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Mardi Dungey
Jan Jacobs
Jing Tian
Simon van Norden

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Research Institute SOM
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University of Groningen

Visiting address:
Nettelbosje 2
9747 AE Groningen
The Netherlands

Postal address:
P.O. Box 800
9700 AV Groningen
The Netherlands

T +31 50 363 7068/3815

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university of groningen / faculty of economics and business

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Mardi Dungey
University of Tasmania, CFAP; University of Cambridge, CAMA

Jan Jacobs
University of Groningen, CAMA and CIRANO
j.p.a.m.jacobs@rug.nl

Jing Tian
University of Tasmania

Simon van Norden
HEC Montréal, CAMA, CIRANO and CIREQ

On Trend-Cycle Decomposition and Data Revision

Mardi Dungey
University of Tasmania, CFAP, University of Cambridge, CAMA
Jan P.A.M. Jacobs
University of Groningen, CAMA and CIRANO
Jing Tian
University of Tasmania
Simon van Norden*
HEC Montréal, CAMA, CIRANO and CIREQ

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Abstract

A well-documented property of the Beveridge-Nelson trend-cycle decomposition is the perfect negative correlation between trend and cycle innovations. This paper gives a novel explanation for this negative correlation originating from the Jacobs-van Norden (2011) data revision model. Trend shocks may enter the equation for the cycle or cyclical shocks may enter the trend equation. We discuss economic interpretations and implications, including filtering and smoothing properties. We illustrate the idea with simulations based on the Morley, Nelson and Zivot (2003) outcomes.

JEL classification: C22, C53, C82
Keywords: trend-cycle decomposition, data revision, state-space form, Kalman filter, Kalman smoother

*Corresponding author: Simon van Norden, HEC Montréal, 3000 Chemin de la Cote Sainte Catherine, Montreal QC, H3T 2A7, 514-340-6781, simon.van-norden@hec.ca.

1 Introduction

Trend-cycle decompositions have a long history in macroeconometrics, with many different methods proposed, including simple moving averages, fitted linear trends and sophisticated linear filters such as the Hodrick-Prescott filter and bandpass filters of Baxter and King (1999) and Christiano and Fitzgerald (2003).[1] Harvey (1989)'s Structural Time Series Analysis takes a different route by modelling the components as uncorrelated Unobserved Components (UC).[2]

The Beveridge-Nelson (1981; BN) decomposition was perhaps the first method specifically designed for series with unit roots and consequently has received much attention from macroeconomists.[3] The BN approach decomposes nonstationary series into a deterministic trend, a random walk and a cycle. Unlike the Structural Time Series approach, where shocks to trends and cycles are typically assumed to be uncorrelated, the BN decomposition assumes that these shocks are perfectly negatively correlated. Morley, Nelson and Zivot (2003) were the first to investigate the equivalence between the UC and BN approaches.

This paper provides a novel interpretation of the negative correlation between trend and cycle innovations which draws on recent contributions to the literature modeling data revision and measurement error. The literature

[1]See Jacobs (1998) or Mills (2003) for overviews.
[2]For a recent description see Harvey (2006).
[3]Although Nelson (2008) notes that it was left on the shelf for nearly a decade.

studying data revision, which began with Persons (1919), has seen many recent contributions (Coushore (2011a,2011b) provides an introduction) and has been influenced by questions of the predictability of data revisions; see Mankiw, Runkle and Shapiro 1984, Croushore and Stark (2003), and recently Aruoba (2008).

The BN trend-cycle decomposition and data revisions can both be cast in state space form. Morley (2002) was first to provide this for the BN decomposition. Recent examples of data revisions models in state-space form are Jacobs and van Norden (2011; JvN), Kishor and Konig (2012) and Cunningham et al. (2012). JvN show how the assumption of unforecastable revisions ("news") implies a negative correlation between the innovations to "true" values and measurement errors. Here we show how this relates to the assumption in BN models of a negative correlation between trend and cycle innovations. We can also distinguish two cases: (i) shocks to the trend have an impact on the cycle; (ii) shocks to the cycle affect the trend.

In addition to comparing the alternative interpretations of the above models, we consider their implications for Kalman filtering and smoothing. The negative correlation between shocks to different components of the state vector can make smoothed estimates more volatile than filtered estimates and observed series. This phenomenon, earlier documented by Proietti (2006), Morley (2011) and Dungey et al. (2012), has important economic implications which may help to determine the most appropriate identification route, based on the underlying economic concepts surrounding the decomposition.

We investigate the sensitivity of this property to different parameter values of the elements in the transition matrix and the size of the shocks to the trend and the cycle.

The remainder of this paper is structured as follows. First, in Section 2 we use a deliberately simple model to illustrate different assumptions that may be used in trend-cycle decompositions. We consider different interpretations associated with these assumptions and suggest directions in which the basic model may be generalized. The three sections thereafter flesh-out the more general case. Section 3 discusses the Beveridge-Nelson decomposition in some detail and its state space form. Section 4 introduces an alternative to obtain negative correlation between trend and cycle innovations, and discussions economic implications. Having set out the general models, Section 5 investigates the Kalman filter/smoother properties of BN trend-cycle decomposition, focusing on the cases considered in Morley et al. (2003). Section 6 concludes.

2 A simple model for decompositions with multiple interpretations

First consider the decomposition

$$y_t = \widetilde{y}_t + e_t,$$

where y_t is observable, $\widetilde{y}_t$ is a latent variable, and $e_t \equiv y_t - \widetilde{y}_t$. We will assume that $\widetilde{y}_t$ is a random walk —which is equivalent to $\Delta\widetilde{y}_t$ being i.i.d.—but we will make no identifying assumptions about e_t for the moment.

We can write one such very simple model in state-space form as

$$\text{Measurement Equation} \qquad y_t = \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_t \\ e_t \end{bmatrix}$$

$$\text{Transition Equation} \qquad \begin{bmatrix} \widetilde{y}_t \\ e_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_{t-1} \\ e_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma_\eta & 0 \\ 0 & \sigma_\nu \end{bmatrix} \cdot \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix},$$

where $\begin{bmatrix} \eta_t & \nu_t \end{bmatrix}' \sim \text{i.i.d.} N\left(\mathbf{0}, \boldsymbol{I}_2\right)$. Note that since y_t is just $\widetilde{y}_t$ plus i.i.d. noise, $\text{var}\left(\Delta y_t\right) > \text{var}\left(\Delta\widetilde{y}_t\right), \ \forall \sigma_\nu > 0$.

We can tweak this simple model in two different directions. First, the model implies $y_t \sim IMA(1,1)$, which might not be realistic. In particular, if y_t is thought to contain cycles we can nest this possibility by allowing e_t to follow an AR(2) process. Now the measurement equation becomes

$$y_t = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_t \\ e_t \\ e_{t-1} \end{bmatrix}$$

with transition equation

$$\begin{bmatrix} \widetilde{y}_t \\ e_t \\ e_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_{t-1} \\ e_{t-1} \\ e_{t-2} \end{bmatrix} + \begin{bmatrix} \sigma_\eta & 0 \\ 0 & \sigma_\nu \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix},$$

where η_t is the 'trend' shock and ν_t is the 'cycle' shock.

We may think of this as a prototypical unobserved components model of the business cycle with *orthogonal* shocks, i.e. the seminal model of Watson (1986). However, orthogonality is not essential; we could instead assume that shocks to the cycle and trend are perfectly correlated, which results in the transition equation

$$\begin{bmatrix} \widetilde{y}_t \\ e_t \\ e_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_{t-1} \\ e_{t-1} \\ e_{t-2} \end{bmatrix} + \begin{bmatrix} \sigma_\eta \\ \sigma_\nu \\ 0 \end{bmatrix} \cdot [\eta_t],$$

or allow for something in between, which yields

$$\begin{bmatrix} \widetilde{y}_t \\ e_t \\ e_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_{t-1} \\ e_{t-1} \\ e_{t-2} \end{bmatrix} + \begin{bmatrix} \sigma_\eta & 0 \\ r & \sigma_\nu \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix},$$

where r is non-zero. These two alternatives are equivalent to the *Beveridge-Nelson trend as definition* and *Beveridge-Nelson trend as estimate* proposals in Morley (2011, Section 2).

The other tweak involves a change in interpretation. Rather than interpret this model as a business cycle model that decomposes output y_t into a latent trend $\widetilde{y}_t$ and a cycle e_t, we can think of our original model as a measurement error model where e_t is just the measurement error in observing our object of interest $\widetilde{y}_t$. Typical measurement error models assume that $E(\widetilde{y}_t \cdot e_t) = 0$; what we observe is the 'truth' + 'noise'. However, we might prefer to think of measurement error as 'news' rather than 'noise', so that $E(y_t \cdot e_t) = 0$. (This would be more consistent with the idea of an "efficient" statistical agency as suggested by Sargent (1989), for example.) In that case, the transition equation of our original model becomes

$$\begin{bmatrix} \widetilde{y}_t \\ e_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_{t-1} \\ e_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma_\eta & \sigma_\nu \\ 0 & -\sigma_\nu \end{bmatrix} \cdot \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix}.$$

If we wanted to allow for those measurement errors to be correlated across time, we could write

$$\begin{bmatrix} \widetilde{y}_t \\ e_t \\ e_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \widetilde{y}_{t-1} \\ e_{t-1} \\ e_{t-2} \end{bmatrix} + \begin{bmatrix} \sigma_\eta & \sigma_\nu \\ 0 & -\sigma_\nu \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \eta_t \\ \nu_t \end{bmatrix},$$

where now η_t is the 'truth' shock and ν_t is the 'news' shock.

As we will see below, the different assumptions and interpretations just described capture the essential differences between a number of important and much more general state-space models. The difference between $E\left(\widetilde{y}_t \cdot e_t\right) = 0$ and $E\left(y_t \cdot e_t\right) = 0$ captures the essential difference between Structural Time Series Models (which use the former assumption) and the BN decomposition (which uses the latter.) The BN decomposition interprets the results as a stochastic trend and a cycle, while the JvN approach interprets them as a "true value" contaminated by measurement error. All of these models also have multivariate extensions that may play important roles in the identification of the model; for example, see Morley (2011).

3 Beveridge-Nelson trend-cycle decomposition

Consider the case where y_t is an I(1) variable with the Wold representation

$$\Delta y_t = \mu + \psi(L)\varepsilon_t,$$

where $\psi(L)$ is a polynominal in the lag operator L with roots outside the unit circle. Using $\psi(L) = \psi(1) + (1 - L)\psi^*(L)$, we obtain

$$\Delta y_t = \mu + \psi(1)\varepsilon_t + (1 - L)\psi^*(L)\varepsilon_t,$$

or

$$y_t = \frac{\mu}{(1 - L)} + \psi(1)\frac{\varepsilon_t}{(1 - L)} + \psi^*(L)\varepsilon_t \equiv \tau_t + c_t.$$

Here the BN permanent component or 'trend' follows a random walk with drift $\tau_t = \mu + \tau_{-1} + \psi(1)\varepsilon_t$, while the BN transitory component or 'cycle' is a zero mean stationary process which can be written as $c_t = \phi_p^*(L)c_t + \theta_q^*(L)\varepsilon_t + (1 - \psi(1))\varepsilon_t$, assuming c_t follows an ARMA(p, q) process. It is immediately clear that

- if $\psi(1) < 1$, then innovations in trend and cycle will have perfect positive correlation and trend and cycle will share the variation in the data
- if $\psi(1) > 1$, then innovations in trend and cycle will have perfect negative correlation, and τ_t will be more volatile than c_t.

Paraphrasing Morley et al. (2003), a positive shock to output can shift the trend, leaving actual output below trend until it catches up – that is trend shocks impact the cycle. This implies a negative contemporaneous correlation, for this positive trend shock is associated with a negative shock to the transitory component of output.

To cast the BN trend-cycle decomposition in state space form Morley et al. (2003) write the measurement equation as

$$y_t = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_t \\ c_t \\ c_{t-1} \end{bmatrix}, \tag{1}$$

with state equation:

$$\begin{bmatrix} \tau_t \\ c_t \\ c_{t-1} \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ c_{t-1} \\ c_{t-2} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_t^* \\ \varepsilon_t^* \end{bmatrix}, \tag{2}$$

where asterisks are used to denote that shocks η_t^* and ε_t^* do not have unit variances. Allowing trend and cycle innovations to be correlated, we get

$$\boldsymbol{Q} \equiv \mathrm{E}(\boldsymbol{R}^* \boldsymbol{\eta}^* (\boldsymbol{\eta} *)' (\boldsymbol{R}^*)') = \begin{bmatrix} \sigma_\eta^2 & \sigma_{\boldsymbol{\eta}\varepsilon} & 0 \\ \sigma_{\boldsymbol{\eta}\varepsilon} & \sigma_\varepsilon^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Alternatively, the single shock of error form of Anderson et al. (2006) has measurement equation:

$$y_t = \mu + \tau_{t-1} + \phi_p^*(L) c_t + \theta_q^*(L) \varepsilon_t + \varepsilon_t$$

and ‘transition’ equations:

$$\begin{aligned} \tau_t &= \mu + \tau_{t-1} + \psi(1) \varepsilon_t \\ c_t &= \phi_p^*(L) c_t + \theta_q^*(L) \varepsilon_t + (1 - \psi(1)) \varepsilon_t. \end{aligned}$$

In this case we have perfect correlation between trend and cycle innovations:

$$\begin{bmatrix} \psi(1) \\ 1-\psi(1) \end{bmatrix} \begin{bmatrix} \psi(1) & 1-\psi(1) \end{bmatrix} = \begin{bmatrix} (\psi(1))^2 & \psi(1)(1-\psi(1)) \\ \psi(1)(1-\psi(1)) & (1-\psi(1))^2 \end{bmatrix}$$

and depending on the sign of $(1-\psi(1))$ we get positive or negative correlation.

4 Data revision and news

While the Beveridge-Nelson decomposition has primarily been presented and used in the context of business cycles and related behaviour (such as consumption or unemployment), a separate literature has examined the revision of macroeconomic data. While this literature has been active for decades, recent contributions (such as those mentioned above) have emphasized state-space formulations in modeling the revision process.Of particular interest in this literature is the case where revisions are "news" in the sense of Mankiw, Runkle and Shapiro (1984): that is, where the expected revision conditional on all available information is zero.

In this section, we explore the relationship between state-space models of "news" and the Beveridge-Nelson decompositions of the previous section. To do so, we focus on a simple special case of the JvN framework with only one vintage and 'news' measure errors. In this case the measurement equation of our state-space model is:

$$\text{observed series}_t = \text{`truth'}_t + \text{`news'}_t,$$

and the state equation:

$$\begin{bmatrix} \text{`truth'}_t \\ \text{`news'}_t \end{bmatrix} = \boldsymbol{T} \begin{bmatrix} \text{`truth'}_{t-1} \\ \text{`news'}_{t-1} \end{bmatrix} + \begin{bmatrix} \text{`truth' shock}_t - \text{`news' shock}_t \\ \text{`news' shock }_t \end{bmatrix}.$$

The introduction of the 'news' shock in the equation for the 'truth' yields negative correlation in the variance-covariance matrix of the state equation, similar to the negative correlation in BN trend-cycle decomposition noted above. See Dungey et al. (2012) for more details.

We can adopt a similar procedure in BN trend-cycle decomposition. Taking measurement equation (1) and writing the state vector as in Section 3 as $\boldsymbol{\alpha} = \begin{bmatrix} \tau_t & c_t & c_{t-1} \end{bmatrix}'$ and adding a drift term to the state equation of the trend, we can distinguish two cases:

1. trend shocks enter the cycle equation. The state equation becomes:

$$\begin{bmatrix} \tau_t \\ c_t \\ c_{t-1} \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ c_{t-1} \\ c_{t-2} \end{bmatrix} + \begin{bmatrix} \sigma_\eta & 0 \\ -\sigma_\eta & \sigma_\varepsilon \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \nu_\eta \\ \nu_\varepsilon \end{bmatrix}, \qquad (3)$$

where $(\nu_\eta, \nu_\varepsilon) \sim N(0, I_2)$. In this case the variance matrix of trend and cycle innovations is given by

$$\mathrm{E}\left(\begin{bmatrix} \sigma_\eta & 0 \\ -\sigma_\eta & \sigma_\varepsilon \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \nu_\eta \\ \nu_\varepsilon \end{bmatrix} \begin{bmatrix} \nu_\eta \\ \nu_\varepsilon \end{bmatrix}' \begin{bmatrix} \sigma_\eta & 0 \\ -\sigma_\eta & \sigma_\varepsilon \\ 0 & 0 \end{bmatrix}'\right) = \begin{bmatrix} \sigma_\eta^2 & -\sigma_\eta^2 & 0 \\ -\sigma_\eta^2 & \sigma_\eta^2 + \sigma_\varepsilon^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

2. cycle shocks enter the trend equation, which yields the following state equation:

$$\begin{bmatrix} \tau_t \\ c_t \\ c_{t-1} \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ c_{t-1} \\ c_{t-2} \end{bmatrix} + \begin{bmatrix} \sigma_\eta & -\sigma_\varepsilon \\ 0 & \sigma_\varepsilon \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \nu_\eta \\ \nu_\varepsilon \end{bmatrix}. \qquad (4)$$

The variance matrix of the trend and cycle innovations is now given by

$$\mathrm{E}\left(\begin{bmatrix} \sigma_\eta & -\sigma_\varepsilon \\ 0 & \sigma_\varepsilon \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \nu_\eta \\ \nu_\varepsilon \end{bmatrix} \begin{bmatrix} \nu_\eta \\ \nu_\varepsilon \end{bmatrix}' \begin{bmatrix} \sigma_\eta & -\sigma_\varepsilon \\ 0 & \sigma_\varepsilon \\ 0 & 0 \end{bmatrix}'\right) = \begin{bmatrix} \sigma_\eta^2 + \sigma_\varepsilon^2 & -\sigma_\varepsilon^2 & 0 \\ -\sigma_\varepsilon^2 & \sigma_\varepsilon^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Comparing these two forms of the BN model with the JvN model of news, several things should be noted.

- Both the BN models as well as the JvN news model yield negative correlation between trend and cycle innovations.

- The size of the negative correlation is driven by the relative sizes of σ_η and σ_ε.

- The news model implies that the "truth" will have a greater innovation variance than the observed series (so long the revision error is non-zero). Hence in the BN model, innovations in the trend will be more volatile than those in the cycle when cycle shocks enter the trend; when trend shocks enter the cycle, innovations in the cycle will be more volatile than those in the trend.

This last point has implications for the correspondence between the JvN and BN models. The truth in the BN model may correspond to the trend in the BN model when cycle shocks enter the trend, or it may correspond to the cycle in the BN model trend shocks enter the cycle.

In addition to comparing the alternative interpretations of the above models, we consider their implications for Kalman filtering and smoothing. The negative correlation between shocks to different components of the state vector can make smoothed estimates more volatile than filtered estimates and observed series. This phenomenon has important economic implications which may help to determine the most appropriate identification route, based on the underlying economic concepts surrounding the decomposition. Below we investigate the sensitivity of this property to different parameter values of the elements in the transition matrix and the size of the shocks to the trend and the cycle.

It has been noted (see Harvey and Koopman (2000)) that negative correlations between the shocks in univariate state-space systems skew the weights of the Kalman smoother towards future values of the observed series while positive correlations skew them towards the past. They further note some parameter values may result in weights that alternate in sign, a result they find strongly counter-intuitive and leads them to recommend the use of uncorrelated shocks in most applications. However, Morley, Nelson and Zivot (2003) argue that business cycle data are better fit by models with negative correlations than conventional models with uncorrelated shocks, a conclusion shared by Oh, Zivot and Creal (2008), Sinclair (2009), Morley (2011), and Jun et al. (2011).[4] Nelson (2008) also finds that models with negatively correlated shocks do as well or better at forecasting cyclic movements than models with uncorrelated shocks. Proietti (2006) considers the effects of such negative correlations; he notes that the high weights assigned to future observations by the Kalman smoother imply that revisions to filtered estimates of cycles will typically be large, that cycles will typically be grossly underestimated by filtered estimates and that smoothed estimates of cycles may be much more variable (but have lower MSE) than filtered estimates.

We think the perspective of "news" models provide a simpler intuitive framework for understanding these results. By itself, the observation of actual output provides relatively little information about the current cycle-

[4] Perron and Wada (2009) take a completely different view, and blame it all on the 2003 break.

trend decomposition. However, as trend movements are more persistent, future observations will help to distinguish the two. Filtered estimates of the cycle, containing relatively little information, will stay close to their unconditional mean of zero. Over time, the arrival of "news" about the decomposition of contemporary output allows us to reduce the MSE of our estimates and increase their variability. By showing a formal link between the standard intuition of rational expectations and data revision on the one hand and the problem of modeling and estimating business cycles on the other, we think news models help make the theoretical case for modeling business cycles with negatively correlated components.

As noted above, such negative correlations can make smoothed estimates more volatile than filtered estimates and observed series. In Section 5 we investigate the sensitivity of this property to different parameter values of the elements in the transition matrix and the size of the shocks to the trend and the cycle.

Economic interpretation

Our approach generalizes two 'classic' workhorses in business cycle research: (i) the propagation-impulse framework of Frisch (1933) in which separate explanations are given for the way shocks are passed on in the economy (propagation mechanism) and for the origins of the shocks (impulse mechanism); and (ii) the decomposition into that of permanent effects and transitory effects, with permanent (transitory) shocks having permanent (transitory) effects. See e.g. Blanchard and Fisher (1989, pp. 7–11). The propagation-impulse framework is maintained, for example, by Arnold (2002) to describe business cycle theories. Pagan (1997) however argues that it is just as useful to work with the latter.

In our framework, the propagation mechanisms stay the same, but the source of the shock is more flexible. Shocks to the trend equation may enter the cycle equation (Equation (3)), or the other way round (Equation (4)). By this, a permanent shock can effect the cycle, and a transitory shock may have a permanent impact.

The implications of the alternative assumptions are non-trivial. For example, Nelson (2008) makes the point that the lack of forecastability of the GDP cycle indicates the strong influence of unpredictable trend shocks, associated with productivity changes in the usual BN decomposition. Morley (2001) further notes that the degree of correlation between the trend and cycle shocks determines the presence of short or long term forecastability in GDP. Evans and Reichlin (1994) note that multivariate BN decompositions typically find greater forecastability, which they attribute to a better information set, and has the direct consequence of greater variability in the cycle shocks (although they directly examine only the ratio of volatilty of the extracted trend and cycles themselves).

Most authors are agreed that shocks to GDP are predominantly permanent and negatively correlated. Recently, Sinclair (2009) has found the same for unemployment, and indeed noted the importance of this commonality between GDP and unemployment for Okun's law. In contrast, the CPI and inventories have been found to have a more volatile smoothed component than the observed data, suggesting that cyclical shocks are dominant for these indicator; see Bradley, Jansen and Sinclair (2009) for CPI and Knetsch (2005) for inventories.

5 Filtering and smoothing

In this section a series of 206 observations is simulated using the estimates reported in Morley et al. (2003).[5] We discuss how Kalman filtering and smoothing properties change under different assumptions of correlations between trend and cycle shocks, and parameter values of the transition matrix. All computations are done in Oxmetrics 6.20, using the SsfPack libraries of Koopman, Shephard and Doornik (1999, 2008).

Figure 1 shows the smoothed and filtered trend and cycle based on an uncorrelated Unobserved Component model. Appendix A describes our procedure. Although the filtered and smoothed components are quantitatively similar, the filtered trend, presented in the top panel, is slightly more volatile than the smoothed trend, while in the bottom panel, the smoothed cycle is slightly higher in amplitude than the filtered cycle except for the first few observations.

[Figure 1 about here.]

[5] We ignore the drift term, as typically done in the literature on correlated components, see e.g. Harvey and Koopman (2000, Section 2) or Proietti (2006, Section 2).

Allowing non-zero correlation between trend and cycle shocks (the covariance equals -0.8391 as in Morley et. al. (2003)), we estimate the trend and the cycle using the Kalman filter and smoother based on the Unobserved Component model that is equivalent to BN trend-cycle decomposition. Figure 2 presents the resulting filtered and smoothed estimates. The bottom panel shows that the smoothed cycle has a much higher amplitude than the filtered cycle. Note that comparison of Figure 1 and Figure 2 shows that the estimated trend and cycle under the uncorrelated UC model are more persistent, implying a longer estimated cycle.

[Figure 2 about here.]

The next two figures show filtered and smoothed estimates for the two cases of interest in this paper. Figure 3 assumes that trend shocks enter the cycle equation , while Figure 4 assumes that cycle shocks enter the equation for the trend. In both cases, the overall dynamics of the estimated changes in trends (upper panels) and estimated cycles (lower panels) are similar. When cycle shocks enter the equation for the trend (Figure 4), we observe that both filtered and smoothed estimates of the cycle have greater amplitude and innovations in the trend appear to be more variable. Differences between the filtered and smoothed estimates appear to be larger in Figure 4, implying that revisions to estimated trends and cycles tend to be larger in the case where cycle shocks enter the trend equation.

[Figure 3 about here.]

[Figure 4 about here.]

The final two figures show how the patterns in the volatility of smoothed and filtered estimates are affected by changes to the unit root assumption in the trend equation and the cycle to trend shock ratio. Figure 5 corresponds to the case when trend shocks enter the cycle equation, while Figure 6 assumes cycle shocks enter the trend equation. We let the AR parameter vary between 0.05 and 1. The standard deviation of cycle shocks is at 0.7487 as in Morley et. al. (2003), and we vary the standard deviation of trend shocks so that the cycle to trend shock ratio $\frac{\sigma_\varepsilon}{\sigma_\eta}$ ranges between around 0.3 to 2.

The top two panels in Figure 5 show that the smoothed trend is more volatile than the filtered trend, and this is particularly evident when the values of the autoregressive parameter and the cycle to trend shock ratio are both low. Under the unit root assumption in the trend equation, the smoothed trend is more volatile than the filtered trend only when the cycle shock size is much smaller than the trend shock size. These two observations can also be seen in the bottom two panels which show the variance of the smoothed and filtered cycles. If the cycle to trend shock ratio is much smaller than 1, smoothed and filtered cycle estimates become more volatile than the simulated data at a relatively high value of the autoregressive parameter for trend. However, the volatilities drop when the persistent level of trend increases further towards a unit root process.

[Figure 5 about here.]

In Figure 6, where cycle shocks enter the trend equation, the smoothed trend is more volatile than the filtered trend only when the autoregressive parameter in the trend equation is low and the cycle to trend shock ratio is high. However, at given levels of the autoregressive parameter and the cycle to trend shock ratio, smoothed cycles are more volatile than filtered cycles. When the autoregressive parameter is high, but lower than 1, and cycle shocks are larger than trend shocks, smoothed cycles can be more volatile than the simulated data.

[Figure 6 about here.]

6 Conclusion

This paper has three contributions. First, it reveals the correspondence between the state-space formulations of the univariate Beveridge-Nelson trend-cycle decomposition and the univariate Jacobs-van Norden data revision model with 'news'. Second, it provides a novel explanation for negative correlation between trend and cycle innovations in Beveridge-Nelson trend-cycle decompositions, with interesting economic implications. Third, it shows the negative correlation between trend and cycle innovations, or more general state equation covariance matrices with negative off-diagonal elements, can make smoothed estimates more volatile than filtered estimates and observed series.

Future research will deal with establishing necessary and sufficient conditions for the property that the smoothed truth may be more volatile than the filtered truth or the observed series as a result, including the impact of random walk drift term. In addition, we will study more general identification schemes for the shocks in the state equation and multivariate trend-cycle decompositions.

Finally, going beyond the BN-JvN interpretation, the state space form we introduced in Section 4 has great potention in (empirical) macroeconomics. Many other economic models can be cast in this framework, varying from models with stationary and non-stationary variables like the Structural Vector AutoRegressive (SVAR) model of Blanchard and Quah (1999) and "common features" of Engle and Kosicki (1993) and Vahid and Engle (1993), to New-Keynesian Phillips curves (Galí and Gertler (1999); Lee and Nelson (2007)) and recent explanations for the growth-employment puzzle (see e.g. Elsby and Shapiro 2012)). Extending our framework for use in multivariate analysis, and particularly for use in determining the most appropriate identification assumptions based on economic theory is scope for future research.

Appendix A

To produce Figure 1, we use a uncorrelated unobserved component model

$$y_t = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_t \\ c_t \\ c_{t-1} \end{bmatrix},$$

with state equation:

$$\begin{bmatrix} \tau_t \\ c_t \\ c_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ c_{t-1} \\ c_{t-2} \end{bmatrix} + \begin{bmatrix} \sigma_\eta & 0 \\ 0 & \sigma_\varepsilon \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \nu_\eta \\ \nu_\varepsilon \end{bmatrix},$$

where $\phi_1 = 1.5303$, $\phi_2 = -0.6098$, $\sigma_\eta = 0.6893$ and $\sigma_\varepsilon = 0.6199$.

The correlated unobserved component model that is used to produce Figure 2 is specified as

$$y_t = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_t \\ c_t \\ c_{t-1} \end{bmatrix},$$

with state equation:

$$\begin{bmatrix} \tau_t \\ c_t \\ c_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ c_{t-1} \\ c_{t-2} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_t^* \\ \varepsilon_t^* \end{bmatrix},$$

and the variance matrix of trend and cycle innovations is given by

$$\boldsymbol{Q} \equiv \mathrm{E}(\boldsymbol{R}^{*}\boldsymbol{\eta}^{*}(\boldsymbol{\eta}*)'(\boldsymbol{R}^{*})') = \begin{bmatrix} \sigma_{\eta}^{2} & \sigma_{\boldsymbol{\eta}\varepsilon} & 0 \\ \sigma_{\boldsymbol{\eta}\varepsilon} & \sigma_{\varepsilon}^{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Here we set parameters $\phi_1 = 1.3418$, $\phi_2 = -0.7059$, $\sigma_\eta = 1.2368$, $\sigma_\varepsilon = 0.7487$ and $\sigma_{\boldsymbol{\eta}\varepsilon} = -0.8391$.

We then respecify the state equation as equation (3) and (4) to produce Figure 3 and 4 respectively. Both simulations use the following parameter setup: $\phi_1 = 1.3418$, $\phi_2 = -0.7059$, $\sigma_\eta = 1.2368$, and $\sigma_\varepsilon = 0.7487$.

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Figure 1: Uncorrelated UC model

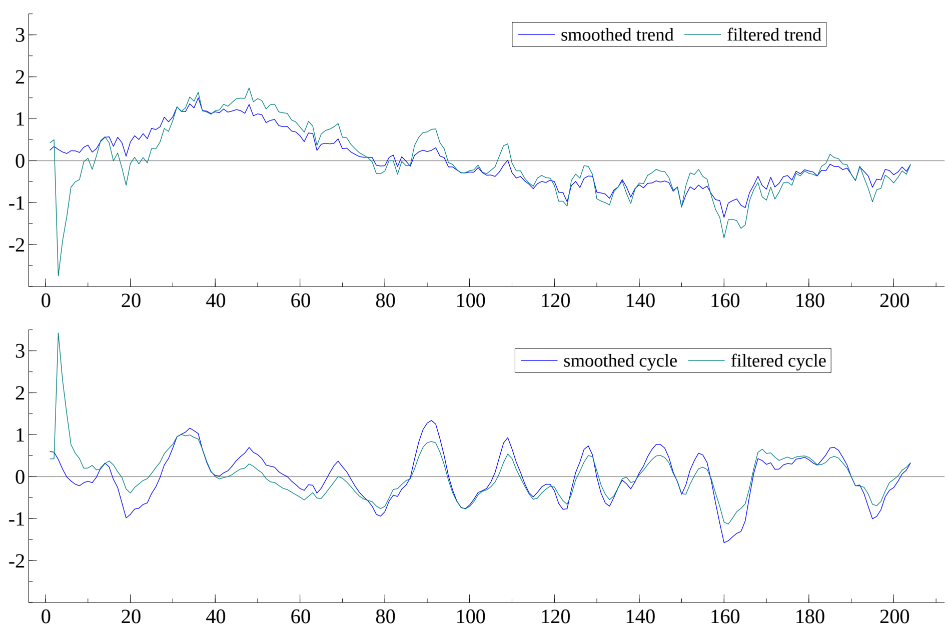


Figure 2: Correlated UC model ≡ BN trend-cycle decomposition

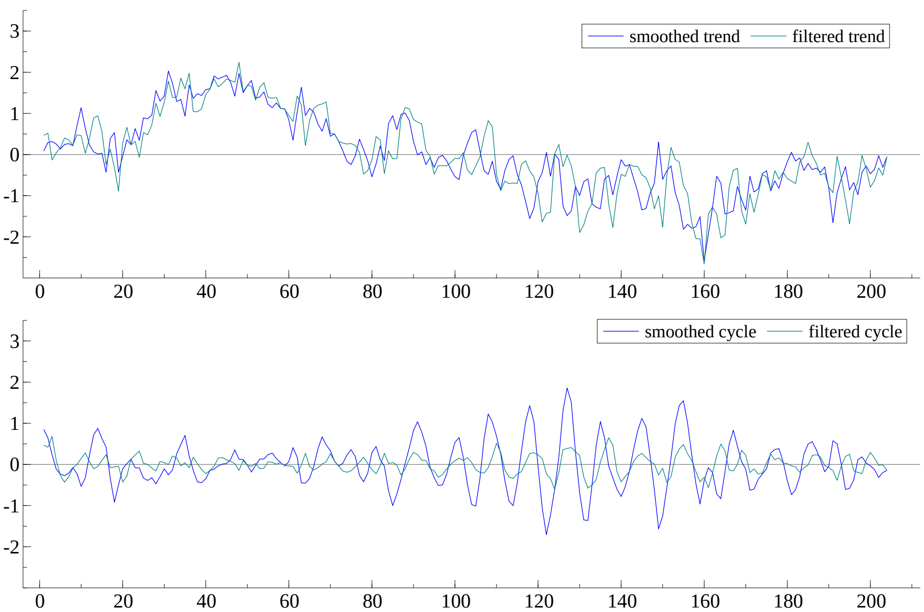


Figure 3: Trend shocks enter the cycle equation

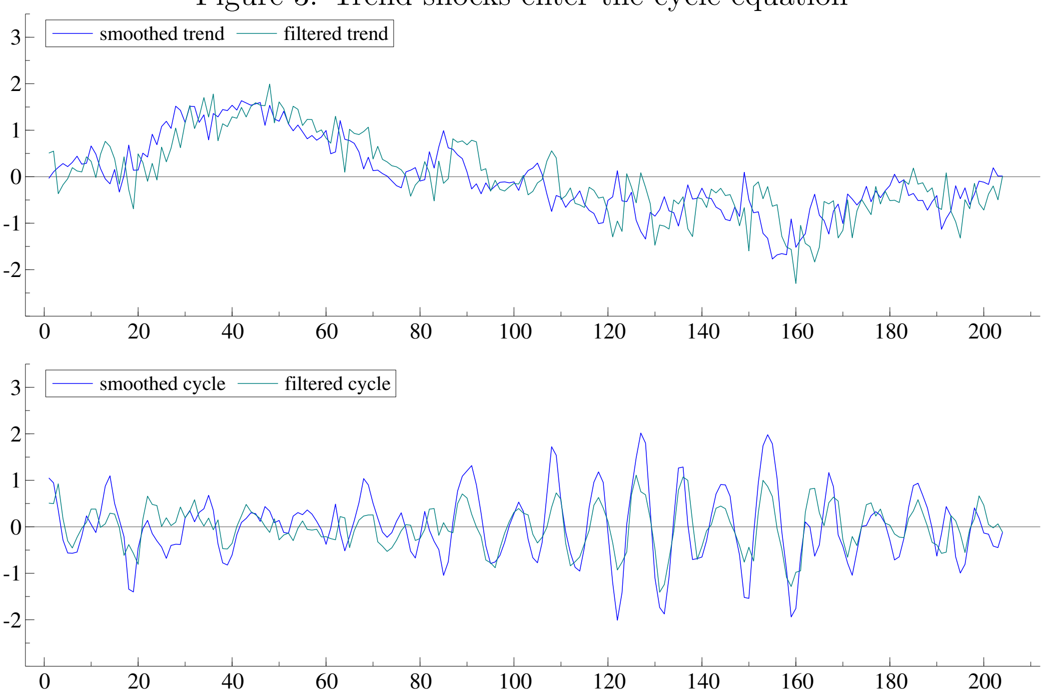


Figure 4: Cycle shocks in the trend equation

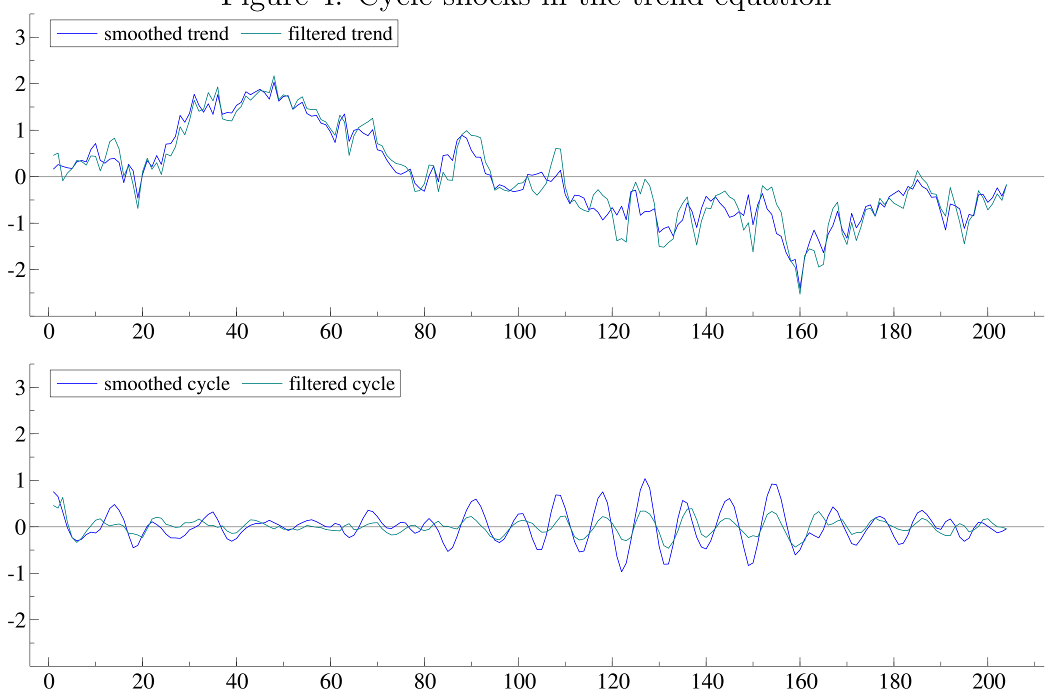


Figure 5: Sensitivity analysis of variances of trend and cycle estimates: Trend shocks enter the cycle equation

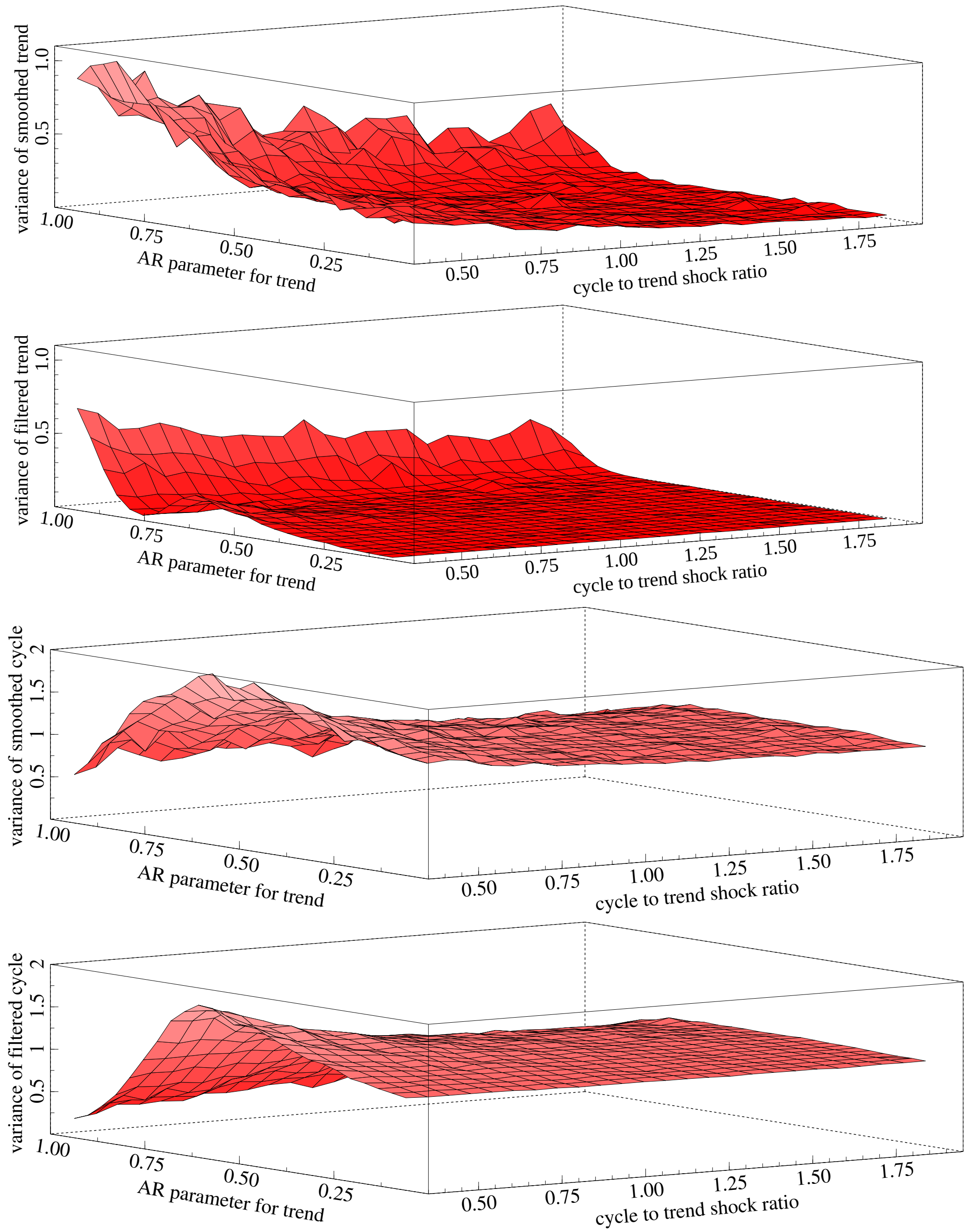
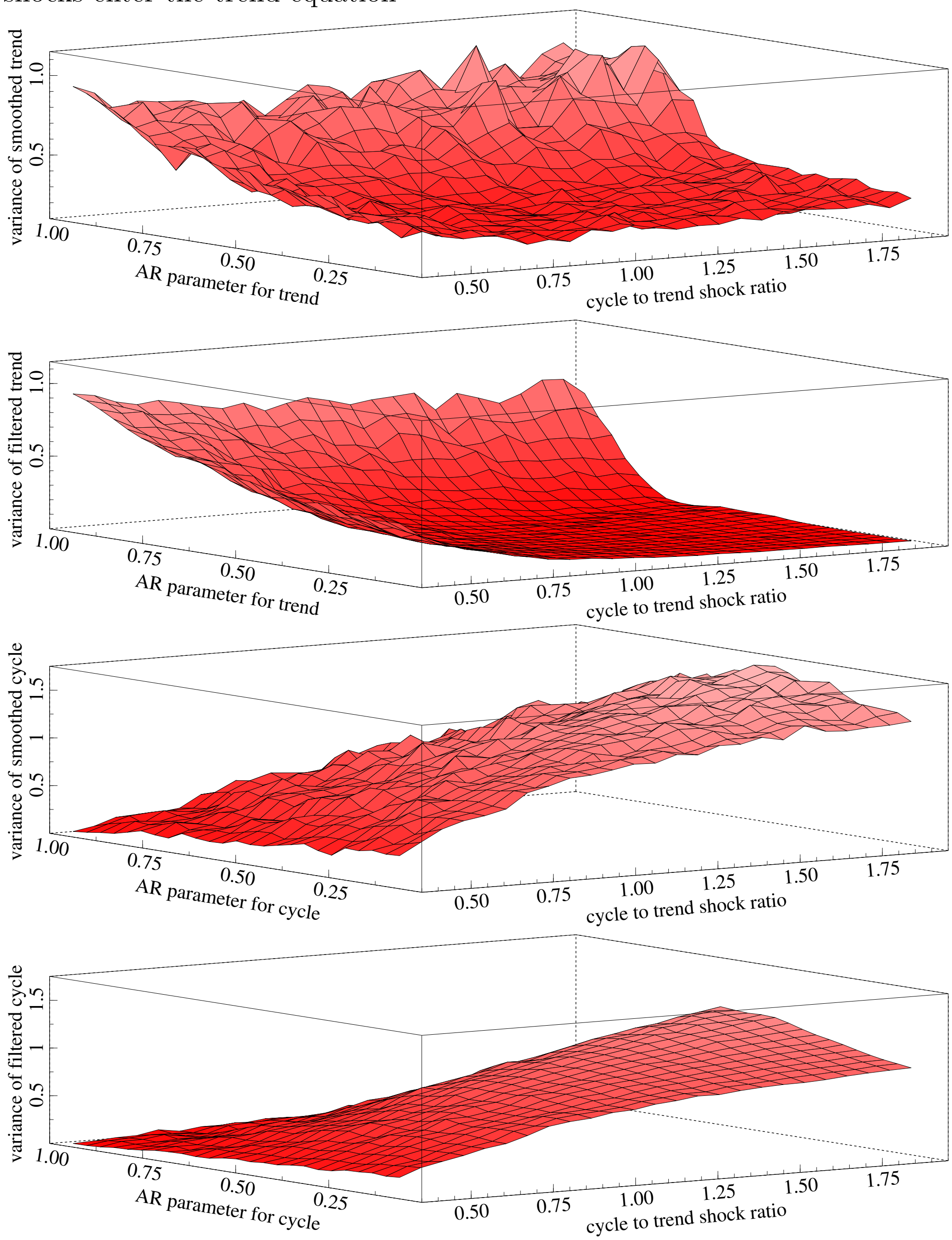


Figure 6: Sensitivity analysis of variances of trend and cycle estimates: Cycle shocks enter the trend equation



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