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# Winning Back the Unfaithful while Exploiting the Loyal: Retention Offers and Heterogeneous Switching Costs 

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# Winning back the unfaithful while exploiting the loyal <br> Retention offers and heterogeneous switching costs 

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#### Abstract

We study retention offers, the practice that firms lower prices to consumers that want to cancel their contract. In a two-period Hotelling model, consumers have either low or high switching costs. In the second period, firms try to poach consumers. Consumers with a poaching offer can solicit a retention offer from their original supplier. In equilibrium, only low switching costs go though the effort of obtaining a poaching offer. Hence, retention offers serve as a mechanism to price discriminate against high switching cost consumers. In our model, the possibility of retention offers increases prices and profits. Consumer surplus decreases.


Keywords: Switching costs, retention offers, behavior-based price discrimination, poaching.

JEL classification: D11, D43, L13.

[^0]
## 1 Introduction

In subscription-type markets, e.g. those for credit cards, cable, telecom, and insurance, firms are often willing to offer a better deal to consumers who indicate that they want to cancel their subscription. These offers are known as retention offers, as firms make them in an attempt to retain fickle consumers. Consumers' reactions to these practices differ. Some seem largely unaware of it, or at least unwilling to exploit such offers. Others actively chase them, sharing details of current offers on websites like flyertalk.com.

In this paper, we analyze retention offers. We assume that there are two types of consumers; those with relatively low, and those with relatively high switching costs. Firms can use retention offers to screen consumers with low switching costs. Consumers that have already gone through the trouble of obtaining an offer from a competing firm, signal that they have low switching costs and hence are likely to switch. Retention offers then effectively serve as a mechanism to price discriminate against consumers with high switching costs.

We thus focus on cases where consumers cancel their current subscription in favor of a competitor. For example, in the UK, Ofcom (2010) reports that in e.g. mobile telephony, consumers that want to switch have to contact their current provider and request a code which they must communicate to their new provider to complete the switch. However, when applying for such a code, the current provider can, and often does, make a retention offer. Indeed, this paper was inspired by a similar experience of one of the authors. After having switched to a cheaper car insurance, he still received a renewal from the old insurer. He phoned them, the company apologized, asked why he cancelled his policy, and what price the new insurer charged. It then offered a price slightly below that - which he was willing to accept. It is exactly this experience that we try to model in this paper.

We study a two-period model with two firms located at the endpoints of a Hotelling line. In the second period, firms set prices based on buying behavior in period 1. In particular, firm B can try to poach consumers from firm A by charging them a lower price. Once a consumer indicates that she
intends to switch from A to B to take advantage of that poaching price, however, firm A can make a retention offer. In the equilibrium of our model, low-switching-cost consumers strategically solicit offers from the competing firm to secure a retention offer from their current provider - even if they have no intention to switch. Soliciting offers requires costly effort, and high-switching-cost consumers do not find making that effort worthwhile. Hence, using retention offers allows firms to price discriminate between the two types.

We find that the possibility of retention offers increases prices. Prices for loyal consumers increase, as this pool of consumers is less likely to switch on average. But poaching prices increase as well; as low-cost consumers have already incurred part of their switching costs, they become easier to poach. Equilibrium prices in the first period also increase. As competition for consumers with low switching costs is fiercer in the second period, firms are less eager to attract these consumers in period 1 . The welfare effects are ambiguous. Firms are better off, while consumers are worse off. The latter applies to all individual high-switching-cost consumers, and to consumers as a whole. The effect on individual low-cost consumers is ambiguous.

This paper clearly fits in the literature on behavior-based price discrimination. Classic references in this field include Chen (1997), Fudenberg and Tirole (2000) and Taylor (2003), that all look at multi-period models in which firms can base the price they charge on a consumer's purchase history. Chen and Pearcy (2010) allow consumer tastes to evolve over the course of the game. Gehrig, Shy and Stenbacka (2011) study the welfare effects of behavior-based price discrimination in the context of entry deterrence. Yet, none of these papers allows for retention offers. Our paper adds to the literature on switching costs, of which overviews can be found in Klemperer (1995) and Farrell and Klemperer (2007).

Two recent papers, developed independently from ours, do look at retention offers. Gnutzmann (2013) extends Chen (1997) by looking at retention offers in a model with homogeneous products and $N \geq 2$ firms. In the second period, consumers can readily observe loyalty prices, poaching prices and retention prices, but have to exert effort 1 to secure the poaching price and effort $\alpha<1$ to secure the retention price. Consumers differ in their cost
of effort, i.e. their switching costs. In this paper, different from ours, firstperiod prices do not affect second-period actions, as consumers only learn their switching costs after the first period. Esteves (2014) looks at model that is similar to ours. She extends the Fudenberg and Tirole (2000) with the possibility of retention offers. Crucially, however, she does not allow consumers to strategically solicit an offer from the competing firm, in an attempt to obtain a better deal from her current supplier. In her model, consumers do not rationally foresee that retention offers will be made.

Finally note that retention offers differ from price-matching policies, in which a supplier is always willing to match a lower price of a competitor. Such price-matching policies do not depend on purchase behavior of consumer. Also, in our model, we will see that the equilibrium retention price is actually higher than the poaching price offered by the competitor, simply because the consumer has already revealed a preference for this supplier by her past buying behavior. Price-matching policies are studied in e.g. Arbatskaya, Hviid and Shaffer (2004) and Corts (1997).

This paper is organized as follows. First, section 2 introduces the model. Section 3 considers a benchmark in which there are no retention offers, but there is poaching and heterogeneous switching costs. The model with retention offers is studied in section 4. We study the effects of the possibility of retention offers in Section 5 and conclude in Section 6 .

## 2 The model

A unit mass of consumers is uniformly distributed on a Hotelling line. Transportation costs are normalized to 1 . Firms $A$ and $B$ are located at 0 and 1 respectively and face marginal costs $c$. There are 2 periods. Consumers have unit demand in each period, and willingness-to-pay $r$, gross of transportation costs. The market is fully covered. Firms and consumers use a common discount factor $\delta \in(0,1)$.

We have two types of consumers: those with high switching costs $z_{H}$, and those with low switching costs $z_{L}<z_{H}$. The share of low types is given by $\lambda \in(0,1)$, independent of location. Switching costs are incurred if a consumer
switches suppliers in period 2. Switching costs consist of two elements. First, a consumer has to prepare for a switch, for example by securing an offer from the competing supplier. Second, she has to effectuate the switch, for example by actually signing a contract with the new supplier.

For our analysis, it is crucial that actions taken to prepare for a switch satisfy three conditions. First, they have to involve sunk costs to the consumer. Second, they have to be revocable, in the sense that after incurring preparation costs, the consumer still has the option to stick to her original supplier. Third, the costly actions have to be observable to her original supplier. For example, consider a consumer that considers to switch car insurers. She secures an offer from the other supplier, and then makes a phone call to her current supplier to cancel her contract. The costs involved with these actions are sunk. However, the switch is revocable: she may still change her mind and stay with the current supplier. Finally, the current supplier observes that this consumer has contacted her and, possibly, also that she has secured a competing offer. Hence, all three conditions are satisfied.

We denote the costs for a type $i \in\{L, H\}$ to prepare for a switch as $z_{i}^{1}$, and the additional costs to perform the switch as $z_{i}^{2}$. We assume that both types of switching costs are higher for the high types. Thus $z_{H}^{1}>z_{L}^{1}$ and $z_{H}^{2}>z_{L}^{2}$, while $z_{L}=z_{L}^{1}+z_{L}^{2}$ and $z_{H}=z_{H}^{1}+z_{H}^{2}$.

The timing of the game is as follows. In period $1, A$ and $B$ simultaneously set prices $p_{A}^{1}$ and $p_{B}^{1}$, respectively. Consumers observe these prices, and decide where to buy in the first period. A fraction $\hat{x}_{i}^{1}$ of type $i$ consumers, to be determined endogenously, will buy from firm $A$. The other $1-\hat{x}_{i}^{1}$ will buy from firm $B$. We will refer to the consumers that buy from $A$ in period 1 as segment $A$, and to the consumers that buy from $B$ in period 1 as segment $B$. In period 2, the following sequence of events unfolds. In the first stage, firms $A$ and $B$ simultaneously each set 2 prices, observable to everyone. Firm $A$ charges a loyalty price $p_{A A}^{2}$ to consumers that bought from $A$ in period 1, and a poaching price $p_{A B}^{2}$ to consumers that bought from $B$. Similarly, $B$ sets prices $p_{B B}^{2}$ and $p_{B A}^{2}$. In the second stage, each consumer decides whether she incurs preparation costs $z_{i}^{1}$. If she does, her original supplier can observe this and can make a retention offer. The retention offer of firm $A$ is denoted
$p_{A}^{R}$, that of firm $B$ is $p_{B}^{R}$.
For the analysis that follows to be valid, we need to impose some parameter restrictions. These restrictions imply that in the benchmark model without retention offers for any value of $\lambda$ some, but not all, low type consumers, and some, but not all, high type consumers are poached in the second period. Moreover, we want the same thing to be true in the model with retention offers. As we will show in the analyses below, this requires the following parameter restrictions to hold;

$$
\begin{align*}
z_{L} & <1  \tag{1}\\
z_{H} & <1 / 3+2 z_{L} / 3  \tag{2}\\
z_{H} & <1 / 2+z_{L}^{2} / 2 \tag{3}
\end{align*}
$$

## 3 Benchmark: no retention offers

Preliminaries We first consider a benchmark without retention offers. In that case, the separation of total switching costs into preparation costs and effectuation costs is immaterial. The timing of this simplified game is thus as follows. In period 1, $A$ and $B$ simultaneously set $p_{A}^{1}$ and $p_{B}^{1}$, and a fraction $\hat{x}_{i}^{1}$ of type $i$ consumers buys from $A$. These consumers comprise segment $A$, the others segment $B$. In period $2, A$ and $B$ simultaneously set poaching prices and loyalty prices. We look for a symmetric equilibrium and solve with backward induction.

Second period In equilibrium at least some type $i$ consumers in segment $A$ will be tempted by the poaching price of firm $B$. The second period will then have some $\hat{x}_{A i}^{2}<\hat{x}_{i}^{1}$ again choosing for firm $A$, while the remaining $\hat{x}_{i}^{1}-\hat{x}_{A i}^{2}$ switch to $B$. Something similar holds for consumers in segment $B$. The indifferent types $i$ on segments $A$ and $B$ are given by

$$
\begin{equation*}
\hat{x}_{A i}^{2}=\frac{1}{2}\left(1+p_{B A}^{2}-p_{A A}^{2}+z_{i}\right) ; \quad \hat{x}_{B i}^{2}=\frac{1}{2}\left(1+p_{B B}^{2}-p_{A B}^{2}-z_{i}\right), \tag{4}
\end{equation*}
$$

provided that these expressions are strictly between 0 and the relevant $\hat{x}_{i}^{1}$. Parameter restrictions (1) and (2) assure that that is the case in equilibrium. ${ }^{1}$


Figure 1: Market segmentation in both periods, Benchmark.
Figure 1 depicts this situation. The top panel reflects consumers with high switching costs, the bottom panel those with low switching costs. In period 1, those to the left of $\hat{x}_{i}^{1}$ buy from firm $A$ and thus comprise segment $A \|^{2}$ Those to the right of $\hat{x}_{i}^{1}$ buy from $B$ and comprise segment $B$. In period 2 , those in segment $A$ that are located to the left of $\hat{x}_{A H}^{2}$ will buy from firm $A$ (as reflected by the arrow), while those to the right will buy from $B$. Something similar applies to those in segment $B$. As $z_{H}>z_{L}$, we have from (4) that $x_{A H}^{2}>x_{A L}^{2}$ and $x_{B H}^{2}>x_{B L}^{2}$ : as their switching costs are higher, fewer high types will switch in period 2 .

[^1]Second-period profits for firm $A$ are given by

$$
\begin{align*}
\Pi_{A}^{2}= & \Pi_{A A}^{2}+\Pi_{A B}^{2} \\
\equiv & \left(p_{A A}^{2}-c\right)\left[\lambda \hat{x}_{A L}^{2}+(1-\lambda) \hat{x}_{A H}^{2}\right] \\
& +\left(p_{A B}^{2}-c\right)\left[\lambda\left(\hat{x}_{B L}^{2}-\hat{x}_{L}^{1}\right)+(1-\lambda)\left(\hat{x}_{B H}^{2}-\hat{x}_{H}^{1}\right)\right] \tag{6}
\end{align*}
$$

where $\Pi_{A A}^{2}$ (the second line) reflects total profits from loyal consumers, and $\Pi_{A B}^{2}$ (the third line) total profits from consumers that are poached. Similarly, firm $B$ 's profits are given by

$$
\begin{align*}
\Pi_{B}^{2} \equiv & \Pi_{B B}^{2}+\Pi_{B A}^{2} \\
\equiv & \left(p_{B A}^{2}-c\right)\left[\lambda\left(\hat{x}_{L}^{1}-\hat{x}_{A L}^{2}\right)+(1-\lambda)\left(\hat{x}_{H}^{1}-\hat{x}_{A H}^{2}\right)\right] \\
& +\left(p_{B B}^{2}-c\right)\left[\lambda\left(1-\hat{x}_{B L}^{2}\right)+(1-\lambda)\left(1-\hat{x}_{B H}^{2}\right)\right] . \tag{7}
\end{align*}
$$

For ease of exposition, we define $\bar{z}$ as the weighted average of switching costs in the population, and $\hat{x}^{1}$ as the weighted average location of indifferent consumers in period 1 :

$$
\begin{align*}
\bar{z} & \equiv \lambda z_{L}+(1-\lambda) z_{H}  \tag{8}\\
\hat{x}^{1} & \equiv \lambda \hat{x}_{L}^{1}+(1-\lambda) \hat{x}_{H}^{1} . \tag{9}
\end{align*}
$$

Plugging in the expressions from (4) into the second line of (6), we have

$$
\begin{equation*}
\Pi_{A A}^{2}=\frac{1}{2}\left(p_{A A}^{2}-c\right)\left[1+p_{B A}^{2}-p_{A A}^{2}+\bar{z}\right] \tag{10}
\end{equation*}
$$

Similarly, for firm $B$, from the second line of (7), and (4),

$$
\begin{equation*}
\Pi_{B A}^{2}=\left(p_{B A}^{2}-c\right)\left[\hat{x}^{1}-\frac{1}{2}\left(1+p_{B A}^{2}-p_{A A}^{2}+\bar{z}\right)\right] \tag{11}
\end{equation*}
$$

Maximizing (10) with respect to $p_{A A}^{2}$ and (11) with respect to $p_{B A}^{2}$ yields the following reaction functions:

$$
p_{A A}^{2}=\frac{1}{2}\left(1+p_{B A}^{2}+c+\bar{z}\right) ; \quad p_{B A}^{2}=\frac{1}{2}\left(2 \hat{x}^{1}-1+p_{A A}^{2}+c-\bar{z}\right) .
$$

Solving the system gives:

$$
\begin{equation*}
p_{A A}^{2}=c+\frac{1}{3}\left(1+2 \hat{x}^{1}+\bar{z}\right) ; \quad p_{B A}^{2}=c+\frac{1}{3}\left(4 \hat{x}^{1}-1-\bar{z}\right) . \tag{12}
\end{equation*}
$$

We then immediately have

$$
\begin{equation*}
\hat{x}_{A i}^{2}=\frac{1}{6}\left(1+2 \hat{x}^{1}+3 z_{i}-2 \bar{z}\right) \tag{13}
\end{equation*}
$$

and

$$
\begin{equation*}
\Pi_{A A}^{2}=\frac{1}{18}\left(1+2 \hat{x}^{1}+\bar{z}\right)^{2} ; \quad \Pi_{B A}^{2}=\frac{1}{18}\left(4 \hat{x}^{1}-1-\bar{z}\right)^{2} . \tag{14}
\end{equation*}
$$

On segment $B$, we can do a similar analysis. Here

$$
\begin{aligned}
\Pi_{B B}^{2} & =\frac{1}{2}\left(p_{B B}^{2}-c\right)\left[1+p_{A B}^{2}-p_{B B}^{2}+\bar{z}\right] \\
\Pi_{A B}^{2} & =\left(p_{A B}^{2}-c\right)\left[1-\hat{x}^{1}-\frac{1}{2}\left(1+p_{A B}^{2}-p_{B B}^{2}+\bar{z}\right)\right] .
\end{aligned}
$$

Hence

$$
\begin{equation*}
p_{A B}^{2}=c+\frac{1}{3}\left(3-4 \hat{x}^{1}-\bar{z}\right) ; \quad \Pi_{A B}^{2}=\frac{1}{18}\left(3-4 \hat{x}^{1}-\bar{z}\right)^{2} . \tag{15}
\end{equation*}
$$

First period We now solve for the first period. Consumers are forwardlooking and rationally take into account the events that will unfold in the second period. A consumer that is indifferent between $A$ and $B$ in period 1 thus anticipates that, whatever she chooses, she will switch in period 2. Denoting the discount factor by $\delta$, the indifferent type $i$ located at $\hat{x}_{i}^{1}$ has

$$
\begin{align*}
r-\hat{x}_{i}^{1}-p_{A}^{1} & +\delta\left(r-\left(1-\hat{x}_{i}^{1}\right)-p_{B A}^{2}-z_{i}\right) \\
& =r-\left(1-\hat{x}_{i}^{1}\right)-p_{B}^{1}+\delta\left(r-\hat{x}_{i}^{1}-p_{A B}^{2}-z_{i}\right) \tag{16}
\end{align*}
$$

where the left-hand side gives her total lifetime utility if she chooses $A$ in period 1 , while the right-hand side gives that of choosing $B$ in period 1 . Note that switching costs $z_{i}$ drop out of this equality; either way, in equilibrium this consumer will always incur switching costs in period 2 , so these do not affect $\hat{x}_{i}^{1}$. This immediately implies $\hat{x}_{L}^{1}=\hat{x}_{H}^{1}=\hat{x}^{1}$. Solving (16) then gives

$$
\begin{equation*}
\hat{x}^{1}=\frac{1+p_{B}^{1}-p_{A}^{1}-\delta\left(1+p_{B A}^{2}-p_{A B}^{2}\right)}{2(1-\delta)} \tag{17}
\end{equation*}
$$

Substituting second-period equilibrium prices from $\sqrt[12]{ }$ and 15 and solving for $\hat{x}^{1}$ yields

$$
\begin{equation*}
\hat{x}^{1}=\frac{1}{2}+\frac{3\left(p_{B}^{1}-p_{A}^{1}\right)}{6+2 \delta} \tag{18}
\end{equation*}
$$

In the first period, firm $A$ sets $p_{A}^{1}$ as to maximize total discounted profits

$$
\begin{align*}
\Pi_{A} & =\left(p_{A}^{1}-c\right) \hat{x}^{1}+\delta \Pi_{A A}^{2}+\delta \Pi_{A B}^{2} \\
& =\left(p_{A}^{1}-c\right) \hat{x}^{1}+\frac{\delta}{18}\left(1+2 \hat{x}^{1}+\bar{z}\right)^{2}+\frac{\delta}{18}\left(3-4 \hat{x}^{1}-\bar{z}\right)^{2} \tag{19}
\end{align*}
$$

Taking the derivative with respect to $p_{A}^{1}$ :

$$
\frac{\partial \Pi_{A}}{\partial p_{A}^{1}}=\left(p_{A}^{1}-c\right) \frac{\partial \hat{x}^{1}}{\partial p_{A}^{1}}+\hat{x}^{1}+\frac{2 \delta}{9}\left(1+2 \hat{x}^{1}+\bar{z}\right) \frac{\partial \hat{x}^{1}}{\partial p_{A}^{1}}-\frac{4 \delta}{9}\left(3-4 \hat{x}^{1}-\bar{z}\right) \frac{\partial \hat{x}^{1}}{\partial p_{A}^{1}}
$$

A symmetric equilibrium requires $p_{A}^{1}=p_{B}^{1}$ hence $\hat{x}=\frac{1}{2}$. From (18), we have $\frac{\partial \hat{x}^{1}}{\partial p_{A}^{1}}=\frac{-3}{6+2 \delta}$. Hence, the first-order condition becomes

$$
\frac{1}{2}-\frac{3}{6+2 \delta}\left(p_{A}^{1}-c+\frac{2 \delta \bar{z}}{3}\right)=0
$$

This yields equilibrium prices

$$
p_{A}^{1}=p_{B}^{1}=c+1+\frac{\delta}{3}(1-2 \bar{z})
$$

We thus have the following $\sqrt{3}^{3}$

[^2]Theorem 1 In the benchmark without retention offers, equilibrium firstperiod, loyalty and poaching prices are given by

$$
\begin{align*}
p_{1}^{b m} & =c+1+\frac{\delta}{3}(1-2 \bar{z}) \\
p_{\text {loyal }}^{b m} & =c+\frac{1}{3}(2+\bar{z}) \\
p_{\text {poach }}^{b m} & =c+\frac{1}{3}(1-\bar{z}) . \tag{20}
\end{align*}
$$

Equilibrium profits are given by

$$
\Pi^{b m}=\frac{1}{2}+\frac{1}{18}(8 \delta-2 \bar{z} \delta(2-\bar{z})) .
$$

In a model with standard Hotelling competition, without poaching or switching costs, we would have $p=p^{h} \equiv c+1$ in each period. From (1), we have $\bar{z}<1$, hence $p^{h}>p_{\text {loyal }}^{b m}>p_{\text {poach }}^{b m}$. Hence, loyal consumers end up paying a higher price than those that are poached by the other firm $\left(p_{\text {loyal }}^{b m}>p_{\text {poach }}^{b m}\right)$.
which is maximized by setting $p_{B A}^{2}=\frac{1}{2}\left(p_{A A}^{2}+c-z_{L}\right)$. At $p_{B A}^{2}=p_{A A}^{2}-z_{H}$, these profits are decreasing whenever $p_{A A}^{2}-z_{H}>\frac{1}{2}\left(p_{A A}^{2}+c-z_{L}\right)$, hence if $z_{H}(\lambda+5)-z_{L}(\lambda+3)<2$. This is most restrictive for $\lambda=1$, so we need $z_{H}<\frac{1}{3}+\frac{2}{3} z_{L}$ which is exactly (2). Hence, we're on the downward sloping part of $\pi_{L}$. Therefore such a defection cannot be profitable.

Alternatively, firm $B$ could set $p_{B A}^{2}$ so low that we serve all the low types, so $\hat{x}_{A L}^{2}=0$. That implies setting

$$
p_{B A}^{2}=p_{A A}^{2}-z_{L}-1=c+\frac{1}{3}(2+\bar{z})-z_{L}-1=c+\frac{1}{3}(\bar{z}-1)-z_{L}<c,
$$

which is clearly unprofitable.
Now consider firm $A$. First, it can defect by setting $p_{A A}^{2}$ so high that it only sells to the high types. That requires setting $p_{A A}^{2}$ such that $\hat{x}_{A L} \leq 0$ or $p_{A A}^{2} \geq 1+p_{B A}^{2}+z_{L}$. Its profits are then given by

$$
\pi_{A H}^{2}=\frac{1}{2}(1-\lambda)\left(p_{A A}^{2}-c\right)\left[1+p_{B A}^{2}-p_{A A}^{2}+z_{H}\right]
$$

which is maximized by setting $p_{A A}^{2}=\frac{1}{2}\left(1+p_{B A}^{2}+c+z_{H}\right)$. At $p_{A A}^{2}=1+p_{B A}^{2}+z_{L}$, these profits are decreasing whenever $1+p_{B A}^{2}+c+z_{H}<2 p_{B A}^{2}+2 z_{L}+2$ or $-2 z_{L}-\frac{4}{3}+z_{H}+\bar{z} / 3<0$, which is always the case. Hence such a defection cannot be profitable.
Finally, firm $A$ can defect by setting a lower $p_{A A}^{2}$, such that it serves all the high types. In that case the profit function we use in the main text overestimates true profits (since it assumes a $\hat{x}_{A H}>1 / 2$ rather than the true $\hat{x}_{A H}=1 / 2$ ). As we cannot find a profitincreasing defection when looking at an inflated profit function, such a profit-increasing defection definitely does not exist when looking at the true profit function.

Also, the possibility of poaching makes competition particularly fierce in the second period ( $p_{\text {loyal }}^{b m}<p^{h}$ ). In the first period, the effect is ambiguous. On the one hand, consumers are less sensitive to first period prices.$^{[7]}$ marginal consumers know that if they are tempted to consume their less-preferred product, that will imply higher prices in period $25^{5}$ On the other hand, as switching costs increase, firms are more eager to attract consumers in period 1, as consumers will be less inclined to switch, so second-period profits increase. ${ }^{[6}$ As a result, first-period prices are higher ( $p_{1}^{b m}>p_{h}$ ) with low switching costs, but lower ( $p_{1}^{b m}<p_{h}$ ) with high switching costs. We also have:

Corollary 1 The total discounted price paid by both loyal and non-loyal consumers is lower than that in a standard Hotelling model. All consumers are strictly better off. Firms are worse off. Total welfare decreases.

Proof. For loyals, the effect on total discounted price is

$$
\Delta P_{\text {loyal }}=p_{1}^{b m}+\delta p_{\text {loyal }}^{b m}-(1+c)(1+\delta)=-\frac{1}{3} \bar{z} \delta<0
$$

hence they are better off. For consumers that are poached

$$
\Delta P_{\text {poach }}=p_{1}^{b m}+\delta p_{\text {poach }}^{b m}-(1+c)(1+\delta)=-\frac{1}{3} \delta(1+3 \bar{z})<0 .
$$

These consumers now incur switching costs and a disutility from no longer consuming their preferred product in period 2. However, if they would choose not to switch, they would still be strictly better off than in a Hotelling model. Revealed preference implies that their net utility from switching is only higher. As total discounted prices decrease an the markets is covered, profits are lower. For total welfare, prices are just a transfer. With poaching, some consumers incur switching costs, and a utility loss from no longer con-

[^3]suming their preferred product. From a welfare perspective, that is a loss.

## 4 Introducing retention offers

Preliminaries We now consider the full model and analyze whether there is an equilibrium in which retention offers occur. We thus look for an equilibrium where low types that do not switch always pay the retention price while high types that do not switch pay the loyalty price.

We solve with backward induction and again focus on segment $A$; consumers that have bought from firm $A$ in period 1. For retention offers to occur in equilibrium, we need that low types that buy again from $A$ go for the retention offer $p_{A}^{R}$, while high types prefer the loyalty price $p_{A A}^{2}$. For the low types, we thus need that the inspection costs $z_{L}^{1}$ are smaller than the difference between $p_{A}^{R}$ and $p_{A A}^{2}$, while for the high types the opposite is true. An equilibrium with retention offers thus requires

$$
\begin{equation*}
z_{L}^{1}<p_{A A}^{2}-p_{A}^{R}<z_{H}^{1} . \tag{21}
\end{equation*}
$$

As we are interested in situations where retention offers indeed occur in equilibrium, below we will derive parameter restrictions such that these conditions are indeed satisfied. Note that in equilibrium all low types incur the inspection costs $z_{L}^{1}$. Hence a low type that decides to switch rather than stay loyal to firm $A$, only incurs additional switching $\operatorname{costs} z_{L}^{2}$. In equilibrium, the loyal high types do not incur inspection costs. Hence, high types that decide to switch incur an additional $z_{H}$. We denote by $\tilde{z}$ the weighted average of these additional switching costs. Hence

$$
\begin{equation*}
\tilde{z} \equiv \lambda z_{L}^{2}+(1-\lambda) z_{H} . \tag{22}
\end{equation*}
$$

Second period, second stage In stage 2 of period 2, firm $A$ sets retention price $p_{A}^{R}$ to maximize profits, given the loyalty price $p_{A A}^{2}$ and the poaching price $p_{B A}^{2}$ that were set in stage 1. All low types have already incurred the
preparation costs $z_{L}^{1}$. A low type that switches thus incurs an additional $z_{L}^{2}$ and would pay $p_{A}^{R}$ when sticking to $A$. A high type that switches incurs an additional $z_{H}$ and would pay $p_{A A}^{2}$ when sticking to $A$. Hence, the indifferent consumers in segment $A$ are given by

$$
\begin{equation*}
\hat{x}_{A L}^{2}=\frac{1}{2}\left(1+p_{B A}^{2}-p_{A}^{R}+z_{L}^{2}\right) ; \quad \hat{x}_{A H}^{2}=\frac{1}{2}\left(1+p_{B A}^{2}-p_{A A}^{2}+z_{H}\right) \tag{23}
\end{equation*}
$$

Firm $A$ 's second-period profits from segment $A$ now equal

$$
\begin{align*}
\Pi_{A A}^{2}= & \lambda\left(p_{A}^{R}-c\right) \hat{x}_{A L}^{2}+(1-\lambda)\left(p_{A A}^{2}-c\right) \hat{x}_{A H}^{2} \\
= & \frac{1}{2} \lambda\left(p_{A}^{R}-c\right)\left(1+p_{B A}^{2}-p_{A}^{R}+z_{L}^{2}\right) \\
& +\frac{1}{2}(1-\lambda)\left(p_{A A}^{2}-c\right)\left(1+p_{B A}^{2}-p_{A A}^{2}+z_{H}\right) . \tag{24}
\end{align*}
$$

Maximizing with respect to $p_{A}^{R}$ yields

$$
\begin{equation*}
p_{A}^{R}=\frac{1}{2}\left(1+p_{B A}^{2}+z_{L}^{2}+c\right) . \tag{25}
\end{equation*}
$$

Second period, first stage Maximizing (24) with respect to $p_{A A}^{2}$ yields the first-stage best-reply function for firm $A$ :

$$
\begin{equation*}
p_{A A}^{2}=\frac{1}{2}\left(1+p_{B A}^{2}+z_{H}+c\right) . \tag{26}
\end{equation*}
$$

Firm $B$ 's second-period profits on segment $A$ are given by

$$
\begin{equation*}
\Pi_{B A}^{2}=\lambda\left(p_{B A}^{2}-c\right)\left(\hat{x}_{L}^{1}-\hat{x}_{A L}^{2}\right)+(1-\lambda)\left(p_{B A}^{2}-c\right)\left(\hat{x}_{H}^{1}-\hat{x}_{A H}^{2}\right) . \tag{27}
\end{equation*}
$$

Firm $B$ anticipates that $A$ will set $p_{A}^{R}$ according to (25). Using (23), we can write

$$
\begin{aligned}
\Pi_{B A}^{2}= & \lambda\left(p_{B A}^{2}-c\right)\left(\hat{x}_{L}^{1}-\frac{1}{4}\left(1+p_{B A}^{2}+z_{L}^{2}-c\right)\right) \\
& +(1-\lambda)\left(p_{B A}^{2}-c\right)\left(\hat{x}_{H}^{1}-\frac{1}{2}\left(1+p_{B A}^{2}-p_{A A}^{2}+z_{H}\right)\right) .
\end{aligned}
$$

Taking the first-order condition yields the reaction function

$$
p_{B A}^{2}=\frac{4 \hat{x}^{1}+2(1-\lambda) p_{A A}^{2}-\tilde{z}-(1-\lambda) z_{H}+2 c}{4-2 \lambda}-\frac{1}{2} .
$$

For the equilibrium, we plug in the reaction function of firm $A,(26)$ to find

$$
p_{B A}^{2}=\frac{4 \hat{x}^{1}+(1-\lambda)\left(1+p_{B A}^{2}\right)-\tilde{z}+(3-\lambda) c}{4-2 \lambda}-\frac{1}{2} .
$$

Hence

$$
\begin{equation*}
p_{B A}^{2}=c+b, \tag{28}
\end{equation*}
$$

with

$$
\begin{equation*}
b \equiv \frac{4 \hat{x}^{1}-\tilde{z}-1}{3-\lambda} \tag{29}
\end{equation*}
$$

the equilibrium price-cost margin on $B$ 's poaching prices. From (25) and (26) we then have

$$
\begin{equation*}
p_{A}^{R}=c+\frac{1}{2}\left(1+z_{L}^{2}+b\right) ; \quad p_{A A}^{2}=c+\frac{1}{2}\left(1+z_{H}+b\right) . \tag{30}
\end{equation*}
$$

while equilibrium market shares follow directly from (23):

$$
\begin{equation*}
\hat{x}_{A L}^{2}=\frac{1}{4}\left(1+z_{L}^{2}+b\right) ; \quad \hat{x}_{A H}^{2}=\frac{1}{4}\left(1+z_{H}+b\right) \tag{31}
\end{equation*}
$$

provided that these expressions are strictly between 0 and the relevant $\hat{x}_{i}^{1}$. Given that we already impose (1), parameter restriction (3) assures that that is the case in equilibrium. 7 For equilibrium profits for $A$, we plug these values

$$
\begin{aligned}
& { }^{7} \text { First note that we immediately have } \hat{x}_{A i}^{2}>0 \text {. To have } \hat{x}_{A i}^{2}<1 / 2 \text {, we need } \\
& \qquad \max \left\{z_{L}^{2}, z_{H}\right\}+b<1
\end{aligned}
$$

Note

$$
b=\frac{1-\tilde{z}}{3-\lambda}=\frac{1-\left(\lambda z_{L}^{2}+(1-\lambda) z_{H}\right)}{3-\lambda}
$$

which is increasing in $\lambda$ (as the numerator is increasing and the denominator decreasing). We want the condition to be satisfied for all $\lambda$. It is most restrictive for $\lambda=1$, so we require

$$
\max \left\{z_{L}^{2}, z_{H}\right\}+\frac{1}{2}\left(1-z_{L}^{2}\right)<1 .
$$

into (24) to find

$$
\begin{equation*}
\Pi_{A A}^{2}=\frac{1}{8} \lambda\left(1+z_{L}^{2}+b\right)^{2}+\frac{1}{8}(1-\lambda)\left(1+z_{H}+b\right)^{2} \tag{32}
\end{equation*}
$$

Similarly, using (27), profits for firm $B$ can be shown to equal

$$
\begin{align*}
\Pi_{B A}^{2} & =b\left(\hat{x}^{1}-\frac{1}{4} \lambda\left(1+z_{L}^{2}\right)-\frac{1}{4}(1-\lambda)\left(1+z_{H}\right)-\frac{1}{4} b\right) \\
& =\frac{1}{4} b\left(4 \hat{x}^{1}-\tilde{z}-1-b\right)=\frac{1}{4}(2-\lambda) b^{2} \tag{33}
\end{align*}
$$

On segment $B$, we have a similar analysis that yields

$$
\begin{equation*}
p_{A B}^{2}=c+a ; \quad \Pi_{A B}^{2}=\frac{1}{4}(2-\lambda) a^{2} \tag{34}
\end{equation*}
$$

with

$$
a \equiv \frac{3-4 \hat{x}^{1}-\tilde{z}}{3-\lambda}
$$

the price-cost margin on $A$ 's poaching prices.

First period Again, the indifferent consumer in period 1 is given by (17): retention prices do not affect first-period market shares, as the marginal consumer will always switch. Substituting for $p_{A B}^{2}$ and $p_{B A}^{2}$ from (28) and (34) into (17):

$$
\hat{x}_{i}^{1}=\frac{1+p_{B}^{1}-p_{A}^{1}-\delta\left(\frac{8 \hat{x}^{1}-1-\lambda}{3-\lambda}\right)}{2-2 \delta}
$$

Using (9), substituting from the above equations and solving for $\hat{x}^{1}$ yields

$$
\hat{x}_{L}^{1}=\hat{x}_{H}^{1}=\hat{x}^{1}=\frac{(3-\lambda)\left(1+p_{B}^{1}-p_{A}^{1}\right)+\delta(1+\lambda)}{8 \delta+(3-\lambda)(2-2 \delta)}
$$

For the low types, this implies $z_{L}^{2}<1$, which is always satisfied given that 1 is satisfied. For the high types we need $z_{H}+\frac{1}{2}\left(1-z_{L}^{2}\right)<1$, which is implied by (3).

Total profits for firm $A$ are now given by

$$
\begin{align*}
\Pi_{A}= & \left(p_{A}^{1}-c\right) \hat{x}^{1}+\delta \Pi_{A A}^{2}+\delta \Pi_{A B}^{2} \\
= & \left(p_{A}^{1}-c\right) \hat{x}^{1}+\frac{\delta}{8} \lambda\left(b+1+z_{L}^{2}\right)^{2} \\
& +\frac{\delta}{8}(1-\lambda)\left(b+1+z_{H}\right)^{2}+\frac{\delta}{4}(2-\lambda) a^{2} . \tag{35}
\end{align*}
$$

Taking the derivative wrt $p_{A}^{1}$ :

$$
\begin{aligned}
\frac{\partial \Pi_{A}}{\partial p_{A}^{1}}= & \hat{x}^{1}+\left(p_{A}^{1}-c\right) \frac{\partial \hat{x}^{1}}{\partial p_{A}^{1}}+\frac{\delta \lambda}{4}\left(b+1+z_{L}^{2}\right) \frac{\partial b}{\partial \hat{x}^{1}} \frac{\partial \hat{x}^{1}}{\partial p_{A}^{1}} \\
& +\frac{\delta}{4}(1-\lambda)\left(b+1+z_{H}\right) \frac{\partial b}{\partial \hat{x}^{1}} \frac{\partial \hat{x}^{1}}{\partial p_{A}^{1}}+\frac{\delta a}{2}(2-\lambda) \frac{\partial a}{\partial \hat{x}^{1}} \frac{\partial \hat{x}^{1}}{\partial p_{A}^{1}} \\
= & \hat{x}^{1}+\left(p_{A}^{1}-c\right) \frac{\partial \hat{x}^{1}}{\partial p_{A}^{1}}+\frac{\delta}{4}(b+1+\tilde{z}) \frac{\partial b}{\partial \hat{x}^{1}} \frac{\partial \hat{x}^{1}}{\partial p_{A}^{1}}+\frac{\delta a}{2}(2-\lambda) \frac{\partial a}{\partial \hat{x}^{1}} \frac{\partial \hat{x}^{1}}{\partial p_{A}^{1}} .
\end{aligned}
$$

With

$$
\begin{aligned}
\frac{\partial b}{\partial \hat{x}^{1}} & =-\frac{\partial a}{\partial \hat{x}^{1}}=\frac{4}{3-\lambda} \\
\frac{\partial \hat{x}^{1}}{\partial p_{A}^{1}} & =\frac{-(3-\lambda)}{8 \delta+(3-\lambda)(2-2 \delta)}
\end{aligned}
$$

the first-order condition becomes $8^{8}$

$$
\hat{x}^{1}-\frac{(3-\lambda)\left(p_{A}^{1}-c\right)+(b+1+\tilde{z}) \delta-2(2-\lambda) a \delta}{8 \delta+(3-\lambda)(2-2 \delta)}=0 .
$$

Equilibrium requires $p_{A}^{1}=p_{B}^{1}$, hence $\hat{x}=1 / 2$ and $a=b=\frac{1-\tilde{z}}{3-\lambda}$. Solving for equilibrium prices then yields:

Theorem 2 With the possibility of retention offers, equilibrium first-period,

[^4]loyalty, poaching and retention prices are given by
\[

$$
\begin{align*}
p_{1}^{\text {ret }} & =c+1-3 \delta \frac{\tilde{z}(2-\lambda)}{(3-\lambda)^{2}}+\delta \frac{\left(3-\lambda^{2}+\lambda\right)}{(3-\lambda)^{2}} \\
p_{\text {loyal }}^{\text {ret }} & =c+\frac{1}{2}\left(1+z_{H}+b\right) ; \\
p_{\text {poach }}^{\text {ret }} & =c+b \\
p_{\text {retent }}^{\text {ret }} & =c+\frac{1}{2}\left(1+z_{L}^{2}+b\right), \tag{36}
\end{align*}
$$
\]

with $b=(1-\tilde{z}) /(3-\lambda)$. Equilibrium profits are given by

$$
\begin{equation*}
\Pi_{A}^{r e t}=\frac{1}{2}\left(p^{1}-c\right)+\frac{\delta}{8} \lambda\left(b+1+z_{L}^{2}\right)^{2}+\frac{\delta}{8}(1-\lambda)\left(b+1+z_{H}\right)^{2}+\frac{\delta}{4}(2-\lambda) b^{2} . \tag{37}
\end{equation*}
$$

In this case, we have $p^{h}>p_{\text {loyal }}^{\text {ret }}>p_{\text {retent }}^{r e t}>p_{\text {poach. }}^{\text {ret }}$. Again, the comparison of $p_{1}^{\text {ret }}$ and $p_{h}$ is ambiguous ${ }^{9}$

Consumers that go for a retention offer thus pay a higher price than what they would pay if they would switch. As their original supplier knows that these consumers have a preference for their product, they do not have to fully compensate for the lower price of the other firm.

Poaching prices are decreasing in switching costs $z_{H}$ and $z_{L}^{2}$ : the higher these, the more of an effort firms have to make to poach consumers. At the same time, an increase in these switching costs increases loyalty prices, as
${ }^{9}$ Clearly $p_{\text {loyal }}^{\text {ret }}>p_{\text {retent }}^{\text {ret }}$. As $b=\frac{1-\tilde{z}}{3-\lambda}<\frac{1}{2}$ we have that $p_{\text {poach }}^{\text {ret }}<p_{\text {retent }}^{\text {ret }}$. Note

$$
p_{\text {logal }}^{\text {ret }}-p_{h}=\frac{1}{2}\left(\frac{\lambda\left(1-z_{L}^{2}\right)-2\left(1-z_{H}\right)}{3-\lambda}\right)
$$

The numerator is given by

$$
\begin{aligned}
\lambda-\lambda z_{L}^{2}+2 z_{H}-2 & <\lambda-\lambda\left(2 z_{H}-1\right)+2 z_{H}-2 \\
& =-2\left(1-z_{H}\right)(1-\lambda)<0
\end{aligned}
$$

where the first inequality follows from (3). Hence $p_{\text {loyal }}^{\text {ret }}<p_{h}$. This establishes the ranking. Finally, note that

$$
p_{1}^{\text {ret }}-p_{h}=-3 \delta \frac{\tilde{z}(2-\lambda)}{(3-\lambda)^{2}}+\delta \frac{\left(3-\lambda^{2}+\lambda\right)}{(3-\lambda)^{2}} .
$$

With $\lambda=0$, this equals $\delta\left(1-2 z_{H}\right) / 3$, the sign of which is ambiguous.
firms have to make less of an effort to retain consumers. These comparative statics are the same to those in the benchmark model. Retention prices increase in $z_{L}^{2}$. Only low types end up paying this price, and an increase in their switching costs makes it easier to retain them. First period prices decrease in $z_{H}$ and $z_{L}^{2}$. As it becomes harder to poach consumers in the second period, it becomes more profitable to attract them in the first period. Hence, an increase in switching costs deceases first-period prices

In the benchmark model (that added switching costs and poaching to a standard Hotelling model), we compared the total discounted price, consumer welfare, profits and total welfare to that in the Hotelling model. It is less straightforward to do that once we also add retention offers; all comparisons then become ambiguous.$^{10}$ Anyhow, it is far more interesting to compare a world in which retention offers are possible to one where they are not; doing so allows us to truly evaluate the welfare effects of retention offers per se. We wil do so in the next section.

As a final technical aside, note that we need some parameter restrictions for our separating condition (21) to be satisfied. From (30), this requires

$$
\begin{align*}
& z_{H}^{1}+z_{H}^{2}>2 z_{L}^{1}+z_{L}^{2}  \tag{38}\\
& z_{H}^{2}-z_{H}^{1}<z_{L}^{2} . \tag{39}
\end{align*}
$$

## 5 The effects of retention offers

Comparing the model with the possibility of retention offers to the benchmark model, we now have the following:

Theorem 3 Introducing the possibility of retention offers in our benchmark model has the following effects on prices:

1. First-period prices, poaching prices, and loyalty prices all increase.
2. New poaching prices are still lower than benchmark loyalty prices. New loyalty prices are higher than benchmark poaching prices.

[^5]3. Retention prices are always higher than benchmark poaching prices, and higher than benchmark loyalty prices if and only if $\lambda$ is high enough.

Summarizing, we have $p_{1}^{r e t}>p_{1}^{b m}$, while the effects on second-period prices are as follows:

|  | $p_{\text {poach }}^{b m}$ | $p_{\text {loyal }}^{b m}$ |
| :---: | :---: | :---: |
| $p_{\text {poach }}^{\text {ret }}$ | $>$ | $<$ |
| $p_{\text {retent }}^{\text {ret }}$ | $>$ | $\gtrless$ |
| $p_{\text {loyal }}^{\text {ret }}$ | $>$ | $>$ |

Proof. In appendix B.
To see what drives these results, note the following. First, for the low types, effective switching costs decrease. Ceteris paribus, when viewed in isolation, this would lead to a lower price charged by firm $A$ and a higher price charged by firm $B$ (see e.g. equation (12)). Second, the loyalty price will now only be paid by high types. These are reluctant to switch, leading to higher loyalty prices. In turn, when viewed in isolation, these higher loyalty prices also allow firm $B$ to charge higher poaching prices. Both channels lead to higher poaching prices. Also, note that average effective switching costs decrease. That implies that firms become less eager to capture consumers in period 1, hence first-period prices increase.

The welfare effects of retention offers are as follows:
Theorem 4 The possibility of retention offers increases equilibrium profits and decreases total consumer surplus. Each high type consumer is worse off. Only some individual low type consumers that end up paying the retention price, may be better off. Total welfare effects are ambiguous.

Proof. In appendix B.
Hence, although the possibility of retention offers may seem at first sight to benefit consumers, that is not the case in our equilibrium analysis. In our model, retention offers serve to screen consumers with high switching costs from those with low switching costs, allowing firms to effectively price discriminate against high cost consumers, which hurts such consumers. Yet,
consumers with low switching costs are often also worse off. They are forced to incur some of their switching costs in order to qualify for the retention offer, even if they do not intend to switch. Moreover, this lowers their effective switching costs, making competition for them less fierce in the first period. As a result, firms benefit from having the possibility of making retention offers.

## 6 Conclusion

In this paper, we studied the practice of retention offers. In a two-period Hotelling model, two firms practice behavior based price discrimination. In the second period, they can try to poach consumers by offering them a better deal. However, firms can retaliate by making a retention offer. Consumers differ in their switching costs. In equilibrium, low-switching-cost consumers always solicit a retention offer, while this is too costly for high-switchingcost consumers. As a result, retention offers allow firms to effectively price discriminate against high-switching-cost consumers.

We find that the possibility of retention offers increases firm profits. All high-cost consumers are worse off, but some low-cost consumers may benefit. Prices increase. From a welfare perspective, more wasteful switching costs are incurred, as all low-cost consumers solicit a costly offer from the competitor in order to secure a retention price.

## Appendix A: Comparing retention offers to prices in a Hotelling model

Corollary 2 In a model with retention offers, compared to a Hotelling model,

1. the total discounted price paid by loyal consumers is lower for low enough $\lambda$ and the comparison is ambiguous otherwise;
2. that paid by consumers that switch is either always lower, or is higher for high enough $\lambda$ and lower otherwise;
3. that paid by consumers that pay the retention price is higher for high enough $\lambda$ and lower otherwise;
4. firms worse off for low enough $\lambda$, and the comparison is ambiguous otherwise;
5. total welfare decreases.

Proof. First consider the loyal consumers. Using (22) and (36),

$$
\begin{aligned}
P_{\text {loyal }}^{\text {ret }} & \equiv p_{1}^{\text {ret }}+\delta p_{\text {loyal }}^{\text {ret }} \\
& =c+1-3 \delta \frac{\tilde{z}(2-\lambda)}{(3-\lambda)^{2}}+\delta \frac{\left(3-\lambda^{2}+\lambda\right)}{(3-\lambda)^{2}}+\delta\left(c+\frac{1}{2}\left(1+z_{H}+b\right)\right) \\
& =P^{h}-3 \delta \frac{\tilde{z}(2-\lambda)}{(3-\lambda)^{2}}+\delta \frac{\left(3-\lambda^{2}+\lambda\right)}{(3-\lambda)^{2}}+\delta\left(\frac{1}{2}\left(z_{H}+b\right)-\frac{1}{2}\right) \\
& =P^{h}+\frac{1}{2} \delta \frac{\lambda\left(16 z_{H}-15 z_{L}^{2}+7\right)-\lambda^{2}\left(6 z_{H}-7 z_{L}^{2}+3\right)-6 z_{H}}{(3-\lambda)^{2}} .
\end{aligned}
$$

With $\lambda=0$, the numerator is $-6 z_{H}<0$. With $\lambda=1$, it is $4 z_{H}-8 z_{L}^{2}+4>0$, which is ambiguous. For consumers that are poached, we have from (22) and (36)

$$
\begin{aligned}
P_{\text {poach }}^{\text {ret }} & \equiv p_{1}^{\text {ret }}+\delta p_{\text {poach }}^{\text {ret }} \\
& =P^{h}-3 \delta \frac{\tilde{z}(2-\lambda)}{(3-\lambda)^{2}}+\delta \frac{\left(3-\lambda^{2}+\lambda\right)}{(3-\lambda)^{2}}+\delta\left(\frac{1-\tilde{z}}{3-\lambda}-1\right) \\
& =P^{h}+\delta \frac{\tilde{z}(4 \lambda-9)+6 \lambda-2 \lambda^{2}-3}{(3-\lambda)^{2}} .
\end{aligned}
$$

For $\lambda=0$, the numerator is $-9 \tilde{z}-3<0$. For $\lambda=1$, it is $1-5 z_{L}^{2}$, which has ambiguous sign. The derivative of the numerator with respect to $\lambda$ is $4 \tilde{z}+6-4 \lambda>0$.

For consumers that pay the retention price, again using (22) and (36),

$$
\begin{aligned}
P_{\text {retent }}^{\text {ret }} & =p_{1}^{\text {ret }}+\delta p_{\text {retent }}^{\text {ret }} \\
& =P^{h}-3 \delta \frac{\tilde{z}(2-\lambda)}{(3-\lambda)^{2}}+\delta \frac{\left(3-\lambda^{2}+\lambda\right)}{(3-\lambda)^{2}}+\delta\left(\frac{1}{2}\left(z_{L}^{2}+\frac{1-\tilde{z}}{3-\lambda}\right)-\frac{1}{2}\right) \\
& =P^{h}+\frac{1}{2} \delta \frac{\lambda\left(22 z_{H}-21 z_{L}^{2}+7\right)-\lambda^{2}\left(7 z_{H}-8 z_{L}^{2}+3\right)+9 z_{L}^{2}-15 z_{H}}{(3-\lambda)^{2}} .
\end{aligned}
$$

For $\lambda=0$, the numerator is $9 z_{L}^{2}-15 z_{H}<0$. For $\lambda=1$ it is $4-4 z_{L}^{2}>0$. The derivative with respect to $\lambda$ is

$$
z_{H}+2 \lambda z_{L}^{2}+7(3-2 \lambda)\left(z_{H}-z_{L}^{2}\right)+7-6 \lambda>0
$$

Profits in a standard Hotelling model are $\Pi^{h}=\frac{1}{2}(1+\delta)$. We thus have

$$
\begin{aligned}
\Pi^{r e t}-\Pi^{h}= & \frac{1}{2}\left(-3 \delta \tilde{z} \frac{(2-\lambda)}{(3-\lambda)^{2}}+\frac{\delta\left(3-\lambda^{2}+\lambda\right)}{(3-\lambda)^{2}}\right)+\frac{\delta}{8} \lambda\left(b+1+z_{L}^{2}\right)^{2} \\
& +\frac{\delta}{8}(1-\lambda)\left(b+1+z_{H}\right)^{2}+\frac{\delta}{4}(2-\lambda) b^{2}-\frac{1}{2} \delta .
\end{aligned}
$$

With $\lambda=0$, we have $\tilde{z}=z_{H}$ and $b=\left(1-z_{H}\right) / 3$, so this expression simplifies to $\delta\left(2\left(z_{H}\right)^{2}-4 z_{H}-1\right) / 18<0$. With $\lambda=1$, we have $\tilde{z}=z_{L}^{2}$ and $b=$ $\left(1-z_{L}^{2}\right) / 2$, so the expression simplifies to $\left(3\left(z_{L}^{2}\right)^{2}-10 z_{L}^{2}+7\right) / 32$, which has ambiguous sign.

This establishes the result on profits. For total welfare, note that prices are just a transfer. With retention offers, however, some consumers incur switching costs, while no longer consuming their preferred product. From a welfare perspective, that is a loss.

## Appendix B: Proofs

## Proof of Theorem 3

To prove the Theorem, we will first establish that $p_{1}^{r e t}>p_{1}^{b m}$ and then go through all cells in the table.
$\mathbf{p}_{1}^{\text {ret }}>\mathbf{p}_{1}^{b m}:$
Consider the difference between $p_{1}^{\text {ret }}$ and $p_{1}^{b m}$ :

$$
\begin{aligned}
\Delta p^{1} \equiv & p_{1}^{r e t}-p_{1}^{b m}= \\
& -3 \delta \frac{\tilde{z}(2-\lambda)}{(3-\lambda)^{2}}+\delta \frac{\left(3-\lambda^{2}+\lambda\right)}{(3-\lambda)^{2}}-\frac{\delta}{3}(1-2 \bar{z}) \\
= & \lambda \delta \frac{6 z_{L}^{1}(3-2 \lambda)-z_{H}(3-5 \lambda)-4 \lambda-2 \lambda^{2}\left(z_{H}-z_{L}\right)+9-3 \lambda z_{L}^{2}}{3(3-\lambda)^{2}} \\
> & \lambda \delta \frac{6 z_{L}^{1}(3-2 \lambda)-(3-5 \lambda)-4 \lambda-2 \lambda^{2}\left(z_{H}-z_{L}\right)+9-3 \lambda}{3(3-\lambda)^{2}} \\
= & \lambda \delta \frac{6 z_{L}^{1}(3-2 \lambda)+6+2 \lambda-2 \lambda^{2}\left(z_{H}-z_{L}\right)}{3(3-\lambda)^{2}}>0 .
\end{aligned}
$$

This establishes the result.

$$
\mathbf{p}_{\text {poach }}^{\text {ret }}>\mathbf{p}_{\text {poach }}^{b m}:
$$

$$
\begin{aligned}
p_{\text {poach }}^{\text {ret }}-p_{\text {poach }}^{b m} & =\frac{1-\tilde{z}}{3-\lambda}-\frac{1}{3}(1-\bar{z}) \\
& >\frac{1}{3}(1-\tilde{z})-\frac{1}{3}(1-\bar{z})=\frac{1}{3} \lambda z_{L}^{1}>0
\end{aligned}
$$

where the first inequality follows from $\lambda>0$. This establishes the result.

$$
\mathbf{p}_{\text {loyal }}^{r e t}>\mathbf{p}_{\text {loyal }}^{b m}:
$$

$$
\begin{aligned}
\Delta p_{\text {loyal }} & \equiv p_{\text {logal }}^{\text {ret }}-p_{\text {loyal }}^{b m}=\frac{1}{2}\left(1+z_{H}+\frac{1-\tilde{z}}{3-\lambda}\right)-\frac{1}{3}\left(2+\tilde{z}+\lambda z_{L}^{1}\right) \\
& >\frac{1}{2}\left(1+z_{H}+\frac{1-\tilde{z}}{3}\right)-\frac{1}{3}\left(2+\tilde{z}+\lambda z_{L}^{1}\right)=\frac{1}{2} z_{H}-\frac{1}{2} \tilde{z}-\frac{1}{3} \lambda z_{L}^{1} \\
& =\frac{1}{6} \lambda\left(3 z_{H}-2 z_{L}^{1}-3 z_{L}^{2}\right)=\frac{1}{6} \lambda\left(3 z_{H}-3 z_{L}+z_{L}^{1}\right)>0
\end{aligned}
$$

as $z_{H}>z_{L}$. This establishes the result.

$$
\mathbf{p}_{\text {poach }}^{\text {ret }}<\mathbf{p}_{\text {loyal }}^{\text {bm }} \text { : }
$$

$$
p_{\mathrm{poach}}^{\mathrm{ret}}-p_{\mathrm{loyal}}^{\mathrm{bm}}=\frac{1-\tilde{z}}{3-\lambda}-\frac{1}{3}(2+\bar{z})<0
$$

as the first term is strictly smaller than $1 / 2$, while the second term is strictly bigger than $2 / 3$. This establishes the result.

$$
\mathbf{p}_{\text {retent }}^{\text {ret }}>\mathbf{p}_{\text {poach }}^{b m}:
$$

Above, we showed that $p_{\text {poach }}^{r e t}>p_{\text {poach }}^{b m}$. From section 4, $p_{\text {retent }}^{r e t}>p_{\text {poach }}^{r e t}$. This establishes the result.

$$
\mathbf{p}_{\text {retent }}^{r e t} \gtrless \mathbf{p}_{\text {loyal }}^{b m}:
$$

$$
\begin{aligned}
p_{\text {retent }}^{r e t}-p_{\text {loyal }}^{\mathrm{bm}} & =\frac{1}{2}\left(1+z_{L}^{2}+\frac{1-\tilde{z}}{3-\lambda}\right)-\frac{1}{3}(2+\bar{z}) \\
& =\frac{\lambda\left(1+11 z_{H}-6 z_{L}^{1}-12 z_{L}^{2}\right)-9\left(z_{H}-z_{L}^{2}\right)-2 \lambda^{2}\left(z_{H}-z_{L}\right)}{6(3-\lambda)} .
\end{aligned}
$$

This expression is positive if and only if the numerator is positive. For $\lambda=0$, it equals $-9\left(z_{H}-z_{L}^{2}\right)<0$. For $\lambda=1$, it equals $2 z_{L}-6 z_{L}^{1}-3 z_{L}^{2}+1=$ $1-z_{L}^{2}-4 z_{L}^{1}$, of which the sign is ambiguous. This establishes the result.
$\mathbf{p}_{\text {loyal }}^{\text {ret }}>\mathbf{p}_{\text {poach }}^{\text {bm }}:$

$$
p_{\mathrm{loyal}}^{\mathrm{ret}}-p_{\mathrm{poach}}^{\mathrm{bm}}=\frac{1}{2}\left(1+z_{H}+\frac{1-\tilde{z}}{3-\lambda}\right)-\frac{1}{3}(1-\bar{z})>0
$$

as the first term is strictly larger than $1 / 2$, while the second is strictly smaller than $1 / 3$. This establishes the result.

## Proof of Theorem 4

We now set about proving Theorem 4. We proceed as follows. First, we compare the total discounted prices that consumers end up paying under different circumstances. These comparisons will prove useful in deriving our results. We then consider how individual consumers are affected, and look at total
welfare. After that, we consider firm profits and total welfare, respectively.

## The effect on total discounted prices

We can establish the following:
Lemma 1 Introducing the possibility of retention offers often increases the total discounted prices paid by consumers. The only exceptions are the case in which a consumer would be loyal in the benchmark, but would either get poached or get a retention offer when retention offers can be made. Such consumers pay a lower total discounted price if $\lambda$ is low enough, but may pay a higher total discounted price if $\lambda$ is high enough.

Summarizing, the effects are as follows:

|  | $P_{\text {poach }}^{b m}$ | $P_{\text {loyal }}^{b m}$ |
| :---: | :---: | :---: |
| $P_{\text {poach }}^{\text {ret }}$ | $>$ | $\gtrless$ |
| $P_{\text {retent }}^{\text {ret }}$ | $>$ | $\gtrless$ |
| $P_{\text {loyal }}^{\text {ret }}$ | $>$ | $>$ |

Proof. Results involving $P_{\text {poach }}^{\mathrm{bm}}$ follow directly from Theorem 3, as does $P_{\text {loyal }}^{\mathrm{ret}}>P_{\text {loyal }}^{\mathrm{bm}}$. Let us now consider the expression $\Delta P_{\mathrm{rr} \text {-bl }} \equiv P_{\text {retent }}^{\mathrm{ret}}-P_{\text {loyal }}^{\mathrm{bm}}$. At $\lambda=0$, we have that $p_{1}^{\text {ret }}=p_{1}^{\mathrm{bm}}$, hence $\Delta P_{\mathrm{rr}-\mathrm{bl}}=p_{\mathrm{retent}}^{\mathrm{ret}}-p_{\mathrm{bm}}^{\text {loyal }}<0 . \Delta P_{\mathrm{rr}-\mathrm{bl}}=$ $27 \delta\left(z_{L}^{2}-z_{H}\right)<0$, while for $\lambda=1$, we have $\Delta P_{\mathrm{rr}-\mathrm{bl}}=\delta\left(z_{L}+3 z_{L}^{1}-3 z_{L}^{2}+6\right) / 12>$ 0 , which implies the statement in the Theorem concerning this case. Finally, consider the expression $\Delta P_{\text {rp-bl }} \equiv P_{\text {poach }}^{\text {ret }}-P_{\text {loyal }}^{\mathrm{bm}}$. It can be shown that for $\lambda=0$, we have that $\Delta P_{\mathrm{rp}-\mathrm{bl}}=-\delta\left(2 z_{H}+1\right) / 3<0$, while for $\lambda=1$, we have $\Delta P_{\text {rp-bl }}=\frac{1}{12} \delta\left(4 z_{L}^{1}-11 z_{L}^{2}+3\right)$, the sign of which is ambiguous.

## The effect on consumer welfare

Lemma 2 The possibility of retention offers makes all consumers strictly worse off, apart possibly from those that pay the retention price in period 2.

Proof. For a single consumer, there are 6 possible options: she is poached both in the benchmark as well as in the scenario with retention offers; she is loyal in both cases, she is poached in the benchmark and loyal in the retention
scenario; she is loyal in the benchmark but poached in the retention scenario; she is loyal in the benchmark, but pays a retention price in the retention scenario, or she is poached in the benchmark and pays a retention price in the retention scenario. We will refer to these 6 options as $P P, L L, P L, L P$, $L R$ and $P R$ respectively. Note that not all 6 options necessarily occur in equilibrium, depending on parameter values, either one may occur.

In all cases, the total discounted price that a consumer ends up paying is a disutility for that consumer. A consumer that is poached in the second period has an additional disutility of, first, the switching costs that she has to incur and, second, the utility mismatch that is caused by the fact that she does no longer consumer her preferred product. A consumer that pays the retention price in the second period has an additional disutility that consist of the additional costs she has to incur to prepare for a switch.

A consumer is worse off with the possibility of retention offers if the total disutility she ends up with then is higher than her total disutility in the benchmark. We will refer to the total disutility in scenario $x$ if a consumer ends up paying a price of type $y$ as $D_{y}^{x}$. Going through all possibilities:
$\mathbf{P P}$ The net difference in disutility in both scenarios equals that in total discounted prices. As $P_{\text {poach }}^{\mathrm{ret}}>P_{\text {poach }}^{\mathrm{bm}}$, we thus have $D_{\text {poach }}^{\mathrm{bm}}<D_{\text {poach }}^{\mathrm{ret}}$.

LL The net difference in disutility in both scenarios equals that in total discounted prices. As $P_{\text {loyal }}^{\text {ret }}>P_{\text {loyal }}^{\mathrm{bm}}$, we thus have $D_{\text {loyal }}^{\mathrm{bm}}<D_{\text {loyal }}^{\mathrm{ret}}$.

PL As this consumer chooses the poaching price in the benchmark, she has $D_{\text {poach }}^{\mathrm{bm}}<D_{\text {loyal }}^{\mathrm{bm}}$. With $D_{\text {loyal }}^{\mathrm{bm}}<D_{\text {loyal }}^{\text {ret }}$, this implies $D_{\text {poach }}^{\mathrm{bm}}<D_{\text {loyal }}^{\text {ret }}$.

LP As this consumer chooses the loyalty price in the benchmark, she has $D_{\text {loyal }}^{\mathrm{bm}}<D_{\text {poach }}^{\mathrm{bm}}$. With $D_{\text {poach }}^{\mathrm{bm}}<D_{\text {poach }}^{\text {ret }}$, this implies $D_{\text {loyal }}^{\mathrm{bm}}<D_{\text {poach }}^{\text {ret }}$.

LR In this case, consider $\Delta D_{\text {rr-bl }} \equiv D_{\text {retent }}^{\mathrm{ret}}-D_{\text {loyal }}^{\mathrm{bm}}=P_{\text {retent }}^{\mathrm{ret}}+\delta z_{L}^{1}-P_{\text {loyal }}^{\mathrm{bm}}$. From the proof of Lemma 1 with $\lambda=0$, we have that $P_{\text {retent }}^{\text {ret }}-P_{\text {loyal }}^{\mathrm{bm}}=$ $27 \delta\left(z_{L}^{2}-z_{H}\right)$, hence $\Delta D_{\text {rr-bl }}=\delta\left(27 z_{L}^{2}+z_{L}^{1}-27 z_{H}\right)<0$. With $\lambda=1$, we have that $P_{\text {retent }}^{\text {ret }}-P_{\text {loyal }}^{\mathrm{bm}}>0$, hence $\Delta D_{\text {rr-bl }}>0$, rendering the net effect ambiguous.

PR In this case, consider $\Delta D_{\text {rr-bp }}(x) \equiv D_{\text {retent }}^{\mathrm{ret}}-D_{\text {poach }}^{\mathrm{bm}}(x)$. As we will see below, it is now important to take into account that the disutility of a consumer that is poached depends on her location, which we denote $x$. We thus have

$$
\begin{aligned}
\Delta D_{\text {rr-bp }}(x) & =P_{\text {retent }}^{\mathrm{ret}}+\delta z_{L}^{F}-P_{\text {poach }}^{\mathrm{bm}}-\delta z_{L}-\delta m(x) \\
& =P_{\text {retent }}^{\mathrm{ret}}-P_{\text {poach }}^{\mathrm{bm}}-\delta z_{L}^{2}-\delta m(x),
\end{aligned}
$$

with $m(x)$ the mismatch of a consumer located at $x \leq 1 / 2$ that gets poached: this consumer's transportation costs are now $1-x$ whereas they would have been $x$ if she consumed her preferred product. Hence $m(x)=1-2 x$. From the proof of Lemma 1, with $\lambda=0$, we have $P_{\text {retent }}^{\mathrm{ret}}-$ $P_{\text {poach }}^{\mathrm{bm}}<0$, hence $\Delta D_{\text {rr-bp }}(x)<0$. With $\lambda=1$, we have $\Delta D_{\text {rr-bp }}(x)=$ $\frac{1}{12} \delta\left(13-6 z_{L}^{2}+12 z_{L}-5 z_{H}\right)-\delta(1-2 x)$, which is ambiguous.

Lemma 3 Total consumer welfare decreases with the possibility of retention offers. This holds both for the high types as well as for the low types.

Proof. For the high types, this follows directly from Lemma 2 (note that high types never pay the retention price). The analysis for the low types is more involved. Consider segment $A$ in the benchmark scenario. Total disutility of the low types that are loyal is given by $\hat{x}_{A L}^{\mathrm{bm}} \cdot D_{\text {loyal }}^{\mathrm{bm}}=$ $\hat{x}_{A L}^{\mathrm{bm}} \cdot P_{\text {loyal }}^{\mathrm{bm}}$. Total disutility of the low types that are poached first consists of $\left(\frac{1}{2}-\hat{x}_{A L}^{\mathrm{bm}}\right)\left(P_{\text {poach }}^{\mathrm{bm}}+\delta z_{L}\right)$, as these consumers pay $P_{\text {poach }}^{\mathrm{bm}}$ and also incur switching costs in the second period. Moreover, each of these consumers incurs a mismatch: her transportation costs are now $1-x$ whereas they would have been $x$ if she consumed her preferred product. Hence $m(x)=1-2 x$, and the total size of this mismatch equals

$$
M_{A L}^{\mathrm{bm}}=\int_{\hat{x}_{A L}^{\mathrm{bm}}}^{1 / 2} m(x) d x=\left(\frac{1}{2}-\hat{x}_{A L}^{\mathrm{bm}}\right)^{2} .
$$

Hence total disutility of the low types in the benchmark is given by

$$
D_{L}^{\mathrm{bm}} \equiv 2 \hat{x}_{A L}^{\mathrm{bm}} \cdot P_{\text {loyal }}^{\mathrm{bm}}+2\left(\frac{1}{2}-\hat{x}_{A L}^{\mathrm{bm}}\right)\left(P_{\mathrm{poach}}^{\mathrm{bm}}+\delta z_{L}\right)+2\left(\frac{1}{2}-\hat{x}_{A L}^{\mathrm{bm}}\right)^{2} .
$$

Along the same lines, with the possibility of retention offers, it is given by

$$
D_{L}^{\mathrm{ret}} \equiv 2 \hat{x}_{A L}^{\mathrm{ret}} \cdot P_{\mathrm{retent}}^{\mathrm{ret}}+2\left(\frac{1}{2}-\hat{x}_{A L}^{\mathrm{ret}}\right)\left(P_{\mathrm{poach}}^{\mathrm{ret}}+\delta z_{L}\right)+2\left(\frac{1}{2}-\hat{x}_{A L}^{\mathrm{bm}}\right)^{2}
$$

Figure 2: Effect on disutility of low types of the possibility of retention offers.


The figure gives the upper and lower bound of the effect of the possibility of retention offers on total disutility of the low types, as a function of lambda.

It turns out to be impossible to compare these two expressions analytically. We therefore resort to a numerical analysis. For all values of $\lambda$, Figure 2 gives the upper and the lower bound on the net welfare effects for the low
types of the possibility of having retention offers (thus on $D_{L}^{\text {ret }}-D_{L}^{\mathrm{bm}}$ as defined above), for all admissible values of $z_{L}^{1}, z_{L}^{2}, z_{H}^{1}$, and $z_{H}^{2}{ }^{11}$

From the figure, we have that welfare of the low types may improve with retention offers for low enough $\lambda$. Only for those $\lambda$, we saw that the low types that buy from A in both scenarios do pay a lower price under retention, while the number of low types that gets poached decreases, lowering their costs of mismatch. For low $\lambda$, these positive effects outweigh the negative effects of a higher poaching price and costs to secure a competing offer.

Note that we have not weighted the loss by the number of low type consumers, which makes the graph easier to read.

## The effect on profits

First note that equilibrium profits would obviously increase if all total discounted prices in the retention scenario would be higher than those in the benchmark. Unfortunately, that is not the case. From Lemma 1, consumers may end up paying a lower price if they are loyal in the benchmark, but are either poached or pay the retention price in the case where retention offers are possible.

With a unit mass of consumers that always buy in equilibrium, finding the scenario with the highest profit is equivalent to finding the scenario with the highest average price. Focusing on segment $A$ without loss of generality, we have that the average price paid in the benchmark is given by

$$
\begin{aligned}
\bar{P}^{\mathrm{bm}} \equiv & 2 \lambda \hat{x}_{A L}^{\mathrm{bm}} \cdot P_{\text {loyal }}^{\mathrm{bm}}+2 \lambda\left(\frac{1}{2}-\hat{x}_{A L}^{\mathrm{bm}}\right) P_{\mathrm{poach}}^{\mathrm{bm}} \\
& +2(1-\lambda) \hat{x}_{A H}^{\mathrm{bm}} \cdot P_{\text {loyal }}^{\mathrm{bm}}+2(1-\lambda)\left(\frac{1}{2}-\hat{x}_{A H}^{\mathrm{bm}}\right) P_{\mathrm{poach}}^{\mathrm{bm}} .
\end{aligned}
$$

[^6]With the possibility of retention, it is given by

$$
\begin{aligned}
\bar{P}^{\mathrm{ret}} \equiv & 2 \lambda \hat{x}_{A L}^{\mathrm{ret}} \cdot P_{\mathrm{retent}}^{\mathrm{ret}}+2 \lambda\left(\frac{1}{2}-\hat{x}_{A L}^{\mathrm{ret}}\right) P_{\mathrm{poach}}^{\mathrm{ret}} \\
& +2(1-\lambda) \hat{x}_{A H}^{\mathrm{ret}} \cdot P_{\text {loyal }}^{\mathrm{ret}}+2(1-\lambda)\left(\frac{1}{2}-\hat{x}_{A H}^{\mathrm{ret}}\right) P_{\mathrm{poach}}^{\mathrm{ret}} .
\end{aligned}
$$

It turns out to be impossible to compare these two expressions analytically. We therefore resort to a numerical analysis. For all values of $\lambda$, Figure 3 gives the upper and the lower bound on the price effect of the possibility of having retention offers (thus on $\bar{P}^{\text {ret }}-\bar{P}^{\text {bm }}$, as defined above), for all admissible values of the parameters $z_{L}^{1}, z_{L}^{2}, z_{H}^{1}$, and $z_{H}^{2}$, using an analysis very similar to that described above.

Figure 3: Effect on average price of the possibility of making retention offers.


The figure gives the upper and lower bound of the effect of the possibility of retention offers on average prices paid in equilibrium, as a function of lambda.

From the figure, it is immediate that average prices with retention are always higher than those in the benchmark. As $\lambda$ approaches zero, the price difference disappears. This is intuitive: with $\lambda=0$, the number of low types is zero, so no retention offers will be made, rendering the case with the possibility of retention offers identical to the benchmark.

Figure 4: Effect on welfare of the possibility of making retention offers.


The figure gives the upper and lower bound of the effect of the possibility of retention offers on total welfare, as a function of lambda.

## The effect on total welfare

Figure 4 reports on an analysis that is very similar to that in Figures 2 and 3, but now for total welfare.

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[^1]:    ${ }^{1}$ To have $\hat{x}_{A i}^{2}>0$, we need $\left(2+3 z_{i}-2 \bar{z}\right)>0$, hence $2 \bar{z}<2+3 z_{i}$. We want this to be satisfied for all $\lambda$. It is most restrictive for $\lambda=0$, in which case it yields

    $$
    \begin{equation*}
    2 z_{H}<2+3 z_{i} . \tag{5}
    \end{equation*}
    $$

    For the high types, this is always satisfied. For the low types, it requires $2 z_{H}-3 z_{L}<2$.
    In a symmetric equilibrium, we will have $\hat{x}_{i}^{1}=1 / 2$. For the second period, we thus need $\hat{x}_{A i}^{2}<1 / 2$, hence $2+3 z_{i}-2 \bar{z}<3$, so $3 z_{i}-2 \bar{z}<1$. We want this to be satisfied for all $\lambda$. It is most restrictive for $\lambda=1$, in which case it yields $3 z_{i}-2 z_{L}<1$. For the low types, this requires (1). For the high types, it requires $3 z_{H}-2 z_{L}<1$, or (2). Note that if this is satisfied, (5) is satisfied as well.
    ${ }^{2}$ We will show below that $\hat{x}_{H}^{1}=\hat{x}_{L}^{1}$, as is depicted in the figure.

[^2]:    ${ }^{3}$ The second-period profit functions (6) and (7) are clearly concave - provided that firms set prices such that the indifferent high and low type consumers are both strictly between 0 and $1 / 2$ in equilibrium. Yet, it may still be profitable to do a large defection. We will show that that is not the case. As in the main text, we focus on segment $A$.

    First consider firm $B$. It can defect to a price $p_{B A}^{2}$ that is so high that it only sells to the low types. In equilibrium, that requires setting $p_{B A}^{2}$ such that $\hat{x}_{A H} \geq 1 / 2$, or $1+p_{B A}^{2}-p_{A A}^{2}+z_{H} \geq 1$. This implies setting $p_{B A}^{2}=p_{A A}^{2}-z_{H}$. Its profits are then given by

    $$
    \pi_{B L}^{2}=\lambda\left(p_{B A}^{2}-c\right)\left[\frac{1}{2}-\frac{1}{2}\left(1+p_{B A}^{2}-p_{A A}^{2}+z_{L}\right)\right]
    $$

[^3]:    ${ }^{4}$ Note from (18) that $\partial \hat{x}_{1} / \partial\left(p_{B}^{1}-p_{A}^{1}\right)=3 /(6+2 \delta)$, whereas in a standard Hotelling model, we would have $\partial \hat{x} / \partial\left(p_{B}-p_{A}\right)=1 / 2$.
    ${ }^{5}$ From (12), an increase in $\hat{x}_{1}$, the size of segment $A$, implies that both $p_{A A}^{2}$ and $p_{B A}^{2}$ increase.
    ${ }^{6}$ From (19), in equilibrium $\partial \Pi_{A} / \partial \bar{z}=\delta(1+2 \bar{z}) / 9>0$.

[^4]:    ${ }^{8}$ It is readily checked that the second-order condition is satisfied as well.

[^5]:    ${ }^{10}$ See Appendix A for details

[^6]:    ${ }^{11}$ The analysis was done in MATLAB. For each of 100 values of $\lambda$ between 0 and 1, we considered 50 values of $z_{L}^{1}, z_{L}^{2}, z_{H}^{1}$, as well as $z_{H}^{2}$ to find the highest and the lowest possible value of the price effect of retention offers, taking into account the conditions that have to be satisfied by our switching cost parameters (thus: $z_{L}^{1}<z_{H}^{1} ; z_{L}^{2}<z_{H}^{2}$, and conditions (1)-(3), (38) and (39)). The analysis took 21 minutes on a 3.30 GHz 4 GB RAM Windows $7 \overline{\mathrm{PC}}$. The MATLAB code is available upon request. Looking at a finer grid did not appreciably affect the outcomes. Note that in all figures, we have taken $\delta=1$. The size of $\delta$ does not affect the qualitative analysis, however, as all comparisons we consider are proportional to $\delta$.

