Summer Course Mathematics

Exercises Wednesday

Chapter 4: Functions of One Variable

§ 2 Definitions

1. Let $f(x) = x^2 + \frac{2}{x^2}$.
   a. Compute, if possible, $f(1),\ f(-1),\ f(0),\ f(-2),\ f(98),\ f(-\frac{1}{2}),\ f(a-2)$, and $2 + f(t^2-2)$.
   b. For what values of x is it true that $f(x) = f(-x)$?
   c. For what values of x is it true that $f(x) = -f(-x)$?
   d. For what values of x is it true that $f(x) = f(\frac{1}{x})$?
2. Suppose $F(x) = 5$ for all $x \geq 0$. Find $F(1),\ F(5),\ F(-1),\ F(5.1) - F(5)$
3. Find the domain of:
   a. $f(x) = x + \frac{1}{x}$ b. $g(x) = \sqrt{4-x}$ **c.** $h(x) = \sqrt{9-x^2}$

§ 3 Graphs of Functions

1. It is important to know the following functions by name and how their graphs look like. In order to sketch the graphs, calculate the outcome of the functions for some well chosen values of the independent variable x. Thereafter, locate these points in an xy plane and draw a smooth curve through the points.
   a. $y = f(x) = -2x + 6$ at $x = 0,\ 1,\ 2,\ -2$
   b. $y = 3x^2$ at $x = -2,\ -1,\ 0,\ 1,\ 2$
   c. $y = -2x^2$ at $x = -2,\ -1,\ 0,\ 1,\ 2$
   d. $y = x^3$ at $x = -2,\ -1,\ 0,\ 1,\ 2,\ 3$
   e. $y = \sqrt{x}$ at $x = -1,\ 0,\ 1,\ 2,\ 4,\ 9$
   f. $y = \dfrac{1}{x}$ at $x = -2,\ -1,\ 0,\ 1,\ 2,\ 3$

§ 4 Linear Functions

1. Find the slopes of the lines passing through the following pairs of points:
   a. $(2,4)$ and $(5,10)$ b. $(5,10)$ and $(3,18)$ **c.** $(2,4)$ and $(6,-12)$
   d. $(\frac{1}{2}, 2\frac{2}{3})$ and $(\frac{1}{3}, 2\frac{2}{3})$ e. $(2,7)$ and $(2,10)$
2. Draw graphs for the following straight lines:
   a. $2x - 3y = 6$ **b.** $\frac{x}{3} + \frac{y}{2} = 1$ c. $x = 4$ **d.** $y = 2$
3. Determine the most simple equations for the following straight lines: a. L_1 passes through $(2,2)$ and has a slope of 4. b. L_2 passes through $(-1,1)$ and $(1,-3)$. **c.** L_3 passes through the origin and is parallel to $2x + 4y = 6$. **d.** L_4 passes through the point on the x-axis with $x = 4$ and has a slope of -2.
4. Sketch in the xy-plane the set of all pairs of numbers (x, y) that satisfy the following inequalities:
   a. $2x + y \geq 0$ **b.** $x > 0$ and $y > 0$ and $2x + 3y \leq 12$

§ 6 Quadratic Functions

1. Find the solutions to the following equations if there are any, where p is a positive parameter:
   a. $f(x) = x^2 + 2px - p^2$. Solve $f(x) = 0$.
   b. $g(x) = x^2 + 4p + 4$. Solve (i) $g(x) = 0$, (ii) $g(x) = 4$ and (iii) $g(x) = 8$.
   c. $h(x) = 3x^2 + 6x + 3p$. Solve (i) $h(x) = 0$ and (ii) $h(x) = 3p$, sketch this situation.

2. ***a.*** The sum of two positive numbers is 12 and their product is 35. Use quadratics to determine these two numbers.
b. The perimeter of a rectangle is 22 and the area is 30. Use quadratics to determine the size of the rectangle.
c. The diagonal of rectangle is 13 and one of the sides 5. What is the length of the other side?

§ 8 Power Functions

1. Determine the domain and sketch the graph of the following functions:
a. $y = f(x) = x^{-2}$ b. $y = g(x) = x^{-1}$ c. $y = h(x) = x^{\frac{1}{2}}$ ***d.*** $y = k(x) = x^{-\frac{1}{2}}$
2. Solve the following equations for x (without using a calculator):
a. $2^{3x} = 8$ b. $5^{2x+1} = \frac{1}{125}$ ***c.*** $3^{x-1} = 27$ ***d.*** $2^{(x^2)} = 16$
3. Solve the following equations for the unknown (without using a calculator):
a. $9 \cdot 3^{x-1} = 27$ ***b.*** $4^{(x^2)} = \sqrt{2}$ ***c.*** $2^t \cdot 3 \cdot 4^{t+1} = 12$

§ 9 Exponential Functions

1. a. Make a table of $y_1 = 3^x$ and $y_2 = 3^{-x}$ for $x = -3,\ -1,\ 0,\ 1,\ 2,\ 3$.
b. Sketch the graphs of the functions y_1 and y_2 in the same xy plane.
c. The value of the important number e is between 2 and 3: $\mathrm{e} \approx 2.718\ldots$. Sketch the graph of the function $y = \mathrm{e}^x$ in the same figure.

§ 10 Logarithmic Functions

1. You should have some idea about the graph of the log function. Sketch the graphs of $y = \log_2 x$, $y = \ln x$, and $y = \log_3 x$ using the values of y for $x = 1$ and the values of x for $y = 1$.

2. Express the following numbers in terms of $\ln 2$:
a. $\ln 8$ ***b.*** $\ln \frac{1}{16}$ ***c.*** $\ln \sqrt[4]{8}$ d. $\ln(\sqrt{25} - 1)$
3. Solve the following equations for x:
a. $2^x = 9$
b. $\ln x = 3$
c. $\ln(x^2 - 3) = 0$, what is the domain of this expression?
d. $\ln(3 - x^2) = 1$, what is the domain of this expression?
e. $\ln(x^2 + 4x + \mathrm{e}^2) = 2$, be careful!
f. $\log_3(2x + 3) = 20$
g. $1500(1.045)^x = 1600$
h. Write as one ln: $\ln(x + 1) + \ln(x - 1) - 2\ln x$

Continue with the following problems in the book:

- Chapter 4, § 2: Problems 3, 7, 8.
- Chapter 4, § 3: Problem 1.
- Chapter 4, § 4: Problem 6.
- Chapter 4, § 6: Problems 2, 4.
- Chapter 4, § 8: Problem 3.
- Chapter 4, § 9: Problems 1, 4, 6.
- Chapter 4, § 10: Problem 3 (a), (b) and (c), 6.