Summer Course Mathematics

Exercises Thursday

Chapter 5: Properties of Functions

§ 1 Shifting Graphs

1. Use the graphs of $y = x,\ y = x^2,\ y = \sqrt{x},$ and $y = \frac{1}{x}$ and the rules for shifting graphs to sketch the graphs of the following functions:
   a. $y = x - 2$ b. $y = x^2 + 3$ c. $y = 2 + \sqrt{x - 2}$ **d.** $y = 1 + \frac{1}{x-1}$
2. The graph of the function $y = f(x) = -(x + 2)^2(x - 1)$ is shown several times on the next page. Use these graphs to sketch the graphs of the following functions:
   a. $y = f(x + 2)$ b. $y = f(x) - 1$ **c.** $y = -f(x)$ **d.** $y = f(-x)$

§ 2 New Functions from Old

1. Sketch the graphs of the following functions:
   a. $y = x + \frac{1}{x}$ b. $y = x + \mathrm{e}^{-x}$ **c.** $y = x + \frac{1}{x^2}$
2. If $f(x) = x^2 + x$ and $g(x) = x^2 - x$, compute:
   a. $(f + g)(x)$ **b.** $(f - g)(x)$ c. $(fg)(x)$ **d.** $(f/g)(x)$, do not forget to simplify
   e. $f(g(1))$ **f.** $g(f(1))$ **g.** $f(g(x))$ h. $g(f(x))$

Chapter 6: Differentiation

§ 2 Tangents and Derivatives

1. The derivative of the function $f(x) = x^2$ equals $f'(x) = 2x$. Use this to determine the equation of the tangent line to the graph of the function $f(x) = x^2$ at the point $(3, 9)$.

2. Consider the function $f(x) = 3x^2$.
a. Determine $f(4 + h) - f(4)$.
b. Use the result to determine $\dfrac{f(4 + h) - f(4)}{h}$.
c. Use the result to determine $f'(4)$.
d. Translate the outcome into words.

§ 3 Increasing and Decreasing Functions

1. The function f is defined as $f(x) = 3x^2 - 2x + 8$.
a. Determine the derivative $f'(x)$ of this function.
b. Use this derivative to determine where the function $f(x)$ is increasing/decreasing.

2. The function f is defined as $f(x) = 1\frac{2}{3}x^3 - 3x^2 + x - 8$.
a. Determine the derivative $f'(x)$ of this function.
b. Use this derivative to determine where the function $f(x)$ is increasing/decreasing.

§ 6 Simple Rules for Differentiation

1. Determine the derivatives $\frac{dy}{dx}$ of the following functions y of x:
   a. $y = 0$ b. $y = x^2$ c. $y = 3x^4$ **d.** $y = 4^2$ **e.** $y = x^2 \cdot 3 \cdot x^5$
2. Determine the derivatives of the following functions. The function $g(x)$ is an arbitrary function that has not been specified yet. Its derivative can be denoted by $g'(x)$.
   a. $f(x) = 3g(x) + 5$ b. $f(x) = ag(x) + b$ c. $f(x) = -\frac{1}{2}g(x) + 2x$
   d. $f(x) = \dfrac{4(g(x) + 12)}{3}$ **e.** $f(x) = ag(x) + bx^p,\ p \neq 0$
3. Determine the derivatives $\frac{dy}{dx}$ of the following functions y of x:
   a. $y = x^5$ **b.** $y = 2x^{10}$ c. $y = x^5 \cdot 7$ d. $y = x^{-2}$
   e. $y = \dfrac{5}{x^3}$ f. $y = \dfrac{5x^6}{12}$ g. $y = -\dfrac{-2}{x^3 \times 5}$ **h.** $y = \dfrac{x^2}{x\sqrt{x}}$
4. Compute the following:
   a. $\dfrac{d}{dr}(2\pi r)$ **b.** $\dfrac{d}{dy}(y\sqrt{4y})$ c. $\dfrac{d}{dt}(5t^2)$

§ 7 Sums, Products, and Quotients

1. Differentiate the following functions:
   a. $f(x) = (3x^2 - 1)(x^4 - 2)$ b. $g(x) = (x^4 + 4)\left(\dfrac{4}{x} + x^4\right)$

2. Differentiate the following functions:
   a. $f(x) = \dfrac{x - 1}{x + 1}$ b. $g(x) = \dfrac{2 + x}{x^6}$ **c.** $h(x) = \dfrac{4x - 7}{3x + 2}$

§ 8 The Chain Rule

1. The function $f(x) = (1 - x^2)^3$ is given.
   a. Differentiate this function using the chain rule. Start with defining a function u of x.
   b. Differentiate this function without using the chain rule. Verify that the derivatives are equivalent.

Continue with the following problems in the book:

- Chapter 5, § 1: Problem 5.
- Chapter 5, § 2: Problem 1.
- Chapter 6, § 1: Problems 1, 2.
- Chapter 6, § 7: Problems 1, 6.