exact expressions of the payoff function are available from authors upon request.

