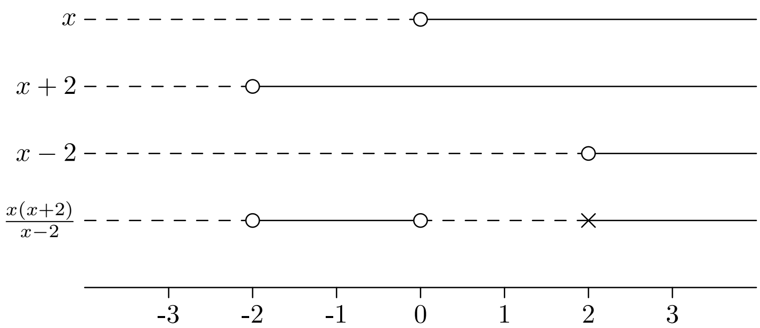
Summer Course Mathematics

Facts Tuesday

1. Rules for inequalities:
   - Rule 1: If $a > 0$ and $b > 0$, then $a + b > 0$ and $a \times b > 0$.
   - Rule 2: If $a > b$, then $a + c > b + c$ for all c.
   - Rule 3: If $a > b$ and $b > c$, then $a > c$.
   - Rule 4: If $a > b$ and $c > 0$, then $ac > bc$.
   - Rule 5: If $a > b$ and $c < 0$, then $ac < bc \rightarrow$ **Sign change:** Multiplying an inequality with a **negative** number **changes the sign** of the inequality.

2. Inequalities with multiplications and divisions on one side and a zero on the other side can be solved using a sign diagram.

   Example: Solve $\dfrac{x(x+2)}{x-2} \geq 0$.

   The solution is $-2 \leq x \leq 0$ or $x > 2$.

3. When solving an inequality: Do not multiply or divide both sides by an expression that contains the variable. We don't know the sign of such an expression, and we therefore don't known whether the sign of the inequality changes.

4. General form of a quadratic equation:

   $$ax^2 + bx + c = 0, \;\; a \neq 0.$$

   By dividing both sides by a, a quadratic equation can be rewritten in the form

   $$x^2 + px + q = 0, \;\; a \neq 0.$$

5. Special cases of quadratic equations that can easily be solved:

   - $c = 0$:

$$
\begin{aligned}
ax^2 + bx &= 0 \\
x^2 + \frac{b}{a}x &= 0 \\
x\left(x + \frac{b}{a}\right) &= 0 \\
x = 0 \text{ or } x + \frac{b}{a} &= 0 \\
x = 0 \text{ or } x &= -\frac{b}{a}
\end{aligned}
$$

   - $b = 0$:

$$
\begin{aligned}
ax^2 + c &= 0 \\
x^2 + \frac{c}{a} &= 0
\end{aligned}
$$

     If $\frac{c}{a} > 0$, there are no solutions.
     If $\frac{c}{a} = 0$, there is one solution: $x = 0$.
     If $\frac{c}{a} < 0$, the equation can be factorized:

$$
\begin{aligned}
\left(x + \frac{c}{a}\right)\left(x - \frac{c}{a}\right) &= 0 \\
x + \frac{c}{a} = 0 \text{ or } x - \frac{c}{a} &= 0 \\
x = -\frac{c}{a} = 0 \text{ or } x &= \frac{c}{a}
\end{aligned}
$$

   - If $a = 1$ and if we can find two numbers p and q that sum to b and have c as their product, the sum-product rule can be used:

$$
\begin{aligned}
x^2 + bx + c &= 0 \\
(x + p)(x + q) &= 0 \qquad \text{(Provided that } pq = b \text{ and } p + q = c\text{)} \\
x + p = 0 \text{ or } x + q &= 0 \\
x = -p \text{ or } x &= -q
\end{aligned}
$$

6. The quadratic formula is a general formula for the solution(s) of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

7. In the quadratic formula, $D = b^2 - 4ac$ is called the ‘discriminant’. If

   - $D > 0$, the equation has two solutions.
   - $D = 0$, the equation has one solution.
   - $D < 0$, the equation has no solutions.

8. If a quadratic equation has two solutions, it can be factorized. If the solutions of $ax^2+bx+c=0$ are $x=x_1$ or $x=x_2$, then

$$ax^2+bx+c=a(x-x_1)(x-x_2).$$

9. Two methods can be used to solve a system of linear equations:
   - The substitution method.
   - The elimination method.

10. A product is zero if one of its terms it zero, i.e., $f(x)g(x)=0$ if $f(x)=0$ or $g(x)=0$. This property can often be used as a first step to solve a nonlinear equation.