Summer Course Mathematics

Exercises Monday

Chapter 2: Algebra

§ 2   Integer Powers

1. Compute:
   a. 10^2 b. $(-0.4)^2$ c. 5^{-2}

2. Write as powers of 3:
   a. 9   b. 81   c. $\frac{1}{3}$ **d.** $\frac{1}{27}$ **e.** 1

3. Write as powers:
   a. $12 \cdot 12 \cdot 12 \cdot 12$ **b.** $-4 \cdot -4 \cdot -4$ **c.** $\frac{1}{100}$ d. 1
   e. 0.0001   f. $a \cdot a \cdot a \cdot a$ **g.** $(a+b)(a+b)$ h. $(-a)(-a)(-a)(-a)$

4. Expand and simplify:
   a. $\dfrac{p^{12} \cdot p^5}{p^2 \cdot p^4}$ b. $\dfrac{a^2 \cdot b^{-2}}{(a^2 \cdot b^{-1})^2}$ c. $\dfrac{3^5 \cdot (3^2)^3}{(-3)^7 \cdot 3^4}$

5. Expand and simplify:
   a. $3^2 \cdot 3^0 \cdot 3^{-1}$ b. $\left(\frac{5}{4}\right)^2$ **c.** $\left(\frac{5}{3}\right)^{-2}$ **d.** $\dfrac{10^2 \cdot 10^{-3}}{10^{-3} \cdot 10^4}$

6. Give economic interpretations in each of the following expressions and use a calculator to find the approximate values:
   a. $100 \cdot (1.09)^8$ b. $100,000 \cdot (1.03)^{30}$ c. $5000 \cdot (1.07)^{-6}$

7. \$12,000 is deposited in an account earning 4% per year. What is the amount after 15 years and after 16 years? What is the amount after 29 and 30 years?
   Compare the increase from the 15th to the 16th year with the increase from the 29th to the 30th year.

§ 3   Rules of Algebra

1. Expand and simplify:
   a. $2 - (-2) + (-5)$ b. $-3(2-3)$ **c.** $(-3)(2 + (-3))$ d. $(2x+3) \cdot 4$
   e. $(-2t+4) \cdot (-2)$ **f.** $2(-1 - 2x) \cdot \frac{-1}{x}$ g. $0 \cdot \frac{1}{x}$

2. Expand:
   a. $(2x + 3y)^2$ b. $\left(-\frac{1}{a} + a\right)^2$ c. $(2x + \frac{1}{x})(2x - \frac{1}{x})$

3. Solve:
   a. $102^2 - 98^2$, don't use a calculator   b. If $x^2 - 2x + 1 = 1$, then $x = \ldots$

4. Factorize:
   a. $t^2 + 6t + 9$ b. $3t^2 s - 12ts^2$ c. $2x^3 y - 8xy^3$ **d.** $8st^2 - 24st + 18s$

§ 4 Fractions

1. Simplify:
   a. $\dfrac{2}{5} - \dfrac{4}{5} + \dfrac{3}{5}$
   b. $2 + \dfrac{1}{7} - \dfrac{6}{7}$
   c. $\dfrac{6}{9} + \dfrac{1}{6}$
   d. $\dfrac{11}{3} - \dfrac{23}{6}$
   e. $\dfrac{2}{9} \cdot \dfrac{3}{4}$
   f. $\left(\dfrac{4}{7} \div \dfrac{8}{21}\right) \cdot \dfrac{2}{3}$
   g. $\dfrac{\frac{1}{2} - \frac{1}{3}}{\frac{2}{3} + \frac{1}{4}}$

2. Simplify:
   a. $\dfrac{4a}{5} - \dfrac{a}{5} + \dfrac{2a}{5}$
   b. $\dfrac{7x}{14} - \dfrac{x}{2} + \dfrac{2x}{7}$
   c. $\dfrac{a}{15} - \dfrac{2a-3}{5} + \dfrac{7a}{105}$
   d. $\dfrac{x+2}{4} - \dfrac{2x-1}{5}$

3. Cancel common factors:
   a. $\frac{126}{429}$
   b. $\dfrac{a^2 - 4b^2}{3a + 6b}$

4. Simplify:
   a. $\dfrac{2}{t+3} - \dfrac{2}{t-3}$
   b. $\dfrac{t}{t+1} - \dfrac{2}{t-1}$
   c. $\dfrac{t-1}{t+1} - \dfrac{t+1}{t-1}$
   d. $\dfrac{\frac{3}{x} - \frac{1}{x}}{\frac{1}{x+1} + \frac{1}{x-1}}$

5. Simplify:
   a. $\left(\dfrac{1}{7} - \dfrac{1}{6}\right)^{-2}$
   b. $1 - \dfrac{1}{1 - \frac{1}{x}} \cdot \dfrac{\frac{4x^2}{x+1}}{\frac{2x}{x^2-1}}$

§ 5 Fractional Powers

1. Solve for the unknown variable. Determine first the domain of the expression:
   a. $\sqrt{x} = 4$
   b. $2\sqrt{t} = \sqrt{25}$
   c. $\sqrt{3-t} = 2$
   d. $\sqrt{t+4} = -2$, be careful!

2. Rationalize the denominator:
   a. $\dfrac{3}{\sqrt{2}}$
   b. $\dfrac{18}{\sqrt{27}}$
   c. $\dfrac{\sqrt{5}}{2\sqrt{3}}$
   d. $\dfrac{\sqrt{90} - \sqrt{40}}{\sqrt{20}}$

3. Rationalize the denominator:
   a. $\dfrac{2}{\sqrt{7} - 2}$
   b. $\dfrac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}$
   c. $\dfrac{\sqrt{3} + 6}{-3 + \sqrt{5}}$

4. Compute without using a calculator:
   a. $\sqrt[3]{27}$
   b. $(32)^{\frac{1}{5}}$
   c. $-625^{\frac{1}{4}}$
   d. $8^{-\frac{4}{3}}$
   e. $125^{\frac{2}{3}}$

5. Simplify:
   a. $\sqrt[3]{216 x^{6p} y^{12p}}$
   b. $\dfrac{\sqrt{12x^3} \cdot \sqrt[3]{2x}}{\sqrt[6]{3x^4}}$

Continue with the following problems in the book:

- Chapter 2, § 1: Problem 1.

- Chapter 2, § 2: Problems 4, 9, 11.

- Chapter 2, § 3: Problems 2, 3, 7, 9, 10, 11.

- Chapter 2, § 4: Problem 4.

- Chapter 2, § 5: Problems 1, 2, 6, 7, 10, 12, 13.