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In forecasting and inventory control textbooks and software applications, the variance of the cumulative lead-time forecast error is, almost invariably, taken as the sum of the error variances of the individual forecast intervals. For stationary demand and a constant lead time, this implies multiplying the single period variance (or Mean Squared Error) by the lead-time. This standard approach is shown in this paper to always underestimate the true lead-time demand variability, resulting in too low safety stocks and poor service. For two of the most widely applied forecasting techniques (Single Exponential Smoothing and Simple Moving Average) we present corrected expressions and show that the error in the standard approach is often considerable. The same fundamental problem exists for all forecasting techniques and all demand processes, and so this issue deserves wider recognition and offers ample opportunities for further research.

Keywords: Demand forecasting; Inventory Control; Safety Stock.

1. Introduction

In any demand forecasting and inventory control application, estimating the variability of the lead-time demand is equally important to estimating the level of demand itself. The former should be expressed through the variance of the lead-time forecast error and relates explicitly to the safety stock investments required to sustain a certain target service level.

If the demand data series do not exhibit any trend, seasonality, or fluctuations of mean demand over time, then a *demand level* model is suitable, i.e. demand fluctuates randomly around a stationary mean. The traditional approach for determining the safety stock in a demand level model, is to multiply the lead-time by the variance (or Mean Squared Error) of the one step ahead forecast error (Axsäter, 2006). The mistake in this approach is to 'forget' that forecast errors are correlated over time, even if the demands are not. Indeed, for the level demand model it is obvious that an under (over) estimation of the unknown *mean* demand for one future period implies an under (over) estimation of the mean demand for any future period, including all periods of the lead-time demand interval. This implies positive covariance of the forecast errors that should be included in the calculation of the lead-time demand variance.

In this paper, we provide corrected expressions for the variance of the lead-time demand forecast error for the demand level model, and show that they often lead to considerably higher safety stocks for two of the most widely used forecasting techniques: Single Exponential Smoothing (SES) and Simple Moving Average (SMA).

Other authors have also argued and shown that the traditional approach can underestimate the variance of (the forecast error of) lead-time demand if the demand process is more variable than assumed, e.g. if the demand level model is assumed but mean demand fluctuates over time. Their results, which will be discussed in Section 2, have great practical relevance since the demand process itself may not be known with certainty, and ignoring this uncertainty may lead to an underestimation of the lead-time demand variance and, consequently, too low safety stocks. It is important to remark, however, that our arguments and insights are fundamentally different. We show that even if the demand process is of the assumed (demand level) type, then forecast errors are still correlated over time and ignoring this leads to under-stocking.

Although we only provide exact correction factors for the demand level model in combination with the SES or SMA forecasting procedure, it will become obvious from our analysis that the same type of correction is needed for other demand processes and other forecasting procedures. In fact, for any real inventory control application where demands are forecasted, the traditional approach for determining safety stocks suffers from the same problem. We therefore suggest that this issue deserves much more consideration.

The remainder of this paper is structured as follows. Next, we discuss the related literature. Then, in Section 3, we formally introduce the level demand model and show how to include auto-correlation in the variance of the cumulative lead-time forecast error for any forecasting procedure. In Section 4, we provide further results for the SES and SMA forecasting procedures, and show for both forecasting techniques under realistic control settings that the traditional approach typically leads to much too small safety stocks. We end in Section 5 with conclusions, insights and directions for future research.

2. Related literature

The fact that lead-time demand forecast errors may be correlated over time has been pointed out by other researchers. Fildes and Beard (1992) argued that for lead-times greater than one period "*the errors will typically be auto-correlated and this issue has received very limited attention* (pp. 13)". Silver et al. (1998) noted that the exact relationship between the variability of the forecast error during the lead-time and that during the forecast interval "*depends in a complicated fashion on the specific underlying demand model, the forecast updating procedure and the values of the smoothing constant used* (pp. 114)". Subsequently they argued that the relationship concerned should be established empirically.

Other authors discussed the related issue of taking into account that the actual demand process may be 'more variable' than assumed. Johnston and Harrison (1986), Graves (1999) and Snyder et al. (1999) showed that fluctuations in the mean demand level (steady state model or an Auto-Regressive Integrated Moving Average process, ARIMA (0,1,1)) lead to additional uncertainty compared to the demand level model, and show how the variance of lead-time demand should be adjusted accordingly. A series of other papers have addressed the unsuitability of the regular lead-time demand variance and safety stock calculations if the demand process is not stationary; see, for example, Harvey and Snyder, 1990; Snyder et al., 2004; Fildes and Kingsman, 2011. These researchers rightly suggested that if there is more variance that should be taken into account, then ignoring it will lead to an underestimation of the safety stocks needed to sustain a certain service level performance.

However, as remarked in Section 1, these arguments and results are fundamentally different from ours. We show that forecast errors are correlated over the lead-time even if demands are not, and therefore that the traditional safety stock calculations that ignore this are flawed even if the demand model is specified correctly. Indeed, our arguments apply to any demand model and any forecasting procedure, although we only derive exact corrections for the level demand model in combination with either SES or SMA.

We hope that our results make clear that the issue of auto-correlation of forecast errors deserves much more attention in the forecasting and inventory control field. This issue remains tacit knowledge for some researchers, whereas it is also true to say that is completely ignored by others. Talluri et al. (2004, pp. 65), for example, considered the traditional calculations, in conjunction with the Mean Absolute Deviation (MAD) approach, for the purpose of modeling and subsequently improving a real-world forecasting and stock control system. Another example of using the traditional calculations as a building block in inventory modelling may be found in the study by Li et al. (1997, pp. 342). It is viewed as imperative to bridge this gap and, conceptually as well as analytically, clarify this issue for the purpose of better informing real world applications and further research in the area of inventory management.

3. Corrected lead-time forecast error variance for any unbiased forecasting technique

We consider a level demand process, that is, demands are independent over time and fluctuate around a constant mean. The analysis in the section will hold for any unbiased forecasting procedure. We remark that the analysis can also be extended to biased forecasting procedures, but this does not provide increased insight into the issue at hand and unbiased procedures are generally preferred. Moreover, since demands are independent over time and have the same mean, we assume that the forecasting procedure produces the same forecast for all future periods (of the lead-time). In other words, we use level forecasts for our level demand model. There is a constant (forecasting and inventory control) lead-time.

Let us introduce the following notation.

L: Lead-time

Y'_t: Demand forecast at the end of period t of demand in any future time period

Y_{t+k}: The actual demand in period $t+k$

e_{t+k}: The forecast error in period $t+k$

$Var[Y]$: Per period demand variance

$Var[Y']$: Per period forecast variance.

The variance of the lead-time forecast error is easily derived (Strijbosch et al., 2000; Syntetos et al., 2005) as

$$
\begin{aligned}
Var\left(\sum_{\kappa=1}^{L} e_{t+\kappa}\right) &= Var\left(\sum_{\kappa=1}^{L}\left(Y_{t+k} - Y'_t\right)\right) \\
&= \sum_{\kappa=1}^{L} Var\left(Y_{t+k}\right) + \sum_{\kappa=1}^{L} Var\left(Y'_t\right) + \sum_{\kappa \neq m}^{L} 2Cov\left(Y'_t, Y'_t\right) \qquad (1) \\
&= L\,Var(Y) + L\,Var(Y') + L(L-1)Var(Y').
\end{aligned}
$$

The three terms in (1) correspond to the total demand variance, forecast variance and forecast covariance, in this order. The latter term is overlooked by the traditional approach that sums the per period variances of forecast, as this gives

$$
\begin{aligned}
\sum_{\kappa=1}^{L} Var\left(e_{t+\kappa}\right) &= \sum_{\kappa=1}^{L} Var\left(Y_{t+k} - Y'_t\right) \qquad (2) \\
&= L\,Var(Y) + L\,Var(Y').
\end{aligned}
$$

Combining (1) and (2), we see that correcting for auto-correlation leads to a relative increase in the standard deviation of the lead-time forecast error of

$$
\sqrt{\frac{L\,Var(Y) + L^2\,Var(Y')}{L\,Var(Y) + L\,Var(Y')}} = \sqrt{\frac{1 + L\,Var(Y')/Var(Y)}{1 + Var(Y')/Var(Y)}} = \sqrt{1 + \frac{(L-1)Var(Y')/Var(Y)}{1 + Var(Y')/Var(Y)}}. \quad (3)
$$

For normally distributed lead-time demand, it is well known (see e.g. Axsäter, 2006) that the safety stock is proportional to the standard deviation of the lead-time demand (forecast error) for any required service level. So, for normally distributed demand, (3) also gives the correct safety stock relative to that resulting from the traditional approach.

Note from (3) that the correction factor increases with both the lead time and the relative variance of the forecast compared to the demand variance. This is intuitive, since a longer lead time implies more auto-correlation effects and increased forecast variance leads to larger (expected) estimation errors and therefore also larger auto-correlation.

In the next section, we further rewrite the correction factor for forecasting procedures SES and SMA, and show that the factor is considerably larger than 1 for many realistic settings of the control parameters.

4. Correction factors for SES and SMA

In this section, the implications of ignoring error auto-correlation are discussed for two of the most popular forecasting methods used in industry: SES and SMA. Of course one may argue that both methods are not suitable for stationary demands in that they are not the minimum variance unbiased estimators. However, small smoothing constant values for the SES estimator introduce minimum deviations from the stationary assumption; the SMA method is indeed optimal over all the data points available so far or, in the case of a structural change, since a demand shift occurred. Moreover, in practice it is typically not known with certainty that the mean demand is indeed constant. In fact, this in part explains the popularity of SES and SMA in practice and their good performance in forecasting competitions (see e.g. Gardner, 1990; 2006; Ali and Boylan, 2011; 2012).

For SES with smoothing constant α, it is well-known (see e.g. Axsäter, 2006) that we have

$$Var(Y')/Var(Y) = \alpha/(2-\alpha)$$

and so the correction factor (3) becomes

$$\sqrt{1+\frac{(L-1)\alpha/(2-\alpha)}{1+\alpha/(2-\alpha)}} = \sqrt{1+\frac{(L-1)\alpha}{2}},$$

which increases in α as expected.

For SMA over N periods, it is well-known (see e.g. Axsäter, 2006) that we have

$$Var(Y')/Var(Y) = 1/N$$

and so the correction factor (3) becomes

$$\sqrt{1+\frac{(L-1)/N}{1+1/N}} = \sqrt{1+\frac{(L-1)}{N+1}},$$

which decreases in N as expected

Please note that the correction factors for SMA and SES are the same if

$$N = \frac{2-\alpha}{\alpha}.$$

This is also the condition for which the average age of the data in the forecasts is the same (Brown, 1963).

In Tables 1 and 2, we show the percentage by which the traditionally calculated safety stocks need to increase for (normally distributed lead-time demand and) the case of SES and SMA, respectively, and for some typical control parameter values.

Table 1: Corrections factors for safety stock calculation with a SES procedure

	Increase in safety stock for		
Lead time	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$
1	0%	0%	0%
2	2%	5%	7%
3	5%	10%	14%
4	7%	14%	20%
5	10%	18%	26%
6	12%	22%	32%

Table 2: Corrections factors for safety stock calculation with an SMA procedure

	Increase in safety stock for			
Lead time	N = 1 (Naïve)	N = 4	N = 12	N = 52
1	0%	0%	0%	0%
2	22%	10%	4%	1%
3	41%	18%	7%	2%
4	58%	26%	11%	3%
5	73%	34%	14%	4%
6	87%	41%	18%	5%

[Under the Naïve forecasting procedure the forecast for the next time period is the last observed demand. This is achieved for N = 1 or $\alpha = 1$.]

It appears from Tables 1 and 2 that the increase in the lead-time forecast variance and therefore in the safety stock that results from auto-correlation is considerable for a wide range of realistic control parameter values of both SES and SMA.

5. Conclusion

Recent work has shown that many assumptions used in inventory applications do not work well in practice (Cattani et al., 2011). We extend this stream of research by arguing that the issue of not knowing mean demand has (largely) been ignored in both theory and practice, and that using the one step ahead Mean Squared Error provides results that are far from optimal. This will always lead to a safety stock that is smaller than what is needed to meet a prescribed target service level, and in many realistic cases it will be far too small.

Simple closed-form expressions were derived for correcting the lead-time forecast error variance and (for normally distributed level demand) corresponding safety stocks, and it transpired that the traditional approach leads to considerable errors for a wide range of realistic forecasting control parameter settings. For other forecasting procedures and other demand processes, our analysis can be extended, although analyzing non-stationary demand processes is considerably more complex.

Another interesting route to explore is to take the lead-time as the forecasting period. By doing so, the lead-time obviously reduces to a single period and therefore the issue of auto-correlated forecast errors over multiple periods is avoided. There are, however, a number of disadvantages of this approach. First, different forecasting periods are needed for different stock keeping units. Second, situations with stochastic lead-times cannot be dealt with in this way. Third, and more technical, calculating the so-called undershoot in inventory control systems (i.e. the amount by which the stock position drops below the reorder level) is not possible using aggregated demand information.

Given the considerable errors that were observed for SES and SMA forecasts under level demand, and the fact that such errors from auto-correlation are present for any demand and any forecast procedure, wider recognition and further exploration are certainly needed.

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