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Opportunistic condition-based maintenance and aperiodic inspections for a two-unit series system

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Abstract

Condition-Based Maintenance (CBM) intends to perform maintenance right before a failure occurs by estimating the pending moment of failure based on monitoring a certain condition, such as vibration or temperature. This paper considers a two-unit series system with economic dependencies. The aperiodic inspection moments are optimized simultaneously with the critical levels at which maintenance is performed in order to minimize cost and/or maximize availability. For this purpose, a stochastic model is developed based on semi-regenerative properties of the maintained system state. We build on the work of Castanier, B., A. Grall, and C. Bérenguer (2005), A condition-based maintenance policy with non-periodic inspections for a two-unit series system, *Reliability Engineering & System Safety 87*(1), 109–120, by fully including all opportunistic maintenance opportunities, determining the system unavailability time more accurately, and providing a more extensive performance evaluation. Results indicate that the accuracy with which the unavailability time is approximated has a great impact on the resulting optimal maintenance strategy.

Keywords: Condition-based maintenance, Multi-unit system, Economic dependencies, Aperiodic inspections, Reliability

1. Introduction

Technical systems are often subject to increasing wear and tear caused by usage, age or random shocks. If ignored, this deterioration may eventually cause a system breakdown, which can lead to high costs, system unavailability and safety hazards. Performing preventive or predictive maintenance can help to prevent failures and their corresponding detriments by repairing or replacing a component before a system breakdown occurs [1]. One type of a predictive maintenance strategy is condition-based maintenance (CBM), which intends to perform maintenance right on time, so just before a failure occurs. The concept of CBM is to monitor a certain condition of the equipment, such as vibration or temperature, and to initiate a maintenance action as soon as the condition reaches a prespecified threshold value. Compared to classical

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Nomenclature

α_i	(Rate) parameter of the Exponential and Erlang distributions f_i and $f_i^{(l)}$
$A(t)$	System availability up to time t
A_∞	Long-run average system availability
$C(t)$	Cumulative operating costs up to time t
C_∞	Long-run average operating costs per time unit
$c_c^{(i)}$	Cost of a corrective replacement for component i
$C_C^{(i)}(t)$	Cumulative corrective replacement costs of component i up to time t
c_d	Unavailability cost rate, per time unit that the system is unavailable
c_n	Cost of a system inspection
$c_p^{(i)}$	Cost of a preventive replacement for component i
$C_P^{(i)}(t)$	Cumulative preventive replacement costs of component i up to time t
c_s	Fixed set-up costs for a replacement
$C_S(t)$	Cumulative set-up costs up to time t
$C_U(t)$	Cumulative system unavailability costs up to time t
$\Delta_{(k,k+1)}X^i$	Degradation increment, $\Delta_{(k,k+1)}X^i = X^i_{k+1} - X^i_k$
$D_U(t)$	Cumulative system unavailability time up to time t
$\hat{D}_U(t)$	Upper bound for $D_U(t)$
$\tilde{D}_U(t)$	Linear approximation of $D_U(t)$
$\mathrm{E}_\pi[\cdot]$	Expected value with respect to the stationary law π
$f_i(\cdot)$	Pdf of the deterioration increments of component i
$f_i^{(l)}(\cdot)$	Pdf of the cumulative deterioration increments over l time units for component i
$F_i(y,l)$	Probability that component i will not fail during the first l time units starting with a deterioration level of y
$h_i(m \mid y)$	Pdf of a failure at time m (under the assumption of a linear increase in deterioration), given a previous deterioration level of y for component i
L_i	Failure-level of component i
n	Number of inspection threshold values for each component
$\pi(x_1, x_2)$	Long-run probability that components 1 and 2 are in states x_1 and x_2, respectively, at the start of an inspection
q	Binary variable indicating whether the system unavailability time is approximated using the upper bound ($q = 0$) or the linear increase in deterioration ($q = 1$)
S	Length of a semi-regeneration cycle in steady state
X_k^i	Condition of component i at time k
$\xi_j^{(i)}$	Inspection thresholds for component i ($j = 0, 1, \ldots, n-1$)
$\xi_n^{(i)}$	Preventive replacement threshold for component i
ζ_i	Opportunistic replacement threshold for component i

maintenance policies, CBM is more efficient [1, 2], since it can postpone maintenance activities, while failures are limited due to the constant monitoring of the condition. Furthermore, CBM has been proved to minimize maintenance costs, improve operational safety, and reduce the number and severity of failures [3].

Over the past decades, a lot of research has been performed in the field of maintenance strategies. A number of surveys have been written, e.g., [4–6]. However, most of this research considers preventive maintenance strategies rather than predictive maintenance strategies such as CBM. Furthermore, most existing literature on CBM focuses on a system consisting of just one component. In case a system consists of two or more components, it does not necessarily hold true that the optimal decision for one component is optimal for the complete system [7]. This depends on the types of dependencies for systems consisting of multiple components: economic dependence, structural dependence, and probabilistic (or stochastic) dependence [8]. In this paper, we focus on the first, which applies when combining maintenance actions on several components yields a lower cost than maintaining each component separately. This is for example the case when shared set-up costs are involved. Since these fixed costs are independent of the number of components that require maintenance, it can be profitable to opportunistically replace components when another component requires immediate maintenance.

Few articles do consider CBM for systems consisting of multiple components subject to economic dependencies [7, 9, 10]. In [9], dynamic maintenance grouping is applied to optimize the maintenance costs on a rolling horizon for a system with periodic inspections. In [10], a CBM policy is proposed based on the proportional hazards model, where the periodic inspection moments are fixed in advance. A CBM policy for a two-unit series system with aperiodic inspections is developed in [7], where the maintenance costs are obtained by using semi-regenerative properties of the maintained system state. The latter model is very advanced, in that the inspection moments and the maintenance thresholds are optimized simultaneously.

In practice, the critical level at which preventive maintenance is initiated is usually based on recommendations of suppliers and manufacturers of condition-monitoring equipment rather than on incentives to save costs or improve reliability [11]. Justifications for the selected inspection moments are also frequently lacking, and usually based on a simple rule-of-thumb. Due to safety concerns, the critical level is likely to be set too low, while inspections may be scheduled more often than actually required. To overcome these problems, most existing literature on CBM considers either a model that optimizes the critical level at which a preventive replacement should be initiated given the (periodic) inspection intervals, or a model that optimizes the inspection intervals given the critical level. Besides Castanier et al. [7], joint optimization has only be studied in [11–13]. A CBM model based on the random coefficient growth model is described in [11], while a multi-threshold CBM policy is considered in [7, 12, 13]. In fact, [7] and [13] both extend the model of [12], where a CBM policy is developed for a system consisting of one component and failures are noticed immediately. Partial repairs and durations of maintenance activities are considered in [13], assuming that failures can only be noticed upon inspection. A system consisting of two components that are functioning in series is analyzed in [7].

In this paper, we extend the work of Castanier et al. [7] that is unique in its simultaneous optimization of inspection and maintenance decisions for a system consisting of multiple components. We determine the unavailability costs more accurately and include all possibilities for opportunistic replacements. In addition, we consider availability as an additional performance criterion and we perform a more extensive comparative cost assessment. The remainder of this article is organized as follows. Section 2 describes the deterioration model, followed by the multi-threshold maintenance policy in Section 3. Next, the definition and evaluation of the performance criteria are given in Section 4, while Section 5 contains the performance assessments and a comparison with classical maintenance policies. Section 6 concludes the paper.

2. System description

2.1. Deterioration model

The discrete-time system under consideration consists of two components, which independently suffer from increasing wear. The condition of component i at time k can be described by a random variable X_k^i, for $i = 1, 2$ and $k \in \mathbb{N}$. A failure of component i will occur as soon as its deterioration level exceeds a preset failure level L_i, but can only be noticed upon inspection. This means that the component remains in the failed state until the next planned inspection.

An inspection can be performed at the start of each time unit. In other words, the time between possible inspection moments serves as the time unit. Note that, for $i = 1, 2$, at the start of the process (at time 0) and after each replacement (say at time t_r), component i is assumed to be as good as new, i.e., $X_0^i = 0$ and $X_{t_r}^i = 0$. The degradation of the global system is given by $(X_k)_{k \in \mathbb{N}} = (X_k^1, X_k^2)_{k \in \mathbb{N}}$. Since we assume that degradation will increase over time, we require the random deterioration increments $\Delta_{(k,k+1)} X^i$ in each time interval to be nonnegative. In addition, we assume that the increments are stationary and exchangeable, so the degradation increments satisfy the memoryless property.

Let f_i denote the probability density function of the deterioration increments $\Delta_{(k,k+1)} X^i$ of component i, for $i = 1, 2$ and for all $k \in \mathbb{N}$. From the assumption that the deterioration increments are stationary and exchangeable, it follows that the distribution functions f_i are infinitely divisible [14]. This is for example the case for all gamma distributions. The property of infinitely divisible increments makes a gamma process suitable for describing deterioration caused by continuous use [15]. In particular, it is very suitable for describing the steady evolution of wear between the start-up period and the wear accumulation at the end of the system's life [6]. As in [7], we select f_i as the exponential distribution with rate parameter α_i for component i, $i = 1, 2$. Since the sum of l exponential distributions with parameter α_i (which is the increase in deterioration during l time units) follows an Erlang distribution with parameters α_i and l, it follows immediately that $f_i^{(l)}$ is an Erlang distribution with parameters α_i and l.

2.2. Maintenance actions and corresponding costs

At each time unit, a decision is needed on whether or not to perform an inspection, while at each inspection, a decision is needed on what components to replace. Obviously, failed components should be replaced correctively. Moreover, a functioning component may also be replaced preventively if it is close to failure, or opportunistically if the other component is replaced as well and simultaneous replacements are cost efficient. In practice, shared set-up costs can arise from traveling to the right location, scheduling personnel, ordering spare parts, or doing paperwork. The costs of the different maintenance operations are given in Table 1.

Table 1: An overview of the costs associated with the different maintenance operations.

Maintenance action	Corresponding costs (for item i)
Inspection (both items together)	c_n
Corrective replacement (per item)	$c_c^{(i)}$
Preventive replacement (per item)	$c_p^{(i)}$
Shared set-up cost replacement	c_s

It is realistic to assume that the fixed cost of an inspection (planning, transportation, and possibly a shutdown) are relatively large compared to the variable cost depending on the number of components that is inspected, as no material/repair costs are involved. For this reason, we assume that all components are inspected at every inspection, i.e., that there are common inspection moments, at which a cost c_n is incurred. Since failures can only be noticed upon inspection, the system may spend some time in the failed state before it will be inspected and maintained. In such a case, the so-called unavailability cost rate c_d is incurred per unit of time that the system is unavailable. The amount of time that a component is unavailable cannot be measured exactly, because failures are only noticed upon inspection. As discussed in detail later, an upper bound for the unavailability time is used in [7], while we also present a more accurate approximation and show that this significantly affects the results.

2.3. Performance criteria

Important performance criteria for the multi-threshold maintenance policy are the long-run average maintenance costs per period and the long-run average availability. Whereas only the former is considered in [7], we consider both. The multi-threshold maintenance policy aims at optimizing all threshold values simultaneously in order to minimize the maintenance costs, or to maximize the availability.

3. Multi-threshold maintenance policy

Let us define $\xi_0^{(i)}, \xi_1^{(i)}, \ldots, \xi_n^{(i)}$ (with $\xi_0^{(i)} \leq \xi_1^{(i)} \leq \ldots \leq \xi_n^{(i)} \leq L_i$) as the threshold values of component i, where n denotes the fixed number of inspection threshold values for each component. The lowest threshold value is that of a new component, and it is normalized to zero, i.e., $\xi_0^{(i)} := 0$. The notation $\xi_0^{(i)}$ is not strictly needed but will turn out to be helpful in representing the maintenance policy. At the moment, say k, of an inspection, the deterioration level X_k^i of each component i is revealed. Comparison of these levels with the threshold values then indicates whether or not a replacement should be performed for each item, and when to perform the next inspection. Exactly how this is done is described in detail in what remains of this section, in three steps.

Step 1: Corrective and preventive replacements

In this step, we determine for each component separately whether it requires maintenance. Inspect component i ($i = 1, 2$), and compare the deterioration level X_k^i observed with the threshold values. This reveals the state of the component.

If $0 \leq X_k^i < \xi_n^{(i)}$**:** Component i does not require maintenance yet.

If $\xi_n^{(i)} \leq X_k^i < L_i$**:** Immediately replace component i preventively.

If $X_k^i \geq L_i$**:** Component i has failed. Replace component i correctively.

Step 2: Opportunistic replacements

If the first step indicates that one component must be replaced, and so set-up costs have to be paid anyway, then it may be cost efficient to replace the other component as well. We therefore introduce additional threshold values ζ_i (with $0 \leq \zeta_i \leq \xi_n^{(i)}$ for $i = 1, 2$) and the following opportunistic replacement strategy.

If $0 \leq X_k^i < \zeta_i$**:** Component i does not require maintenance yet.

If $\zeta_i \leq X_k^i < \xi_n^{(i)}$ **and** $X_k^j \in [\xi_n^{(j)}, \infty)$ **(**j **requires a replacement) for** $j \neq i$**:** Replace component i preventively (opportunistically).

Step 3: Next inspection moment

The third step consists of determining the next inspection moment. The following policy is used to determine when the next inspection should be scheduled, taking into account the decisions taken in steps 1 and 2. After component i has been replaced, it is as good as new again, and X_k^i becomes zero ($i = 1, 2$).

If $\xi_{l_1}^1 \leq X_k^1 < \xi_{l_1+1}^1$ **and** $\xi_{l_2}^2 \leq X_k^2 < \xi_{l_2+1}^2$ **for** $l_1, l_2 \in \{0, 1, \ldots, n-1\}$**:** The system needs to be inspection again at $\min\{n - l_1, n - l_2\}$ periods from now.

So the system will be inspected more often when one of its components is approaching failure.

In Figure 1, an example of the wear patterns and corresponding inspection and maintenance actions of a two-unit series system is given. Here $n = 2$, implying that the next inspection is always scheduled at most two time units later. At time 0, both components are as good as new, and the next inspection is scheduled two time units later. As soon as at least one component exceeds its inspection threshold $\xi_1^{(i)}$, the next inspection is scheduled one time unit later. At times 6 and 13, one of the components requires a preventive replacement, while the other component is replaced opportunistically. This results in a complete system replacement. Furthermore, at time 10, only component 1 is replaced, since the deterioration level of component 2 does not exceeds its opportunistic replacement threshold ζ_2.

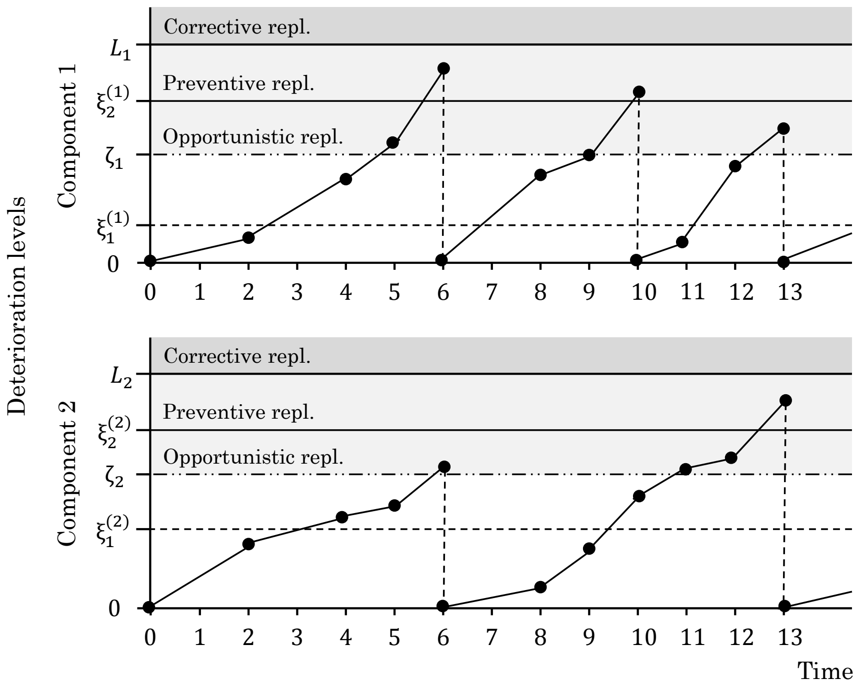

Figure 1: Example evolution of the wear patterns of two components under the multi-threshold maintenance policy with $n = 2$.

4. Evaluation of the performance criteria

The deterioration process contains (semi-)regenerative properties, implying that the costs (and system availability) can be evaluated during a single inspection cycle [7, 13]. To this end, a stationary law $\pi(x_1, x_2)$ needs to be constructed, denoting the probability density function of being in state (x_1, x_2) at the start of an inspection.

4.1. Stationary law

The stationary law $\pi(x_1, x_2)$ can be obtained as the probability density of being in state (y_1, y_2) multiplied by the probability of moving from state (y_1, y_2) to state (x_1, x_2) during an inspection cycle, integrated over all possible states (y_1, y_2). Since these probabilities depend on whether or not components 1 and/or 2 are replaced after the previous inspection, we distinguish four cases:

Both components replaced. If $y_1 \in [\xi_n^1, \infty)$ and $y_2 \in [\zeta_2, \infty)$ or if $y_1 \in [\zeta_1, \xi_n^1)$ and $y_2 \in [\xi_n^2, \infty)$, then both components are replaced. Moreover, the next inspection is scheduled n periods later, which implies that the increases in deterioration levels from 0 to x_1 and from 0 to x_2 have densities of $f_1^{(n)}(x_1)$ and $f_2^{(n)}(x_2)$, respectively.

Only component 1 replaced. If $y_1 \in [\xi_n^1, \infty)$ and $y_2 < \zeta_2$, then only component 1 will be replaced and hence start with a deterioration level of zero. Moreover, if $y_2 \in [\xi_k^2, \xi_{k+1}^2)$ with $k \in \{0, 1, \ldots, n-1\}$, then the next inspection is scheduled $n-k$ periods later. The increases in deterioration levels from 0 to x_1 and from y_2 to x_2 have densities of $f_1^{(n-k)}(x_1)$ and $f_2^{(n-k)}(x_2 - y_2)$, respectively.

Only component 2 replaced. If $y_2 \in [\xi_n^2, \infty)$ and $y_1 < \zeta_1$, then only component 2 will be replaced and hence start with a deterioration level of zero. Moreover, if $y_1 \in [\xi_k^1, \xi_{k+1}^1)$ with $k \in \{0, 1, \ldots, n-1\}$, then the next inspection is scheduled $n-k$ periods later. The increases in deterioration levels from y_1 to x_1 and from 0 to x_2 have densities of $f_1^{(n-k)}(x_1 - y_1)$ and $f_2^{(n-k)}(x_2)$, respectively.

No replacement. If $y_1 \in [\xi_k^1, \xi_{k+1}^1)$ and $y_2 \in [\xi_l^2, \xi_{l+1}^2)$, with $k, l \in \{0, 1, \ldots, n-1\}$, then no replacement will be performed, and the next inspection is scheduled $\min\{n-k, n-l\} = n - \max\{k, l\}$ periods later. The increases in deterioration levels from y_1 to x_1 and from y_2 to x_2 have densities of $f_1^{(n-\max\{k,l\})}(x_1 - y_1)$ and $f_2^{(n-\max\{k,l\})}(x_2 - y_2)$, respectively.

Using this information, the probability law $\pi(x_1, x_2)$ can be constructed as follows.

$$\begin{aligned}
\pi(x_1, x_2) \quad = \quad & \left(\int_{\xi_n^1}^{\infty} \int_{\zeta_2}^{\infty} \pi(y_1, y_2) \mathrm{d}y_2 \mathrm{d}y_1 + \int_{\zeta_1}^{\xi_n^1} \int_{\xi_n^2}^{\infty} \pi(y_1, y_2) \mathrm{d}y_2 \mathrm{d}y_1 \right) f_1^{(n)}(x_1) f_2^{(n)}(x_2) \\
& + \sum_{k=0}^{n-1} \int_{\xi_n^1}^{\infty} \int_{\min\{\xi_k^2, \zeta_2\}}^{\min\{\xi_{k+1}^2, \zeta_2\}} \pi(y_1, y_2) f_1^{(n-k)}(x_1) f_2^{(n-k)}(x_2 - y_2) \mathrm{d}y_2 \mathrm{d}y_1 \\
& + \sum_{k=0}^{n-1} \int_{\min\{\xi_k^1, \zeta_1\}}^{\min\{\xi_{k+1}^1, \zeta_1\}} \int_{\xi_n^2}^{\infty} \pi(y_1, y_2) f_1^{(n-k)}(x_1 - y_1) f_2^{(n-k)}(x_2) \mathrm{d}y_2 \mathrm{d}y_1 \\
& + \sum_{k=0}^{n-1} \sum_{l=0}^{n-1} \int_{\xi_k^1}^{\xi_{k+1}^1} \int_{\xi_l^2}^{\xi_{l+1}^2} \pi(y_1, y_2) f_1^{(n-\max\{k,l\})}(x_1 - y_1) f_2^{(n-\max\{k,l\})}(x_2 - y_2) \mathrm{d}y_2 \mathrm{d}y_1
\end{aligned}$$

This expression can be rewritten as a nonhomogeneous linear Fredholm integral equation of the second kind:

$$\pi(x_1, x_2) \quad = \quad F(x_1, x_2) + \int \int_S \pi(y_1, y_2) K(x_1, x_2, y_1, y_2) \mathrm{d}y_2 \mathrm{d}y_1,$$

where

$$F(x_1,x_2)=f_1^{(n)}(x_1)f_2^{(n)}(x_2),$$

$$S_i^1=\left\{(y_1,y_2):y_1\in[\min\{\xi_i^1,\zeta_1\},\min\{\xi_{i+1}^1,\zeta_1\})\cap y_2\in[\xi_n^2,\infty)\right\},\ \text{ for } i=0,1,\ldots,n-1,$$

$$S_i^2=\left\{(y_1,y_2):y_1\in[\xi_n^1,\infty)\cap y_2\in[\min\{\xi_i^2,\zeta_2\},\min\{\xi_{i+1}^2,\zeta_2\})\right\},\ \text{ for } i=0,1,\ldots,n-1,$$

$$S_i^3=\left\{(y_1,y_2):\left(y_1\in[0,\xi_i^1)\cap y_2\in[\xi_i^2,\xi_{i+1}^2)\right)\cup\left(y_1\in[\xi_i^1,\xi_{i+1}^1)\cap y_2\in[0,\xi_{i+1}^2)\right)\right\},\ \text{ for } i=0,1,\ldots,n-1,$$

$$S=\bigcup_{i=0}^{n-1}\left(S_i^1\cup S_i^2\cup S_i^3\right),$$

$$K(x_1,x_2,y_1,y_2)=\begin{cases} f_1^{(n-i)}(x_1-y_1)f_2^{(n-i)}(x_2)-F(x_1,x_2) & \text{if } (y_1,y_2)\in S_i^1, i=0,1,\ldots,n-1,\\ f_1^{(n-i)}(x_1)f_2^{(n-i)}(x_2-y_2)-F(x_1,x_2) & \text{if } (y_1,y_2)\in S_i^2, i=0,1,\ldots,n-1,\\ f_1^{(n-i)}(x_1-y_1)f_2^{(n-i)}(x_2-y_2)-F(x_1,x_2) & \text{if } (y_1,y_2)\in S_i^3, i=0,1,\ldots,n-1. \end{cases}$$

The densities $f_i^{(l)}$ are regular [7], which implies that we can solve the equation above by applying the method of successive approximations [16]. Due to the high level of complexity, we will approximate the integrals numerically in our experiments in Section 5 by applying the extended midpoint rule [17] to the two-dimensional case, by dividing the area into 30×30 parts. Note that we cannot use infinite upper bounds while using the extended midpoint rule, but instead assume that the deterioration level of component i will never exceed a value of 1.5 times L_i, $i=1,2$. Initial testing showed that this value is sufficiently large to ensure that the results are not affected.

4.2. Long-run average cost per time unit and system availability

Let $C(t)$ denote the cumulative operating costs up to time t, consisting of costs arising from maintenance activities and down-time of the system, and let $A(t)$ denote the total system availability up to time t. Because of the semi-regenerative properties of the multi-threshold maintenance policy, we can consider the long-run average maintenance costs $C(S)$ during an inspection cycle, with length S. These costs can then be divided by the long-run average length of an inspection cycle to obtain the long-run average maintenance costs per period. Similar logic applies to the availability criterion. In the following, let $\mathrm{E}_\pi[\cdot]$ denote the expected value with respect to the stationary law π. The different components of the cost and availability function are specified below.

The long-run average length of an inspection cycle can be obtained by again distinguishing four different cases; both components replaced, only component 1 replaced, only component 2 replaced, and no replacement, respectively:

$$\mathrm{E}_\pi[S] \quad = \quad n\left(\int_{\xi_n^1}^{\infty}\int_{\zeta_2}^{\infty}\pi(x_1,x_2)\mathrm{d}x_2\mathrm{d}x_1+\int_{\zeta_1}^{\xi_n^1}\int_{\xi_n^2}^{\infty}\pi(x_1,x_2)\mathrm{d}x_2\mathrm{d}x_1\right)$$

$$
\begin{aligned}
&+\sum_{k=1}^{n} k\left[\int_{\xi_n^1}^{\infty}\int_{\min\{\xi_{n-k}^2,\zeta_2\}}^{\min\{\xi_{n-k+1}^2,\zeta_2\}}\pi(x_1,x_2)\mathrm{d}x_2\mathrm{d}x_1\right.\\
&+\int_{\min\{\xi_{n-k}^1,\zeta_1\}}^{\min\{\xi_{n-k+1}^1,\zeta_1\}}\int_{\xi_n^2}^{\infty}\pi(x_1,x_2)\mathrm{d}x_2\mathrm{d}x_1\\
&\left.+\int_{\xi_{n-k}^1}^{\xi_{n-k+1}^1}\int_{0}^{\xi_{n-k+1}^2}\pi(x_1,x_2)\mathrm{d}x_2\mathrm{d}x_1+\int_{0}^{\xi_{n-k}^1}\int_{\xi_{n-k}^2}^{\xi_{n-k+1}^2}\pi(x_1,x_2)\mathrm{d}x_2\mathrm{d}x_1\right].
\end{aligned}
$$

Furthermore, the long-run average inspection costs per semi-regeneration cycle, which are incurred exactly once per inspection cycle, are equal to c_n. The long-run average preventive maintenance costs of component i per inspection cycle (excluding the set-up costs) are given by

$$
\mathrm{E}_\pi[C_P^{(i)}(S)] = c_p^{(i)}\left(\int_{\xi_n^i}^{L_i}\int_0^{\infty}\pi(x_1,x_2)\mathrm{d}x_j\mathrm{d}x_i+\int_{\zeta_i}^{\xi_n^i}\int_{\xi_n^j}^{\infty}\pi(x_1,x_2)\mathrm{d}x_j\mathrm{d}x_i\right),
$$

while the long-run average corrective maintenance costs of component i per inspection cycle (excluding the set-up costs) can be obtained as

$$
\mathrm{E}_\pi[C_C^{(i)}(S)] = c_c^{(i)}\int_{L_i}^{\infty}\int_0^{\infty}\pi(x_1,x_2)\mathrm{d}x_j\mathrm{d}x_i.
$$

In case at least one component is replaced, either preventively or correctively, the set-up costs for a replacement c_s need to be paid once. The long-run average set-up costs per inspection cycle are given by:

$$
\mathrm{E}_\pi[C_S(S)] = c_s\left(\int_{\xi_n^1}^{\infty}\int_0^{\infty}\pi(x_1,x_2)\mathrm{d}x_2\mathrm{d}x_1+\int_0^{\xi_n^1}\int_{\xi_n^2}^{\infty}\pi(x_1,x_2)\mathrm{d}x_2\mathrm{d}x_1\right).
$$

The long-run average costs incurred for the time that the system spends in the failed state during one inspection cycle are given by:

$$
\mathrm{E}_\pi[C_U(S)] = c_d\mathrm{E}_\pi[D_U(S)],
$$

where $\mathrm{E}_\pi[D_U(S)]$ denotes the long-run average time that the system is unavailable during an inspection cycle. Since by assumption failures can only be noticed upon inspection, which are scheduled at discrete points in time, the exact moment at which a failure occurs is unknown. As an alternative to using an upper bound for the unavailability time [7], we suggest to assume a linear increase in deterioration between two consecutive inspection moments. This provides a better approximation of the down-time of the system as will be further explained in Section 4.2.1.

The long-run average maintenance costs per time unit can now be obtained as

$$
\begin{aligned}
C_\infty &= \lim_{t\to\infty}\frac{\mathrm{E}[C(t)]}{t}=\frac{\mathrm{E}_\pi[C(S)]}{\mathrm{E}_\pi[S]}\\
&= \frac{c_n+\sum_{i=1}^2\mathrm{E}_\pi[C_P^{(i)}(S)]+\sum_{i=1}^2\mathrm{E}_\pi[C_C^{(i)}(S)]+\mathrm{E}_\pi[C_S(S)]+\mathrm{E}_\pi[C_U(S)]}{\mathrm{E}_\pi[S]},
\end{aligned}
$$

while the long-run average system availability can be obtained as

$$
A_\infty = \frac{\mathrm{E}_\pi[S]-\mathrm{E}_\pi[D_U(S)]}{\mathrm{E}_\pi[S]}.
$$

4.2.1. Long-run average system unavailability time

Suppose that component i fails at time $t_f^{(i)}$, with $t_f^{(i)} \in [k-1, k)$ and $1 \leq k \leq S$. Since the deterioration level of a component, and hence whether or not it has failed, can only be observed at the discrete inspection moments, the exact moment of failure $t_f^{(i)}$ is unknown. Castanier et al. [7] suggest to approximate the component unavailability time $D_U^{(i)}(S)$ by assuming that the failure occurred at time $k-1$. However, this approximation is obvious positively biased and indeed an upper bound, which the authors acknowledge. Moreover, since maintenance policies typically try to achieve a high up-time, failures that do occur are likely to occur towards the end of a period. We therefore use an alternative, linear approximation of the unavailability time, as is illustrated in Figure 2.

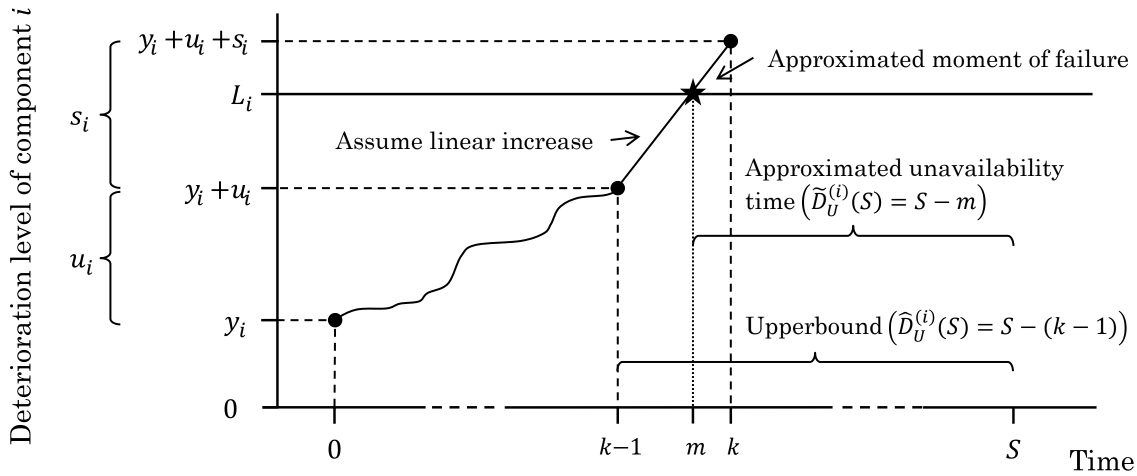

Figure 2: Approximating the unavailability time $D_U^{(i)}(S)$ of component i.

Note that in order for $\tilde{D}_U^{(i)}(S)$ to equal $S - m$, we require an increase in deterioration of $s_i = \frac{L_i - y_i - u_i}{m-(k-1)} = \frac{L_i - y_i - u_i}{m - \lfloor m \rfloor}$ between times $k-1 (= \lfloor m \rfloor)$ and $k (= \lceil m \rceil)$.

For presentational ease, we introduce the binary variable q, indicating whether the system unavailability time is approximated by using [7]'s upper bound ($q = 0$), or by assuming a linear increase in deterioration between two consecutive inspection moments ($q = 1$). In other words,

$$\mathrm{E}_{\pi,q}[D_U(S)] = \begin{cases} \mathrm{E}_\pi[\hat{D}_U(S)] & \text{if } q = 0, \\ \mathrm{E}_\pi[\tilde{D}_U(S)] & \text{if } q = 1. \end{cases}$$

Next, we define the functions $u_{1,q}(m, l)$ as the system unavailability time if only one component fails, (approximately) at time m, in an inspection cycle of length l using method q, and $u_{2,q}(m_1, m_2, l)$ as the system unavailability time if components 1 and 2 fail at (approximate) times m_1 and m_2, respectively, in an inspection cycle of length l using method q. Then

$$
\begin{aligned}
u_{1,q}(m,l) &= \begin{cases} \lceil l-m \rceil & \text{if } q=0, \\ l-m & \text{if } q=1, \end{cases} \\
u_{2,q}(m_1,m_2,l) &= \begin{cases} \max\{\lceil l-m_1 \rceil, \lceil l-m_2 \rceil\} & \text{if } q=0, \\ \max\{l-m_1, l-m_2\} & \text{if } q=1. \end{cases}
\end{aligned}
$$

For the derivation of $\mathrm{E}_{\pi,q}[D_U(S)]$, we define the function $h_i(m|y)$ as the pdf of a failure of component i ($i=1,2$) at time m (under the assumption of a linear increase in deterioration), given a previous deterioration level of y. This function consists of two parts; we multiply the probability that a failure occurs during the $\lceil m \rceil$-th time unit with the probability of a failure at time m given a failure during the $\lceil m \rceil$-th time unit. We find that

$$
h_i(m|y) = \begin{cases} \int_{L_i-y}^{\infty} f_i(s)\mathrm{d}s \cdot \frac{f_i\left(\frac{L_i-y}{m-\lfloor m \rfloor}\right)}{\int_{\lfloor m \rfloor}^{\lceil m \rceil} f_i\left(\frac{L_i-y}{t-\lfloor m \rfloor}\right)\mathrm{d}t} & \text{if } \lceil m \rceil = 1, \\ \int_0^{L_i-y} f_i^{(\lfloor m \rfloor)}(u) \left(\int_{L_i-y-u}^{\infty} f_i(s)\mathrm{d}s\right) \mathrm{d}u \cdot \frac{\int_0^{L_i-y} f_i^{(\lfloor m \rfloor)}(u)\cdot f_i\left(\frac{L_i-y-u}{m-\lfloor m \rfloor}\right)\mathrm{d}u}{\int_{\lfloor m \rfloor}^{\lceil m \rceil}\int_0^{L_i-y} f_i^{(\lfloor m \rfloor)}(u)\cdot f_i\left(\frac{L_i-y-u}{t-\lfloor m \rfloor}\right)\mathrm{d}u\mathrm{d}t} & \text{if } \lceil m \rceil > 1. \end{cases}
$$

Furthermore, we define the function $F_i(y,l)$ as the probability that component i will not fail during the first l time units starting with a deterioration level of y:

$$
F_i(y,l) = \int_0^{L_i-y} f_i^{(l)}(s)\mathrm{d}s.
$$

We can now obtain an expression for the approximated system unavailability times for both methods by distinguishing two cases: either both components fail, or just one component fails. If both components fail, the average system unavailability time is obtained by multiplying $u_{2,q}(m_1,m_2,\cdot)$ with the pdfs $h_1(m_1|\cdot)$ and $h_2(m_2|\cdot)$ and integrating the resulting expression with respect to m_1 and m_2. Similarly, in case only component 1 (2) will fail, the average system unavailability time is obtained by multiplying $u_{1,q}(m,\cdot)$ with the pdf $h_1(m|\cdot)$ ($h_2(m|\cdot)$) and the probability that component 2 (1) will not fail $F_2(\cdot,\cdot)$ ($F_1(\cdot,\cdot)$) and integrating this expression with respect to m. Similar to obtaining an expression for the probability law, this is done separately for each of the following four cases: both components are replaced, only component 1 is replaced, only component 2 is replaced, and no replacement is performed during the previous inspection. This gives

$$
\begin{aligned}
\mathrm{E}_{\pi,q}[D_U(S)] = &\left(\int_{\xi_n^1}^{\infty}\int_{\zeta_2}^{\infty} \pi(y_1,y_2)\mathrm{d}y_2\mathrm{d}y_1 + \int_{\zeta_1}^{\xi_n^1}\int_{\xi_n^2}^{\infty} \pi(y_1,y_2)\mathrm{d}y_2\mathrm{d}y_1\right)\cdot \\
&\left(\int_0^n\int_0^n u_{2,q}(m_1,m_2,n)\cdot h_1(m_1|0)\cdot h_2(m_2|0)\mathrm{d}m_2\mathrm{d}m_1 + \int_0^n u_{1,q}(m,n)\cdot(h_1(m|0)\cdot F_2(0,n) + \right. \\
&\left. F_1(0,n)\cdot h_2(m|0)\,)\,\mathrm{d}m\,\right) + \\
&\sum_{l_2=1}^{n}\int_{\xi_n^1}^{\infty}\int_{\min\{\xi_{n-l_2}^2,\zeta_2\}}^{\min\{\xi_{n-l_2+1}^2,\zeta_2\}} \pi(y_1,y_2)\cdot\left(\int_0^{l_2}\int_0^{l_2} u_{2,q}(m_1,m_2,l_2)\cdot h_1(m_1|0)\cdot h_2(m_2|y_2)\mathrm{d}m_2\mathrm{d}m_1 + \right.
\end{aligned}
$$

$$
\begin{aligned}
&\quad \int_0^{l_2} u_{1,q}(m,l_2)\cdot\left(h_1(m|0)\cdot F_2(y_2,l_2)+F_1(0,l_2)\cdot h_2(m|y_2)\right)\mathrm{d}m\Bigg)\,\mathrm{d}y_2\mathrm{d}y_1+\\
&\sum_{l_1=1}^{n}\int_{\min\{\xi^1_{n-l_1},\zeta_1\}}^{\min\{\xi^1_{n-l_1+1},\zeta_1\}}\int_{\xi^2_n}^{\infty}\pi(y_1,y_2)\cdot\Bigg(\int_0^{l_1}\int_0^{l_1}u_{2,q}(m_1,m_2,l_1)\cdot h_1(m_1|y_1)\cdot h_2(m_2|0)\mathrm{d}m_2\mathrm{d}m_1+\\
&\quad \int_0^{l_1} u_{1,q}(m,l_1)\cdot\left(h_1(m|y_1)\cdot F_2(0,l_1)+F_1(y_1,l_1)\cdot h_2(m|0)\right)\mathrm{d}m\Bigg)\,\mathrm{d}y_2\mathrm{d}y_1+\\
&\sum_{l_1,l_2=1}^{n}\int_{\xi^1_{n-l_1}}^{\xi^1_{n-l_1+1}}\int_{\xi^2_{n-l_2}}^{\xi^2_{n-l_2+1}}\pi(y_1,y_2)\cdot\Bigg(\int_0^{\min\{l_1,l_2\}}\int_0^{\min\{l_1,l_2\}}u_{2,q}(m_1,m_2,\min\{l_1,l_2\})\cdot h_1(m_1|y_1)\cdot\\
&\quad h_2(m_2|y_2)\mathrm{d}m_2\mathrm{d}m_1+\int_0^{\min\{l_1,l_2\}}u_{1,q}(m,\min\{l_1,l_2\})\cdot\left(h_1(m|y_1)\cdot F_2(y_2,\min\{l_1,l_2\})+\right.\\
&\quad \left.F_1(y_1,\min\{l_1,l_2\})\cdot h_2(m|y_2)\ \right)\mathrm{d}m\ \Bigg)\,\mathrm{d}y_2\mathrm{d}y_1. \qquad (1)
\end{aligned}
$$

5. Numerical investigation

For presentational purposes, we consider a system consisting of two identical components. In this way, the threshold values (which are the same for both components) are easier to optimize and the results are easier to interpret than for non-identical components. It also allows us to omit the superscripts denoting to which component a certain cost or threshold value corresponds. Figure 3 shows the failure probability over time of a component with $\alpha = 3.5$ and $L = 2$, provided that it does not undergo any maintenance actions and that it is as good as new at time 0.

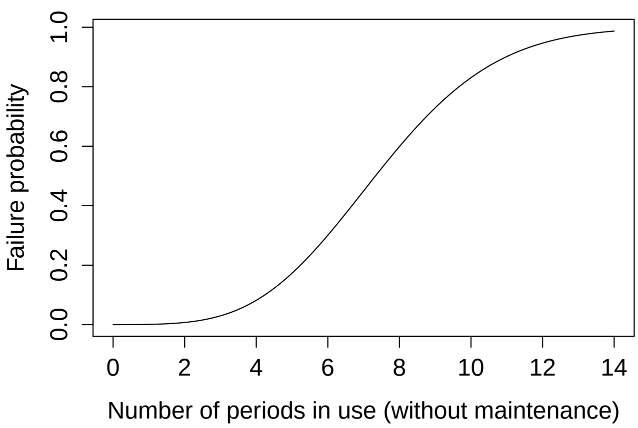

Figure 3: Failure probability over time of a new and unmaintained component with $\alpha = 3.5$ and $L = 2$.

From Figure 3, it follows that n (the number of inspection thresholds) should not be chosen too large, especially when considering relatively high unavailability costs as is typically the case in practice. In our experiments, we will assume an unavailability cost of at least 100 times the inspection cost. This implies roughly that the probability of failure in the next period should not exceed one percent. From Figure 3, we observe that the failure probability is much more than one percent for n larger than two. Hence, we set the number of inspection thresholds to $n = 2$, which means that the next inspection is always scheduled either one or two periods later.

Note that our computations are made using `R` [18] on a computer with a 3.30 GHz quad core processor and 8.00 GB of RAM. Due to the high complexity and considerable computing time, it is convenient to precalculate the integrals with respect to m_1, m_2, and m in Equation (1). Because these are independent from the threshold values and the cost parameters, we only need to calculate them once for all $y_i \in [0, L_i]$, $i = 1, 2$ (which takes about 55 hours) for this specific setting of $n = 2$, $L = 2$, and $\alpha = 3.5$. If we also calculate π (which is independent of the cost parameters too) for all combinations of the threshold values (approximately 87 hours), the cost and availability criteria can be calculated for all possible combinations of the threshold values within 40 seconds for any setting of the cost parameters.

The cost scenario that we consider is partially based on [7]; inspection costs normalized at $c_n = 1$, preventive replacement costs $c_p = 40$ per component, corrective replacement costs $c_c = 100$ per component, set-up costs $c_s = 35$ per (system) replacement, and unavailability cost rate $c_d = 150$ per time unit. We do a full grid search, and calculate the value of C_∞ for all $\xi_1, \xi_2, \zeta \in \{0, 0.1, \ldots, L\}$, with $0 \leq \xi_1 \leq \xi_2$ and $0 \leq \zeta \leq \xi_2$ in order to obtain the cost-minimizing threshold values.

We assess the performances using both the upper bound on the unavailability time, and our linear approximation. We refer to these as 'upper bound' and 'linear approximation', respectively, in the remainder of this section. Results indicate that the minimal long-run average cost per period is located somewhere around 29.96 when using the upper bound, and around 25.99 when using the linear approximation. We remark that these cost figures are not readily comparable (for determining cost savings) as they are based on different cost approximations, but the large difference does show that using a more accurate approximation significantly alters the results. Furthermore, the corresponding optimal threshold values are given by $\xi_1 = 0$, $\xi_2 = 1.0$, and $\zeta = 1.0$ for the upper bound, and by $\xi_1 = 1.3$, $\xi_2 = 1.3$, and $\zeta = 0.8$ for the linear approximation. These two maintenance policies differ in many aspects. Whereas by using the upper bound it is optimal to inspect each time unit, to replace a component at a deterioration level of 1, and to never perform opportunistic replacements, by using the linear approximation we find that the system is inspected every other period, a component is replaced preventively at a level of 1.3, and opportunistic replacements are performed at a deterioration level of 0.8. This implies that the accuracy of approximating the unavailability time has a great impact on the resulting optimal maintenance strategy as well. To gain more insight into the behavior of C_∞ and the differences between the upper bound and the linear approximation, Figure 4 shows the minimal value of C_∞ for different (fixed) values of each one of the threshold values.

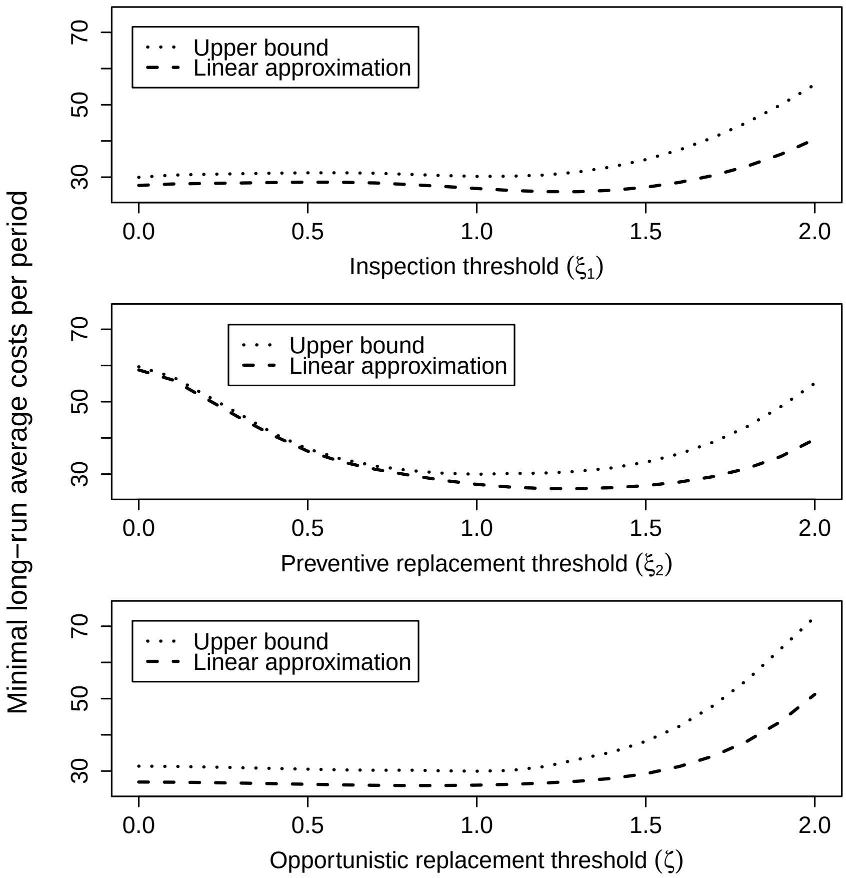

Figure 4: The minimal value of C_∞ for different values of ξ_1, ξ_2, and ζ.

It appears from Figure 4 that the minimal costs estimated by using both the upper bound and the linear approximation behave quite similarly, although the minimal costs obtained by the upper bound are clearly higher than those obtained by the linear approximation. Furthermore, we observe that increasing any one of the thresholds ξ_1, ξ_2, and ζ has a greater impact on the minimal costs based on the upper bound than those based on the linear approximation. This emphasizes the importance of approximating the unavailability time accurately. Besides, Figure 4 illustrates that both the inspection threshold ξ_1 and the opportunistic replacement threshold ζ should not be set too high. This can be explained by the fact that the preventive replacement threshold ξ_2 should exceed these two thresholds, forcing the number of preventive replacements to decrease as well. On the other hand, setting the preventive replacement threshold ξ_2 too low causes the maintenance costs to increase, as maintenance is then performed too often.

5.1. Comparison to classical maintenance policies

As noted in [7], many classical maintenance policies can be viewed as special cases of our multi-threshold maintenance policy. We compare the results of our policy with several of these for the particular example that we consider.

No opportunistic replacements: Set ζ equal to ξ_2 to omit opportunistic replacements.

Periodic inspections: Inspections are performed periodically (with periodicity $n = 2$) by setting ξ_1 equal to ξ_2.

Failure-based maintenance: Under the assumption that failures can only be noticed upon inspection, failure-based maintenance can be achieved by setting both ξ_2 and ζ equal to the failure level L such that no preventive maintenance is performed.

Block replacement: Block replacement is a strategy in which all components are replaced periodically. This policy can be obtained by setting all thresholds to zero.

The minimal long-run average costs along with the optimal threshold values for each of the above maintenance strategies are shown in Table 2 and summarized in Figure 5. If it is optimal to perform periodic inspections, the corresponding periodicity is also presented in the table.

Table 2: Minimal C_∞, optimal threshold values, and (if optimal) periodicity for different maintenance policies using the linear approximation (upper bound).

Maintenance policy	C_∞	ξ_1	ξ_2	ζ	Periodicity
Multi-threshold policy	25.99 (29.96)	1.3 (0.0)	1.3 (1.0)	0.8 (1.0)	2 (-)
No opportunistic replacements	26.76 (29.96)	1.2 (0.0)	1.2 (1.0)	1.2 (1.0)	2 (-)
Periodic inspection	25.99 (30.25)	1.3 (1.1)	1.3 (1.1)	0.8 (0.7)	2 (2)
Failure-based maintenance	51.17 (72.53)	1.9 (2.0)	2.0 (2.0)	2.0 (2.0)	- (2)
Block replacement	58.78 (59.64)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	2 (2)

Note that for the upper bound, the optimal solution obtained with our multi-threshold maintenance policy does not deviate from the one obtained by omitting opportunistic replacements. This is due to the fact that in this particular example it is optimal to not include opportunistic replacements. Similarly, for the linear approximation it turns out that periodic inspections are optimal in this particular case.

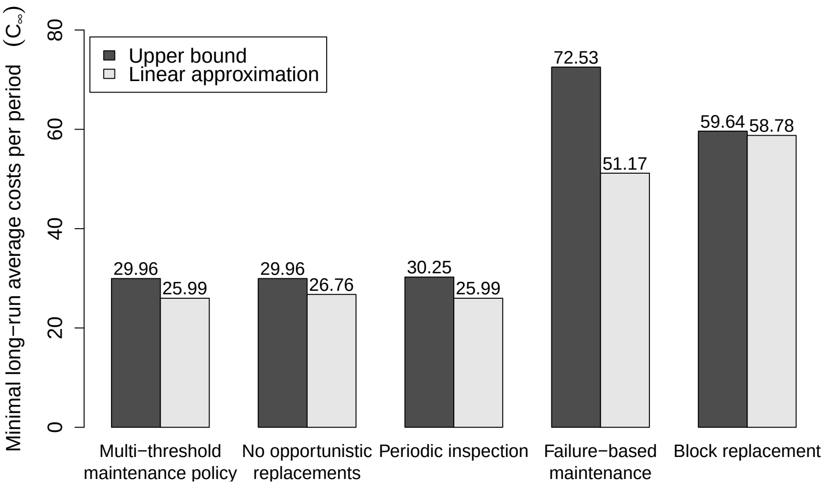

Figure 5: The minimal value of C_∞ compared to classical maintenance policies.

So, including opportunistic maintenance, and therefore performing a system-wide optimization, is essential for this example.

5.2. Sensitivity analysis

5.2.1. Influence of the set-up cost

So far, we assumed a set-up cost of $c_s = 35$. This cost will now be varied from 0 to 50. Figure 6 shows the minimal long-run average costs per period for these values of the set-up costs. As expected, the costs obtained by the linear approximation are lower than those obtained by using the upper bound. Furthermore, increasing the set-up costs has a larger effect on the costs obtained with the upper bound than with the linear approximation, because higher set-up costs imply less preventive maintenance, and hence a higher unavailability time. In addition, Figure 7 shows the long-run average availability A_∞ corresponding to the cost-minimizing threshold values for different values of the set-up costs. It decreases as the set-up costs increase, because maintenance is then performed less often.

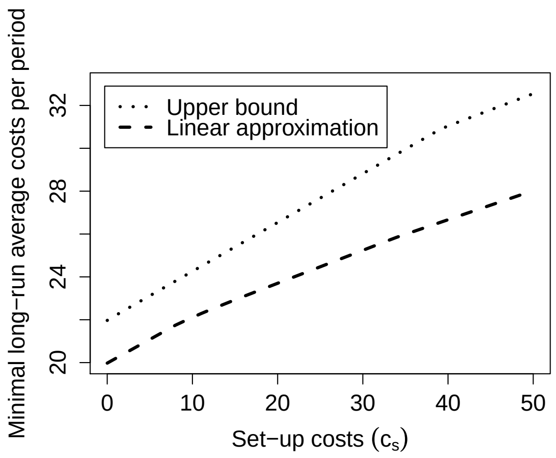

Figure 6: The minimal value of C_∞ for different values of the set-up cost.

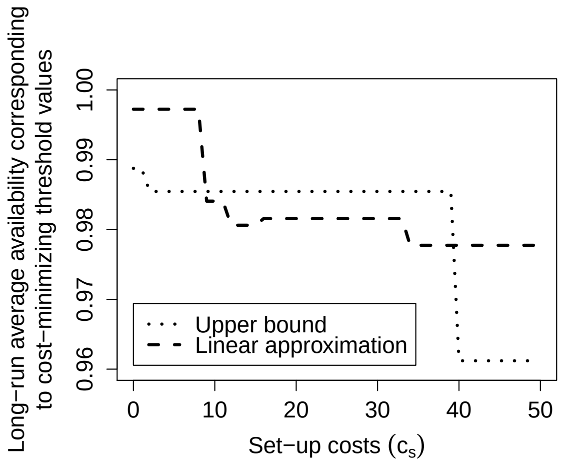

Figure 7: A_∞ with the cost-minimizing threshold values for different values of c_s.

Furthermore, Figure 8 shows the threshold values that minimize the long-run average costs for different values of the set-up costs, both using the upper bound and the linear approximation.

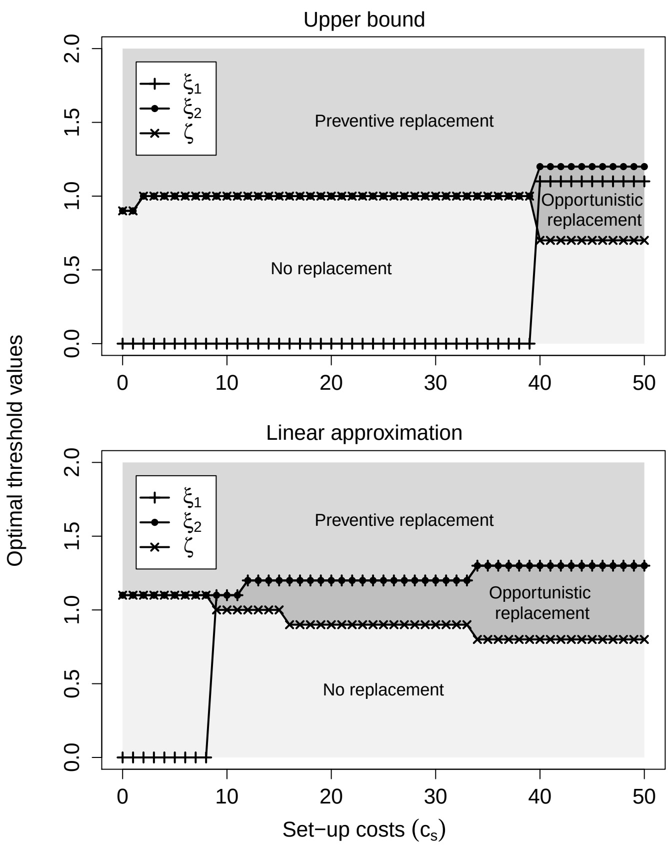

Figure 8: Cost-minimizing threshold values for different values of the set-up cost.

Figure 8 confirms that the preventive replacement threshold ξ_2 is consistently higher for the linear approximation than for the upper bound. It also appears that the inspection threshold ξ_1 is equal to ξ_2 for a wider range of set-up costs under the linear approximation, meaning that fewer inspections are performed. Related, there are more opportunistic replacements under the linear approximation.

5.2.2. Influence of the unavailability cost rate

Next, we vary the unavailability cost rate instead, from 100 to 200. This results in the minimal costs C_∞ shown in Figure 9 with the corresponding availability shown in Figure 10.

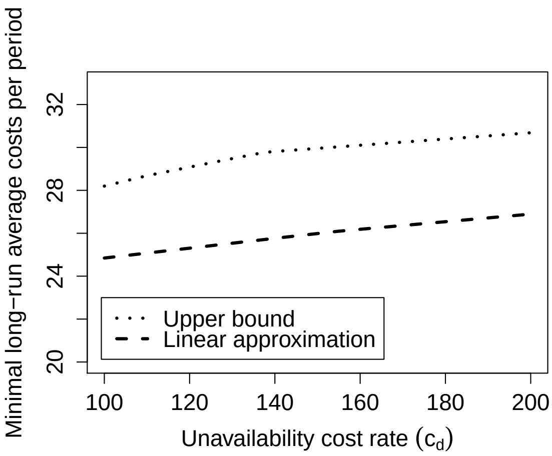

Figure 9: The minimal value of C_∞ for different values of the unavailability cost rate.

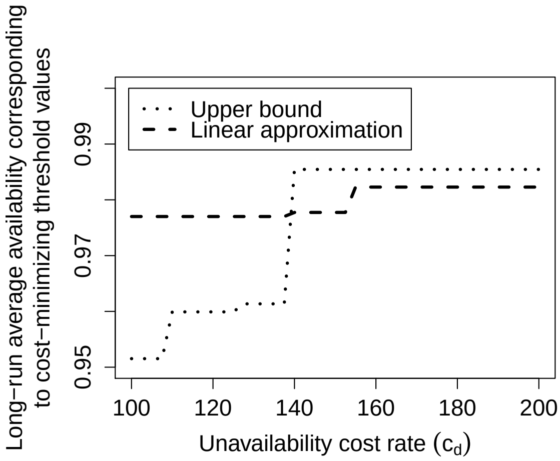

Figure 10: A_∞ with the cost-minimizing threshold values for different values of c_d.

Both figures imply that as the unavailability cost rate increases, the availability corresponding to the cost-minimizing solution increases as well. Furthermore, Figure 11 shows the threshold values that minimize the long-run average costs per period for the different values of the unavailability cost rate. Similar to the case where we varied the set-up costs, the preventive replacement threshold is set higher in case the linear approximation is used than with the upper bound. The same holds for the opportunistic replacements, as long as the unavailability cost rate does not exceed 140. This causes the inspection threshold ξ_1 to drop to zero for the upper bound, implying no opportunistic replacements. For the linear approximation, however, inspections are performed every other time unit, and both preventive and opportunistic replacements are performed more often when the unavailability cost rate increases.

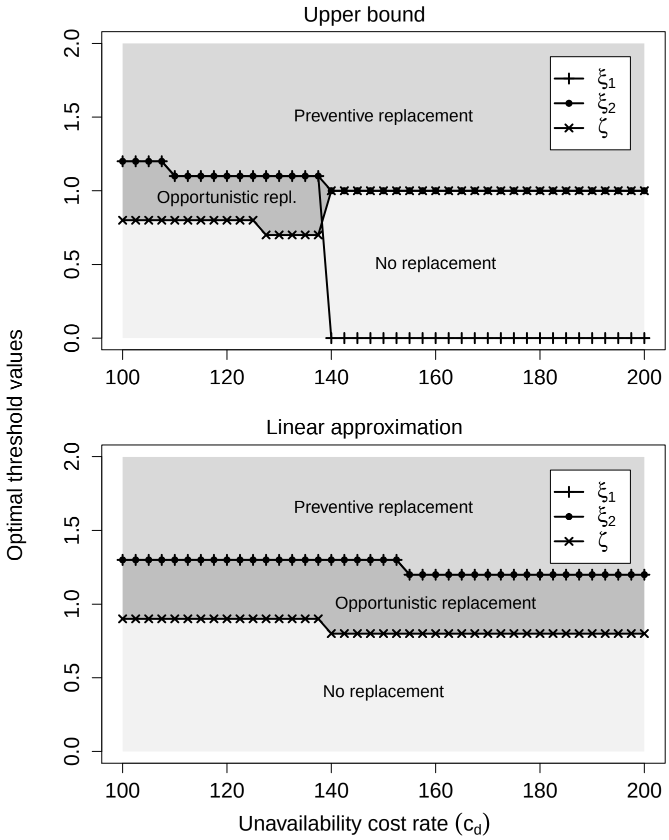

Figure 11: Cost-minimizing threshold values for different values of the unavailability cost rate.

We remark that these sensitivity results were also observed for other values of the deterioration parameters α_i, the failure levels L_i, and the cost parameters, for $i = 1, 2$.

6. Conclusion

In this paper, we built on the work of Castanier et al. [7], who developed an advanced CBM policy for a two-unit series system with economic dependencies, where the aperiodic inspection moments are optimized simultaneously with the critical condition levels at which maintenance is performed.

Whereas only the long-run average maintenance costs per period was considered as a performance criterion in [7], we considered the long-run average availability as well. Since the deterioration level of a component, and hence whether or not a failure has occurred, can only be observed at the inspection moments, the amount of time that a component is unavailable cannot be measured exactly. An upper bound is used in [7], but we approximate it more accurately by assuming a gradual, linear increase in deterioration between two consecutive time units. Results indicate that this greatly influences the resulting optimal maintenance strategy. Using an upper bound, the overestimated unavailability time causes both inspections and preventive replacements to be performed too often, reducing the profitability of opportunistic replacements.

A numerical sensitivity study revealed insights on the trade-off between different types of maintenance actions. Both the inspection thresholds and the opportunistic replacement threshold should not be set too high, as this forces the number of preventive replacements to reduce as well. At the same time, the preventive replacement threshold should not be set too low, since maintenance is then performed too often, which increases the maintenance costs. By selecting the right thresholds, our policy was shown to outperform simpler, classical maintenance policies. In fact, a number of these classical policies can be viewed as special cases of our policy, making it widely applicable and of value to the maintenance literature.

Since in practice systems often contain more than two components, for which different structural relations exist, a direction for future research is to extend this model to a k-out-of-N-system, i.e., the case where a system consisting of N components functions as long as at least k components function [19]. Other relevant extensions of the system considered here include uncertain deterioration failure levels, dependent deterioration processes for the different components, and the inclusion of predetermined periods during which maintenance activities are preferably scheduled such as turn arounds. However, the current analysis is already complex and has a considerable computing time. This is partly due to the fact that no efficient optimization approach exists to find the optimal threshold values, forcing us to do a full grid search. Although we can deal with the long computing time by dividing the calculations into different parts and running them separately, future research could address alternative ways to analyze the stationary law as well.

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