

Sample Entrance Exam Mathematics

Duration: 3 hours

Note: This test exam does not contain all subjects that have to be studied. Furthermore, the actual exam might contain subjects that are not covered by this test exam.

For entrance to studies Economics or Business: Solve the problems 1-5. The weights of the respective problems are 26, 23, 15, 6, and 20 points. The total number of points is 90. The grade equals the number of points earned divided by 10 plus 1, rounded.

For entrance to the study Econometrics and Operations Research: Solve the problems 1-6. The total number of points is 115. The grade equals the number of points earned multiplied by $\frac{9}{115}$ plus 1, rounded. At least 13 points for problem 6 and a sufficient grade for the entire exam are required to pass the exam.

It is not allowed to use a sheet with formulas, a graphical calculator, a symbolic calculator or a calculator with an alpha-numeric keyboard. A simple calculator is allowed and is recommended.

A good or wrong answer is only a small part of the solution. The quality and completeness of your detailed solutions determine the points you will get. You should end a solution with a conclusion or an answer.

If something is not clear or when you have the idea that a problem contains a mistake, please ask! It is quite possible that in your book a subject has a different name than used in this exam.

1. Basics, I.

- a. (3) Solve $7(x - 3) - 2 = -3(x + 1)$ for x .
- b. (4) Given is the inequality $-1(x - 2) < 2x - 4$.
 - (i) Check whether $x = 0$ solves the inequality.
 - (ii) Solve the inequality for x .
- c. (4) Solve $x^2 - 3x + 2 = 0$ for x .
- d. (5) Solve the following system of equations and check your answer.
$$\begin{cases} 2x + 3y = 7 \\ 3x - 2y = 4 \end{cases}$$
- e. (4) Factorize:
 - (i) $4x^3 - 100x$.
 - (ii) $x^2 + 5x + 6$.
- f. (3) Solve without using your calculator:
 - (i) $16x^2 = 64$.
 - (ii) $27^{2x-2} = 81^x$.
 - (iii) $\ln(x) + \ln(2x) = \ln(8)$, $x > 0$.
- g. (3) Solve and use your calculator to approximate in two decimals:
 - (i) $x^{-5} = 10$.
 - (ii) $7^{3x+1} = 98$.
 - (iii) $\log_3 x = 5.5$.

2. Basics, II.

- a. (4) Solve for x :
$$\frac{2}{x+3} + \frac{7}{x+2} = -1.$$
- b. (4) Determine the domain and solve for x :
$$(x-2)\sqrt{x-1} = 0.$$
- c. (4) Show that:
$$\ln\left(\frac{1}{3}x^{-2}\right) = -\ln 3 - 2\ln x \text{ for } x > 0.$$
- d. (5) The graph of a linear function passes through the points (9,9) and (14,19). Give the equation of this linear function.
- e. (6) Solve for x :
$$\frac{x-4}{x-1} \geq 2.$$

3. Differentiation and shifting graphs.

- a. (4) Determine the derivative $f'(x)$ of the function $f(x)$ below.
$$f(x) = 5 + \sqrt{x} + x^4, \quad x \geq 0.$$

Is f increasing or decreasing at $x = 1$?
- b. (4) Determine the derivative $g'(x)$ of the function $g(x)$ below. Do not simplify the derivative.
$$g(x) = (x^3 + 2x + 1)e^{2x+1}, \quad x \in \mathbb{R}.$$
- c. (4) Determine the derivative $h'(x)$ of the function $h(x)$ below. Do not simplify the derivative.
$$y = h(x) = (x^4 + 4x^2 + 1)^3, \quad x \in \mathbb{R}.$$

Is h increasing or decreasing at $x = 0$?
- d. (3) Explain in words how you can obtain the graph of the function $y = 2 + 3(x+3)^2$ from the graph of the function $y = x^2$. You do not have to sketch the graphs.

4. Growth processes.

Your deposit in a bank at this moment is 15,000 euro.

- a. (2) The interest rate is 2.4% per year. How much should you have deposited in a bank 10 years ago to have this deposit now?
- b. (2) The interest rate changes to 2.55% per year. Calculate the value of your deposit 5 years from now.
- c. (2) You buy a device for 25,500 euro. It depreciates 12% per year. What is its value 5 years later?

5. Extremes.

Given is the function $y = f(x) = x(x-3)(x+3)$, $x \in \mathbb{R}$.

- a. (4) Make a sign diagram of the function f and use it to explain in which interval(s) you expect at least one maximum or one minimum.
- b. (2) Show that:
$$x(x-3)(x+3) = x^3 - 9x$$
- c. (4) Determine the derivative $f'(x)$ of the function $f(x)$.
Determine where $f'(x) = 0$ (use your calculator for it, 1 decimal is sufficient).
- d. (4) Where is the function f increasing and where is it decreasing?
- e. (3) Use d. to classify the extremes (maximum or minimum).
- f. (3) Sketch the graph of f .

6. For aspirant students Econometrics and Operations Research only!

- a. (2) Determine using your calculator:
(i) $\sin(212^\circ)$.
(ii) $\cos(\frac{1}{3}\pi)$.
- b. (4) Explain why the equation $5 - 3 \sin(t - 2) = 0$ does not have a solution.
Determine at least one solution of the equation $5 - 6 \sin(t - 2) = 0$.
- c. (3) Determine the domain of $-2\sqrt{x+2} = 1 - x$ and solve the equation for x .
- d. (4) Solve for x :
$$\frac{(x+2)\ln(x-1)}{x^2-1} = 0.$$
- e. (4) Given is the function $y = f(x) = xe^x$. Determine where the function is convex/concave. Use this to classify the stationary point ($f'(x) = 0$).
- f. (4) Determine:
(i) $\lim_{x \rightarrow 1} \frac{x-1}{x^2-1}$.
(ii) $\lim_{x \rightarrow -\infty} \frac{x|x|-2}{x^2+2}$.
- g. (4) Determine:
(i) $\int (6x^2 + 5) dx$.
(ii) $\int_0^2 (6x^2 + \sqrt{x}) dx$.
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