Summer Course Mathematics

Facts Monday

1. Definitions:

$$\begin{aligned} &a^0 = 1, \qquad a \neq 0. \\ &0^n = 0, \qquad n \in \mathbb{N}. \\ &0^0 \text{is not defined.} \end{aligned}$$

2. Definition: $a^{-n} = \dfrac{1}{a^n}$.

3. Rules:

$$\begin{aligned} a^r \cdot a^s &= a^{r+s} && \text{(bases are the same)} \\ \frac{a^r}{a^s} &= a^{r-s} && \text{(bases are the same)} \\ (a^r)^s &= a^{r \cdot s} \\ (ab)^r &= a^r b^r && \text{(exponents are the same)} \\ \left(\frac{a}{b}\right)^r &= \frac{a^r}{b^r} = a^r b^{-r} && \text{(exponents are the same)} \end{aligned}$$

4. Order of priorities: Brackets, Exponentiation, Division, Multiplication, Addition, Subtraction (BEDMAS).

   Division and multiplication have the same priority.

   Addition and subtraction have the same priority.

5. Important quadratic identities:

$$\begin{aligned} (a+b)^2 &= a^2 + 2ab + b^2 \\ (a-b)^2 &= a^2 - 2ab + b^2 \\ (a+b)(a-b) &= a^2 - b^2 \end{aligned}$$

   Remember that

$$(a+b)^2 \neq a^2 + b^2$$

6. Rules of algebra (most of them are obvious):
   - $a + b = b + a$
   - $(a + b) + c = a + (b + c)$
   - $a + 0 = a$
   - $a + (-a) = 0$
   - $ab = ba$
   - $(ab)c = a(bc)$
   - $1 \cdot a = a$
   - $aa^{-1} = 1$ for $a \neq 0$
   - $(-a)b = a(-b) = -ab$
   - $(-a)(-b) = ab$
   - $a(b + c) = ab + ac$
   - $(a + b)c = ac + bc$
7. Rules for fractions:
   - $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$.
   - $\frac{-a}{-b} = \frac{a}{b}$.
   - $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$.
   - $\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}$.
   - $\frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}$.
   - $a + \frac{b}{c} = \frac{a \cdot c + b}{c}$.
   - $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$.
   - $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$.
   - $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$.
8. The fifth rule for fractions is important. It states that fractions can only be summed after their denominators are made equal, this can be done using the least common denominator (LCD).

   Example: $\frac{1}{3x} + \frac{1}{2x^2}$. The LCD is $6x^2$. The sum equals

   $$\frac{1}{3x} + \frac{1}{2x^2} = \frac{2x}{6x^2} + \frac{3}{6x^2} = \frac{2x + 3}{6x^2}.$$
9. Square root:

   If $a > 0$ then $\sqrt{a}$ is the positive number b that multiplied by itself gives a ($b^2 = a$).

   If $a = 0$ then $\sqrt{a} = \sqrt{0} = 0$.

   If $a < 0$ then $\sqrt{a}$ is not defined.

10. Notation:

    $\sqrt{a}$ is the same as $a^{\frac{1}{2}}$.

11. Rules:

    $$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$
    $$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

12. Remember that the square root of a sum cannot be expanded:

    $$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}.$$